TRANSMISSION LINES OF FINITE LENGTH

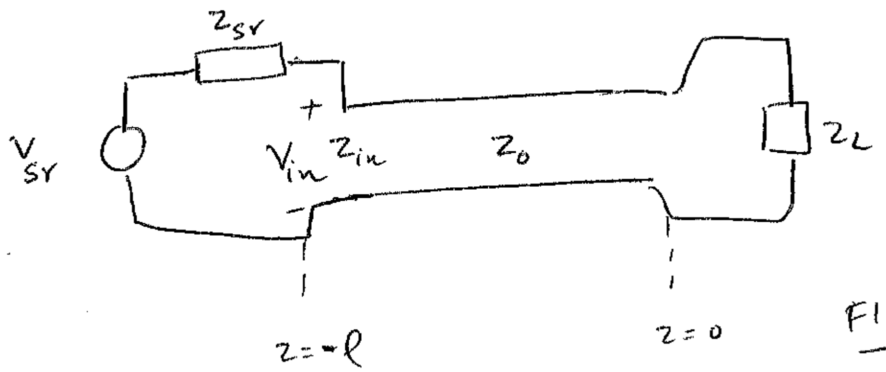


FIG. 1

CONSIDER A LOSSLESS TRANSMISSION LINE OF LENGTH l & IT HAS A CHARACTERISTIC IMPEDANCE Z_0. THE LOAD IS LOCATED AT $z=0$ & THE TR. LINE IS LOCATED ALONG THE −VE z AXIS. THE PHASOR VOLTAGE & CURRENT ARE

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \quad \rightarrow (1)$$

$$I_s(z) = \frac{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}}{Z_0} \quad \rightarrow (2)$$

(∵ FOR THE LOSSLESS CASE, $\alpha = 0$; $\gamma = \alpha + j\beta = j\beta$)

THE WAVE IMPEDANCE IS DEFINED AS

$$Z_w(z) \equiv \frac{V_s(z)}{I_s(z)} = \left(\frac{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}} \right) Z_0 \quad \rightarrow (3)$$

NOTE : Z_W IS DIFFERENT FROM Z_0. IF THERE IS ONLY A FORWARD PROPAGATING WAVE, $Z_W = Z_0$. IF THERE IS ONLY A BACKWARD PROP. WAVE, $Z_W = -Z_0$.

$$\cancel{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\Gamma \equiv \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \longrightarrow (4)$$

$$\therefore\ Z_W(z) = Z_0 \frac{\cancel{V_0^+}\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)}{\cancel{V_0^+}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)} \longrightarrow (5)$$

CONSIDER THE NUMERATOR OF (5),

$$N = Z_0\left(e^{-j\beta z} + \frac{Z_L - Z_0}{Z_L + Z_0} e^{j\beta z}\right) = \frac{Z_0\left[(Z_L + Z_0)e^{-j\beta z} + (Z_L - Z_0)e^{j\beta z}\right]}{Z_L + Z_0}$$

$$= \frac{Z_0}{Z_L + Z_0}\, Z_L\left(e^{-j\beta z} + e^{j\beta z}\right) - Z_0\left(e^{j\beta z} - e^{-j\beta z}\right)$$

Euler identities,

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad ; \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$N = \frac{2Z_0}{Z_L + Z_0}\left[Z_L \cos(\beta z) - j Z_0 \sin(\beta z)\right]$$

Illy, THE DENOMINATOR OF (5) IS

$$D = \frac{2}{Z_L + Z_0}\left[Z_0 \cos(\beta z) + j Z_L \sin(\beta z)\right]$$

$$\therefore \quad Z_w(z) = Z_0\left[\frac{Z_L \cos(\beta z) - j Z_0 \sin(\beta z)}{Z_0 \cos(\beta z) + j Z_L \sin(\beta z)}\right]$$

At $z = -\ell$, THE WAVE IMPEDANCE AT THE LINE INPUT, Z_{in} is

$$\boxed{Z_{in} = Z_w(-\ell) = Z_0\left[\frac{Z_L \cos(\beta \ell) + j Z_0 \sin(\beta \ell)}{Z_0 \cos(\beta \ell) + j Z_L \sin(\beta \ell)}\right]} \rightarrow (6)$$

$$V_s(z=-\ell) = V_{in}\,; \qquad I_s(z=-\ell) = I_{in}\,.$$

THE EQUIVENT CIRCUIT OF FIG. 1 ~~(AS BE WRITTEN AS)~~ IS

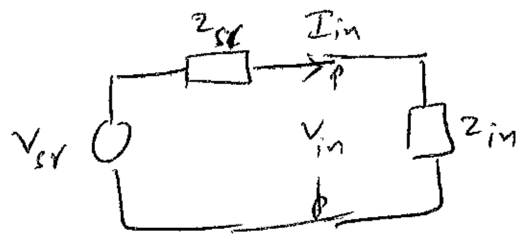


FIG 2

CASE (i): ~~[illegible]~~ ℓ is HALF-WAVELENGTH (or ITS INTEGER MULTIPLE ~~HALF-WAVE~~)

$$\ell = \frac{m\lambda}{2}, \quad m = 1, 2, \ldots \rightarrow (7)$$

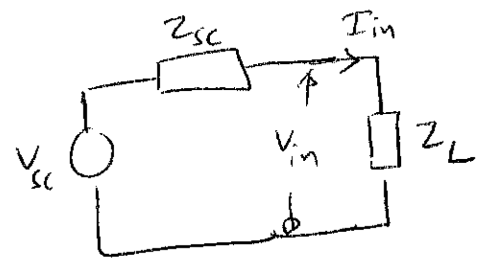
$$\beta \ell = \frac{2\pi}{\lambda} \cdot \frac{m\lambda}{2} = m\pi$$

$$\cos(m\pi) = \pm 1\,; \quad \sin(m\pi) = 0$$

$$Z_{in} = Z_0 \frac{Z_L \cos(m\pi)}{Z_0 \cdot \cos(m\pi)}$$

$$\boxed{Z_{in} = Z_L} \quad \text{WHEN} \quad l = m\lambda/2 \longrightarrow (8)$$

THE EQUIVALENT CIRCUIT OF FIG. 1 IS



FOR A HALF-WAVE LINE (i.e. $l = m\lambda/2$), ~~LINE~~ LOAD IMPEDANCE IS PLACED DIRECTLY ACROSS THE INPUT TERMINALS & THE LINE IS COMPLETELY REMOVED.

NOTE: THIS SIMPLIFICATION IS VALID AT THESE WAVELENGTHS ($m\lambda$) ONLY. IF THE FREQUENCY (EQUIVALENTLY, WAVELENGTH) VARIES CONDITION (7) IS NO LONGER SATISFIED & EQ. (8) IS NOT VALID

CASE (ii): <u>l IS QUARTER-WAVELENGTH (OR ITS ODD MULTIPLE)</u>:

$$l = (2m+1)\frac{\lambda}{4} \qquad m = 0, 1, 2, \cdots$$

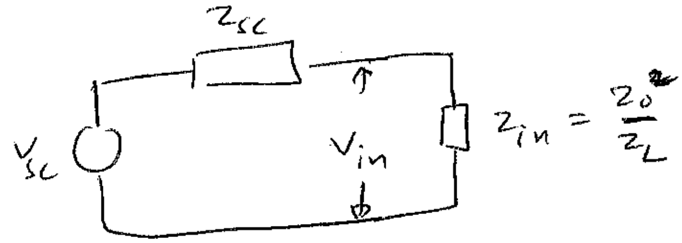
$$\beta = \frac{2\pi}{\lambda}$$

$$\beta l = \frac{2\pi}{\cancel{\lambda}} \cdot (2m+1)\frac{\cancel{\lambda}}{\cancel{4}\,2} = \left(\frac{2m+1}{2}\right)\pi$$

~~$\cos(\beta l) =$~~

$$\cos\left((2m+1)\frac{\pi}{2}\right) = 0 \qquad \sin\left((2m+1)\frac{\pi}{2}\right) = 1$$

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}} \quad \text{WHEN} \quad l = (2m+1)\frac{\lambda}{4} \longrightarrow (9)$$



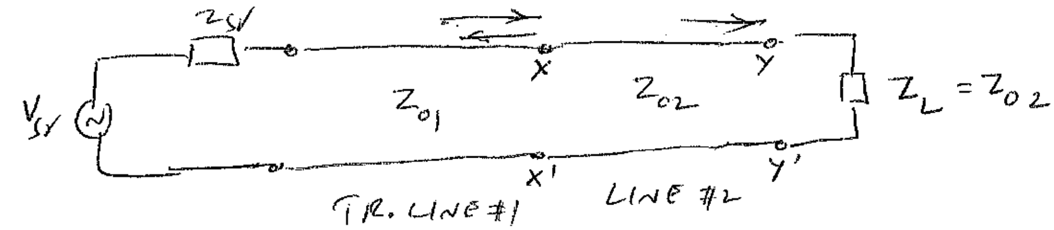
~~IN THIS CASE, THE IMPEDANCE AT THE LINE INPUT IS~~

~~THE GEOMETRIC MEAN OF~~

IN THIS CASE, Z_0 IS THE GEOMETRIC MEAN OF Z_{in} & Z_L, i.e.

$$Z_0 = \sqrt{Z_{in} Z_L} \longrightarrow (10)$$

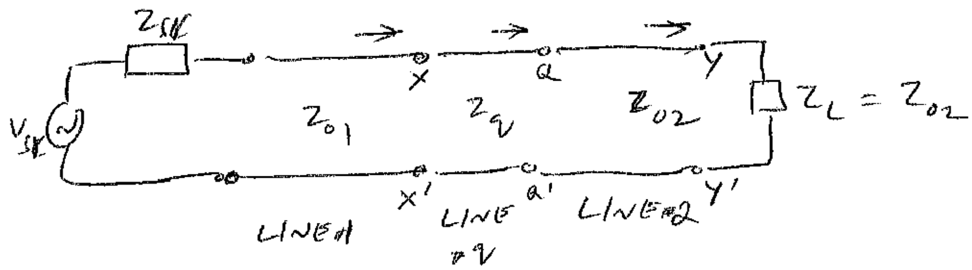
<u>APPLICATION OF QUARTER-WAVELENGTH LINE</u>: SUPPOSE YOU NEED TO CONNECT TWO DISSIMILAR TRANSMISSION LINES OF CHARACTERISTIC IMPEDANCES Z_{01} & Z_{02}. IF THEY ARE CONNECTED DIRECTLY AS SHOWN BELOW,



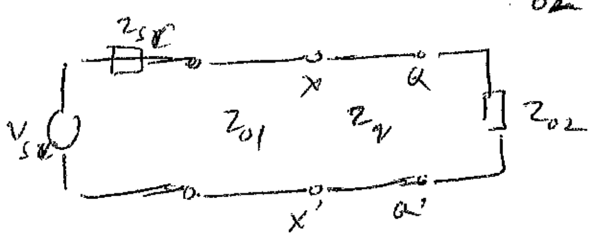
THERE WOULD BE REFLECTION AT X IF $Z_{01} \neq Z_{02}$.

THERE IS NO REFLECTION AT Y SINCE LINE #2 IS MATCHED TO LOAD ($Z_{02} = Z_L$). BECAUSE OF THE REFLECTION AT X, THE POWER TRANSFER TO THE LOAD IS NOT EFFICIENT.

TO IMPROVE THE EFFICIENCY, A QUARTER WAVELENGTH TRANSMISSION LINE IS SANDWICHED BETWEEN LINE #1 & LINE #2. THE CHARACTERESTIC IMPEDANCE, Z_q OF THIS MIDDLE LINE NEEDS TO BE CALCULATED.

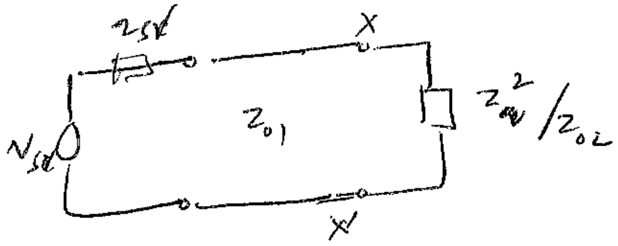


SINCE THE LINE #2 IS MATCHED TO LOAD, THE EFFECTIVE IMPEDANCE BETWEEN Q & Q' IS Z_{02}.



SINCE LINE q IS A $\lambda/4$ line, THE INPUT IMPEDANCE ACROSS X & X' is (SEE (9))

$$Z_{in} = \frac{Z_q^2}{Z_{02}}$$

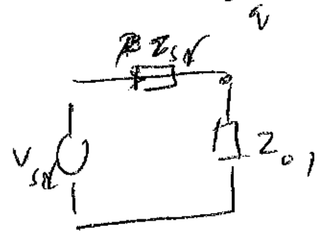


TO AVOID REFLECTION AT X (OR X'), THE EFFECTIVE LOAD IMPEDANCE Z_q^2/Z_{02} SHOULD BE EQUAL TO THE CHARACTERISTIC IMPEDANCE Z_{01},

$$Z_{01} = Z_q^2/Z_{02}$$

or

$$Z_q = \sqrt{Z_{01}Z_{02}}$$



SO, THE CHARACTERISTIC IMPEDANCE OF THE ~~MIDDLE~~ ~~L~~ LINE q SHOULD BE THE GEOMETRIC MEAN OF Z_{01} & Z_{02}. THIS TECHNIQUE IS CALLED <u>QUARTER-WAVE MATCHING</u>. AGAIN IT IS LIMITED TO THE FREQUENCY (OR NARROW BAND OF FREQUENCIES) SUCH THAT $(2n+1)\lambda/4 = l$.