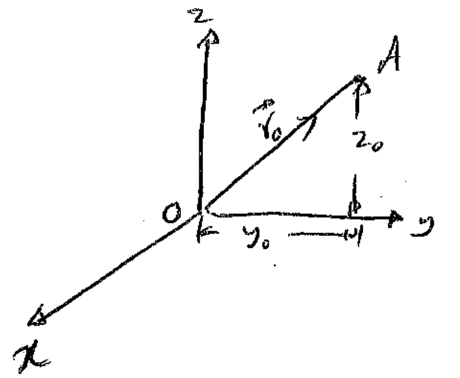
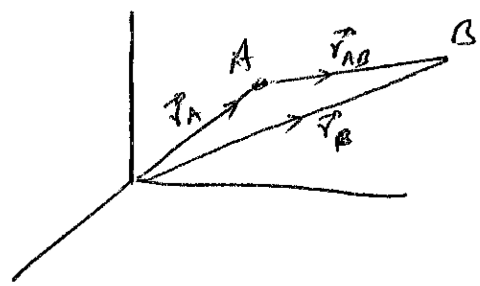
REVIEW OF VECTOR CALCULUS:

VECTOR COMPONENTS:



$$\vec{r}_0 = x_0 \vec{x} + y_0 \vec{y} + z_0 \vec{z} \quad \rightarrow (1)$$



FROM THE VECTOR ADDITION RULE, WE HAVE

$$\vec{r}_A + \vec{r}_{AB} = \vec{r}_B \quad \rightarrow (2)$$

or

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A .$$

FOR EXAMPLE, IF

$$\vec{r}_A = 2\vec{x} + 3\vec{y} + 4\vec{z} \quad \rightarrow (3)$$

$$\vec{r}_B = 4\vec{x} + 6\vec{y} + 5\vec{z} \quad \rightarrow (4)$$

$$\vec{r}_{AB} = (4-2)\vec{x} + (6-3)\vec{y} + (5-4)\vec{z}$$

$$= 2\vec{x} + 3\vec{y} + 1\vec{z} \quad \rightarrow (5)$$

HERE $\vec{r}_0$ or $\vec{r}_{AB}$ ARE EXAMPLES FOR DISPLACEMENT-TYPE VECTORS. FOR VECTORS SUCH AS ELECTRIC/MAGNETIC FIELD INTENSITY ($\vec{E}$ or $\vec{H}$), THE VECTOR HAS COMPONENTS IN $\vec{x}$, $\vec{y}$ & $\vec{z}$ DIRECTIONS, i.e.

$$\vec{E} = E_x \vec{x} + E_y \vec{y} + E_z \vec{z}. \longrightarrow (6)$$

THE MAGNITUDE OF A VECTOR:

FOR A DISPLACEMENT-TYPE VECTOR, r_0 (SEE EQ. 1)

$$|r_0| = \sqrt{x_0^2 + y_0^2 + z_0^2} \longrightarrow (7)$$

HERE, $|r_0|$ IS THE LENGTH OF THE SEGMENT JOINING O AND A. IT IS ALSO CALLED THE MAGNITUDE OF THE VECTOR.

$$\vec{r}_{AB} = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14} \text{ IS THE LENGTH}$$

(SEE EQ. (5))

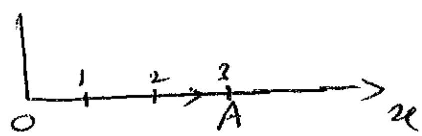
OF THE SEGMENT $\vec{AB}$.

THE MAGNITUDE OF $\vec{E}$ IS

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2}. \longrightarrow (8)$$

~~UNIT~~ UNIT VECTOR:

CONSIDER A VECTOR ~~AB~~ IN THE DIRECTION OF x-AXIS.



LET THE LENGTH OF THE VECTOR BE 3 UNITS.

$$\vec{r}_{0A} = 3\vec{x}$$

HERE, $\vec{x}$ IS THE UNIT VECTOR IN x-DIRECTION.

THE MAGNITUDE OF $\vec{r}_{OA}$ IS

$$|\vec{r}_{OA}| = \sqrt{3^2+0^2+0^2} = 3$$

IF WE DIVIDE $\vec{r}_{OA}$ BY $|\vec{r}_{OA}|$, WE GET THE UNIT VECTOR THAT IS IN THE SAME DIRECTION AS $\vec{r}_{OA}$, i.e.

$$\frac{\vec{r}_{OA}}{|\vec{r}_{OA}|} = \frac{3\vec{x}}{3} = \vec{x} \quad \rightarrow (9)$$

EQ.(9) HOLDS FOR ANY VECTOR POINTING IN ANY DIRECTION. ~~EQ~~ THE UNIT VECTOR IN THE DIRECTION OF ~~AB CORRESPONDING TO~~ ~~AB~~ $\vec{r}_{AB}$ (SEE EQ.(5)) IS

$$\vec{a}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{2\vec{x}+3\vec{y}+1\vec{z}}{\sqrt{14}} = \left(\frac{2}{\sqrt{14}}\right)\vec{x} + \left(\frac{3}{\sqrt{14}}\right)\vec{y} + \left(\frac{1}{\sqrt{14}}\right)\vec{z} \quad \rightarrow (10)$$

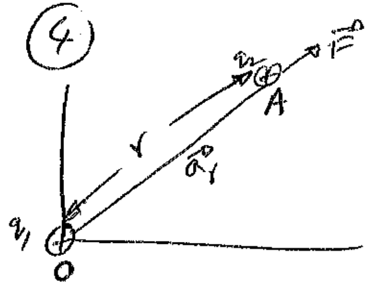
FOR THE ELECTRIC FIELD INTENSITY, $\vec{E}$ (EQ.(6)), THE UNIT VECTOR IS

$$\vec{a}_E = \frac{\vec{E}}{|\vec{E}|} = \frac{E_x\vec{x} + E_y\vec{y} + E_z\vec{z}}{\sqrt{E_x^2+E_y^2+E_z^2}} \quad \rightarrow (11)$$

(4)

q_1 IS AT ORIGIN.

q_2 IS AT A, AT DISTANCE r FROM q_1.



THE FORCE OF ATTRACTION OR REPULSION BETWEEN CHARGES IS GIVEN BY COULOMB'S LAW,

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon r^2} \cdot \vec{a_r} \longrightarrow (12)$$

WHERE $\vec{a_r}$ IS THE UNIT VECTOR IN THE DIRECTION OF $\vec{OA}$.

LET $\vec{OA} = \vec{r}$.

$$|\vec{OA}| = |\vec{r}| = r$$

$$\vec{a_r} = \frac{\vec{OA}}{|\vec{OA}|} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r} \longrightarrow (13)$$

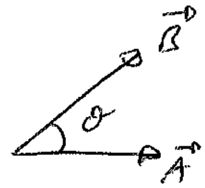
$$\therefore \quad \vec{F} = \frac{q_1 q_2}{4\pi\epsilon r^3} \vec{r} \longrightarrow (14)$$

$$\vec{E} = \frac{\vec{F}}{q_2} = \frac{q_1}{4\pi\epsilon r^3} \cdot \vec{r} \longrightarrow (15)$$

DOT PRODUCT:

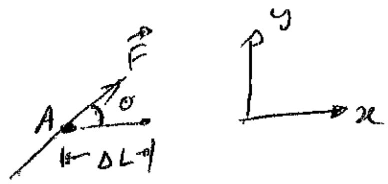
THE DOT PRODUCT IS DEFINED AS

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta \rightarrow (16)$$



$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}.$$

EXAMPLE:



SUPPOSE AN OBJECT, A IS ALLOWED TO MOVE ONLY IN x-DIRECTION. A FORCE $\vec{F}$ IS APPLIED ON THE OBJECT WHICH MOVES A DISTANCE ΔL IN x-DIRECTION. THE WORK DONE IS

$$\Delta W = \vec{F} \cdot \Delta\vec{L} = |\vec{F}|\,|\Delta\vec{L}| \cos\theta \rightarrow (17)$$

$$= F \cos\theta \, \Delta L$$

NOTE THAT IF $\vec{F}$ IS IN y-DIRECTION, THE OBJECT WILL NOT MOVE & NO WORK IS DONE. IN EQ. (17), $\theta = 90°$ IN THIS CASE, $\cos 90° = 0$, SO, $\Delta W = 0$.

IF $\vec{F}$ IS IN X-DIRECTION, THE WORK DONE IS MAXIMUM SINCE $\cos 0 = 1$.

LET $\vec{A} = A_x \vec{x} + A_y \vec{y} + A_z \vec{z}$

$\vec{B} = B_x \vec{x} + B_y \vec{y} + B_z \vec{z}$

$$\vec{A}\cdot\vec{B} = \left(A_x \vec{x} + A_y \vec{y} + A_z \vec{z}\right)\cdot\left(B_x \vec{x} + B_y \vec{y} + B_z \vec{z}\right).$$

NOTE THAT $\vec{x}\cdot\vec{y} = |\vec{x}||\vec{y}|\cos 90^\circ = 0$

III'ly, $\vec{y}\cdot\vec{z} = 0 = \vec{x}\cdot\vec{z}$

$\vec{x}\cdot\vec{x} = \overset{1}{|\vec{x}|}\overset{1}{|\vec{x}|}\cos 0 = 1$

$\vec{y}\cdot\vec{y} = \vec{z}\cdot\vec{z} = 1$

$$\therefore \vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z. \longrightarrow (18)$$

DIVERGENCE:

THE DEL OPERATOR IS DEFINED AS

$$\nabla = \frac{\partial}{\partial x}\vec{x} + \frac{\partial}{\partial y}\vec{y} + \frac{\partial}{\partial z}\vec{z}$$

THE DIVERGENCE OF A VECTOR IS DEFINED AS

$\cancel{\nabla \;(\nabla A = DIV(\vec{A}) = \frac{\partial}{\partial x} x}$

THIS IS JUST LIKE $\vec{B}$ OF EQ(18)

$$DIV(\vec{A}) = \nabla\cdot\vec{A} = \left(\frac{\partial}{\partial x}\vec{x} + \frac{\partial}{\partial y}\vec{y} + \frac{\partial}{\partial z}\vec{z}\right)\cdot\left(A_x \vec{x} + A_y \vec{y} + A_z \vec{z}\right)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$