1.

WORKED EXAMPLES

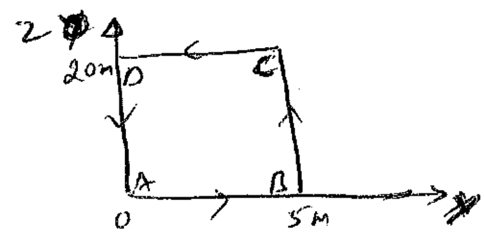
1. IN A DIELECTRIC MEDIUM, $\epsilon_r = 10$, $\mu_r = 1$. IF

$$B_x = 10^{-3} \cos(\omega t) \sin(\beta y), \quad \omega = 10^6 \text{ rad/s} \; \& \; \beta$$

IS UNKNOWN, $B_y = B_z = 0$.

(i) USE $\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$ to FIND $\vec{E}$.

(ii) FIND THE MAGNETIC FLUX LINKED WITH A SURFACE $x = 0$, $0 < z \leq 20$ m, $0 < y < 5$ m, FORMED BY A RECTANGULAR COIL, SHOWN BELOW.



(iii) FIND THE EMF USING FARADAY'S LAW

(iv) EVALUATE THE CLOSED LOOP INTEGRAL OF $\vec{E}$ AROUND THE PERIMETER OF THE SURFACE (ABCD) & DETERMINE β. ~~BY~~

~~(v) FIND THE DISPLACEMENT CURRENT DENSITY~~

(vi) VERIFY THAT $\vec{E}$ SATISFIES THE FARADAY'S LAW IN THE POINT FORM

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

SOLUTION:

(i) $H_x = \frac{B_x}{M}$

H_x DEPENDS ON y; IT DOES NOT DEPEND ON x OR z.

SO $(\nabla \times \vec{H})_z = -\frac{\partial H_x}{\partial y} = \frac{-10^{-3}}{M} \cos(\omega t) \cos(\beta y)\, \beta \quad A/m^2$

x y z x y

$(\nabla \times \vec{H})_x = (\nabla \times \vec{H})_y = 0$

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$$

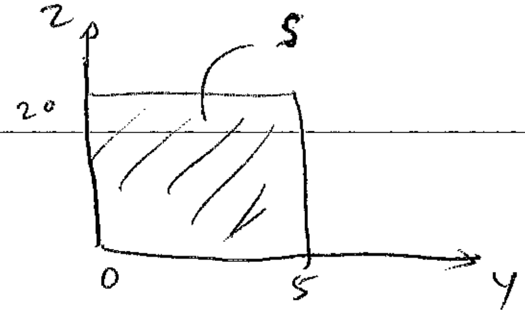
$\frac{\partial \vec{E}}{\partial t}$ HAS ONLY z-COMPONENT ($\because$ $\nabla \times \vec{H}$ HAS ONLY z-COMPONENT)

$$\therefore \epsilon \frac{\partial E_z}{\partial t} = (\nabla \times \vec{H})_z = \frac{-10^{-3}}{M} \cdot \cos(\omega t) \cos(\beta y) \cdot \beta$$

$$E_z = \frac{-10^{-3} \beta \cos \beta y}{M\epsilon} \int \cos(\omega t)\, dt \quad V/m$$

$$= \frac{-10^{-3} \beta \cos(\beta y) \cdot \sin(\omega t)}{\omega M \epsilon} \quad V/m$$

(ii)



MAGNETIC FLUX $= \Phi = \iint_S \vec{B} \cdot d\vec{s}$

$|d\vec{s}| = ds = dy\,dz$. THE DIRECTION OF $d\vec{s}$ IS ALONG THE +VE X-AXIS (NORMAL TO THE SURFACE ELEMENT ds).

$$\vec{B} \cdot d\vec{s} = \left(B_x \vec{x}\right) \cdot \left(dy\,dz\,\vec{x}\right)$$

$$= B_x\,dy\,dz$$

$$\Phi = \int_0^5 \int_0^{20} B_x\,dy\,dz$$

$$= 10^{-3} \cos\omega t \int_0^{20} dz \int_0^5 \sin\beta y \cdot dy \quad \text{Wb}$$

$$= \frac{20 \times 10^{-3}}{\beta} \cos\omega t \cdot (-\cos\beta y) \Big|_0^5 \quad \text{Wb}$$

$$= \frac{20 \times 10^{-3} \cos\omega t \,(1 - \cos\beta 5)}{\beta} \quad \text{Wb}$$

c(ii) USING FARADAY'S LAW

$$emf = \frac{-d\varphi}{dt} = \frac{-20 \times 10^{-3} \left[(-\sin\omega t)(1 - \cos\beta 5) \cdot \omega\right]}{\beta}$$

$$= \frac{20 \times 10^{-3}\,\omega\,[1 - \cos(\beta 5)]\,\sin(\omega t)}{\beta} \quad V \longrightarrow (1)$$

(iv) THE LINE INTEGRAL IS TAKEN ALONG THE COUNTER-CLOCKWISE DIRECTION.

$$emf = \oint \vec{E}\cdot d\vec{l} = \int_A^B (\ ) + \int_B^C (\ ) + \int_C^D (\ ) + \int_D^A (\ )$$

$$= \cancel{\int_0^5 (E_z \hat{z})\cdot(dy\,\hat{y})} + \int_0^{20} E_z(y=5)\,dz$$

$$+ \cancel{\int_5^0 (E_z \hat{z})\cdot(dy\,\hat{y})} + \int_{20}^0 E_z(y=0)\,dz$$

NOTE: $\vec{E}$ HAS ONLY z-COMPONENT. SO, THE FIRST & THIRD LINE INTEGRALS DO NOT CONTRIBUTE SINCE $\vec{E}$ IS ORTHOGONAL TO THE LINE OF INTEGRATION.

$$E_z(y=0) = \frac{-10^{-3}\beta \sin \omega t}{\omega \mu \epsilon}$$

$$E_z(y=5) = \frac{-10^{-3}\beta \cos(5\beta)\cdot\sin(\omega t)}{\omega \mu \epsilon}$$

$$2^{nd}\text{ Integral} = \int_0^{20} dz \times \left(\frac{-10^{-3}\beta \cos(5\beta)\sin \omega t}{\omega \mu \epsilon}\right)$$

LAST INTEGRAL $= \displaystyle\int_{20}^{0} dz \times \left(\frac{-10^{-3} \beta \sin \omega t}{\omega \mu \epsilon} \right)$

$$\therefore \text{emf} = \oint \vec{E} \cdot d\vec{L} = \frac{-10^{-3} \beta \sin \omega t}{\omega \mu \epsilon} \left\{ 20 \cos(5\beta) - 20 \right\} \ \text{V}$$

$$= \frac{20 \times 10^{-3} \beta \left[1 - \cos(\beta 5) \right]}{\omega \mu \epsilon} \cdot \sin(\omega t) \ \text{V} \longrightarrow (2)$$

COMPARING EQNS. (1) & (2), EMFS ARE EQUAL ONLY IF

$$\cancel{\frac{\omega}{\beta} = \frac{\beta}{\omega \mu \epsilon}}$$

$$\frac{\omega}{\beta} = \frac{\beta}{\omega \mu \epsilon}$$

$$\beta^2 = \omega^2 \mu \epsilon \longrightarrow (3)$$

$\omega = 10^6$ rad/s

$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7}$ F/m $\quad (\because \mu_r = 1)$

$\epsilon = \epsilon_0 \epsilon_r = 8.854 \times 10^{-12} \times 10$ H/m

$= 8.854 \times 10^{-11}$ H/m

$\beta = \pm \omega \sqrt{\mu \epsilon} = \pm 10^6 \sqrt{8.854 \times 10^{-11} \times 4\pi \times 10^{-7}}$ rad/m

$= \pm 1.054 \times 10^{-2}$ rad/m

$x\ y\ \hat{z}\ x\ y$

$$\cancel{\frac{\partial E_z}{\partial y} = -\mu \cdot \frac{\partial H_x}{\partial t} - \frac{\partial B_x}{\partial t}}$$

$$\cancel{RHS = +10^{-3} \cdot \sin(\omega t) \cdot \sin(\beta y) \cdot \omega}$$

$$\cancel{LHS = \frac{\partial E_z}{\partial y} = +10^{-3}\beta \sin(\beta y) \sin(\omega t)}$$

(v) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$RHS = -\frac{\partial B_x}{\partial t}\vec{x} = 10^{-3}\sin(\omega t)\sin(\beta y)\,\omega \longrightarrow (4)$$

~~LHS~~ $\cancel{(\nabla \times E)_x =}$

$$\vec{E} = E_z \vec{z}$$

$$(\nabla \times E)_x = +\frac{\partial E_z}{\partial y} \qquad x\ y\ \overset{+}{\hat{z}}\ x\ y$$

$$= \frac{+10^{-3}\beta^2 \sin(\beta y)\sin(\omega t)}{\omega\mu\epsilon} \quad (\text{SEE p.2 for } E_z)$$

FROM (3), ~~USING~~ $\beta^2 = \omega^2\mu\epsilon$

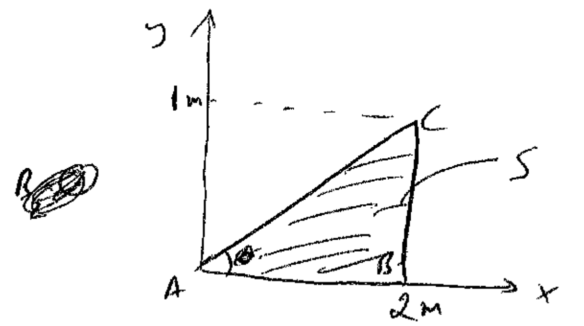
$$\therefore (\nabla \times E)_x = 10^{-3}\,\omega \sin(\beta y)\sin(\omega t)$$

$$= RHS$$

SHOW THAT $(\nabla \times E)_y = (\nabla \times E)_z = 0.$

2.1

2.



CALCULATE THE MAGNETIC FLUX LINKED WITH THE COIL ABC SHOWN ABOVE. THE MAGNETIC FLUX DENSITY IS

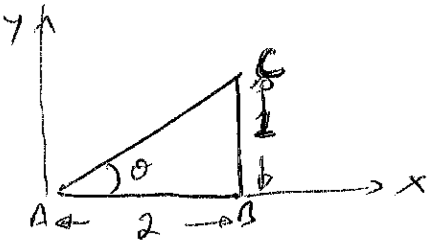
(a) $\vec{B} = B_z \vec{z}$ (b) $\vec{B}$ MAKES AN ANGLE 45° WITH THE NORMAL TO THE SURFACE S. $|\vec{B}| = 10^{-6} \sin(10^2 x) \sin(10^3 y)$ T

(a)

$$\Phi = \iint_S \vec{B} \cdot d\vec{s}$$

$d\vec{s} = dx\,dy\ \vec{n}$ WHERE $\vec{n}$ IS THE NORMAL VECTOR THAT MAKES THE CLOSEST ANGLE WITH $\vec{B}$. SO, $\vec{n} = \vec{z}$.

$$\vec{B} \cdot d\vec{s} = (B_z \vec{z}) \cdot (dx\,dy\,\vec{z}) = B_z\,dx\,dy.$$



$\tan\theta = \frac{1}{2} = m =$ SLOPE

SO, THE EQUATION FOR THE LINE AC IS

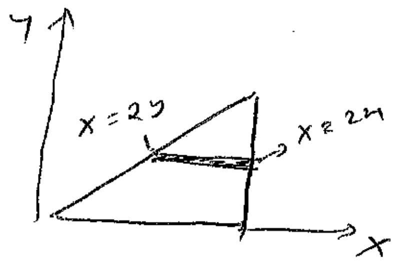
$$y = mx = \frac{x}{2} \longrightarrow (1)$$

or

$$x = 2y$$

(2.2)

LET US FIRST CONSIDER THE LIMITS OF INTEGRATION FOR X.

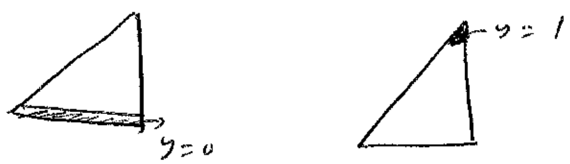


CONSIDER A THIN HORIZONTAL STRIP AT ANY y. ITS LEFT END IS AT $X = 2y$ & THE RIGHT END IS AT $X = 2$ m.

~~SO, LIMIT~~

SO, THE LIMITS FOR X IS FROM $2y$ to 2.

NEXT CONSIDER THE LIMITS OF INTEGRATION FOR y.



IF THE STRIP SLIDES FROM THE LINE $y = 0$ to THE LINE $y = 1$, ENTIRE TRIANGLE IS COVERED. SO, THE LIMITS FOR Y IS FROM 0 to 1.

$$\phi = \int_{0}^{1} \int_{2y}^{2} B_2 \, dx \, dy$$

$\uparrow$ y $\quad$ $\uparrow$ x

2.3

$$\phi = 10^{-6} \int_0^1 \sin(10^3 y) \int_{2y}^{2} \sin(10^2 x)\, dx\, dy \quad \text{Wb}$$

$$= 10^{-6} \int_0^1 dy\, \sin(10^3 y) \cdot \left(-\frac{\cos(10^2 x)}{10^2} \right) \Bigg|_{2y}^{2} \quad \text{Wb}$$

$$= -10^{-8} \int_0^1 dy\, \sin(10^3 y) \left[\cos(2\times 10^2) - \cos(2y \times 10^2) \right] \quad \text{Wb}$$

$$= -10^{-8} \cdot \cos(2\times 10^2) \left(-\frac{\cos(10^3 y)}{10^3} \right) \Bigg|_0^1$$

$$+ 10^{-8} \int_0^1 dy \cdot \sin(10^3 y) \cos(2y \times 10^2) \quad \text{Wb}$$

SINCE

$$\sin u \cos v = \frac{1}{2} \left[\sin(u+v) + \sin(u-v) \right],$$

$$\phi = +10^{-11} \cos(2\times 10^2) \left\{ \cos(10^3 \cdot 1) - 1 \right\}$$

$$+ \frac{10^{-8}}{2} \cdot \int_0^1 dy \left[\sin(10^3 y + 2y \times 10^2) + \sin(10^3 y - 2\times 10^2 y) \right]$$

Wb

(2.4)

$$\phi = -2.13\times10^{-12} + \frac{10^{-8}}{2}\left\{ -\frac{\cos(1200y)}{1200}\Big|_0^1 + -\frac{\cos(800y)}{800}\Big|_0^1 \right\} \text{ Wb}$$

$$= -2.13\times10^{-12} + \frac{10^{-8}}{2}\left\{ \frac{3.94\times10^{-3}}{1200} + \frac{1.44}{800} \right\} \text{ Wb}$$

$$= 6.93\times10^{-12} \text{ Wb.}$$

(b)

$$\vec{B}\cdot d\vec{S} = |\vec{B}|\,|d\vec{S}|\cos 45^\circ$$

$$= \frac{1}{\sqrt{2}}\cdot 10^{-4}\cdot \sin(10^2 x)\sin(10^3 y)\,dx\,dy$$

So, in this case, the flux linked is $\frac{1}{\sqrt{2}}$ times that of case (a), i.e. $\phi = 4.905\times10^{-12}$ Wb.

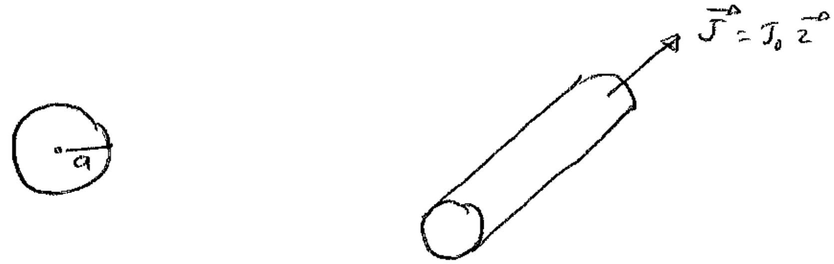
3. CONSIDER A CONDUCTOR OF RADIUS 'a' CARRYING A UNIFORM CURRENT DENSITY, $\vec{J} = J_0 \, \hat{z}$ (J_0 IS A CONSTANT).

(i) FIND THE MAG. FIELD INTENSITY INSIDE & OUTSIDE THE CONDUCTOR

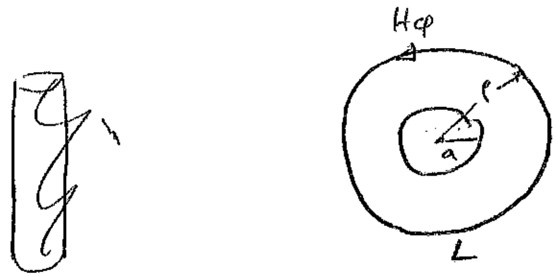
(ii) VERIFY THAT

$$\nabla \times \vec{H} = \vec{J}$$

IS SATISFIED INSIDE & OUTSIDE THE CONDUCTOR.



SOLUTION: DUE TO CYLINDRICAL SYMMETRY OF THE PROBLEM, WE USE CYLINDRICAL COORDINATES (ρ, Φ & z).



CONSIDER A CIRCULAR PATH L WITH RADIUS $\rho > a$.

$$\oint_L \vec{H} \cdot d\vec{l} = I$$

H_φ

$\vec{H}$ IS IN A TANGENTIAL DIRECTION AT EVERY POINT ON THE CIRCLE, L & ITS MAGNITUDE IS CONSTANT DUE TO SYMMETRY. SO,

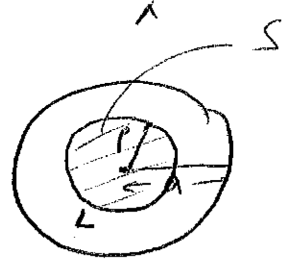
$$\oint \vec{H}\cdot d\vec{L} = H_\varphi \oint_L dL = H_\varphi \cdot 2\pi\rho = I$$

SINCE J IS UNIFORM,

$$I = JA = J \times \pi a^2 = J_0 \pi a^2$$

$$\therefore \quad H_\varphi = \frac{J_0 \pi a^2}{2\pi\rho} \longrightarrow ① \quad \text{OUTSIDE THE CONDUCTOR.}$$

NEXT, CONSIDER A CIRCLE OF RADIUS ρ INSIDE THE CONDUCTOR ($0 < \rho \le a$)



THE CURRENT ENCLOSED BY THE SURFACE S (WHOSE PERIMETER IS L) IS

$$I_S = J_0 \cdot \pi\rho^2 = \frac{I}{\pi a^2}\cdot \pi\rho^2$$

2.3

USING AMPERE'S LAW,

$$\oint_L \vec{H}\cdot d\vec{l} = H_\varphi \cdot 2\pi\rho = I_s = \frac{I\rho^2}{a^2}$$

$$H_\varphi = \frac{I\rho}{2\pi a^2} \quad (2) \quad : \text{ INSIDE THE CONDUCTOR}$$

(ii) FIRST CONSIDER THE REGION OUTSIDE THE CONDUCTOR $(\rho > a)$.

IN THIS REGION, $\vec{J} = 0$. (CURRENT & CURRENT DENSITY ARE CONFINED TO THE REGION INSIDE THE CONDUCTOR).

IN CYLINDRICAL CO-ORDINATES,

$$(\nabla \times \vec{H})_z = \frac{1}{\rho}\left[\frac{\partial(\rho H_\varphi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \varphi}\right]$$

SINCE, $H_\rho = 0$,

$$(\nabla \times \vec{H})_z = \frac{1}{\rho}\left[\frac{\partial(\rho H_\varphi)}{\partial \rho}\right]$$

~~USING (1), $(\nabla \times H)_z = \frac{I_0 \pi a^2}{2\rho}$~~

USING (1), $\rho H_\varphi = I_0 a^2/2$

$\cancel{(\nabla \times \vec{H})_z}$

$$\therefore \frac{\partial(\rho H_\varphi)}{\partial \rho} = 0 \Rightarrow (\nabla \times \vec{H})_z = 0 = \vec{J}$$

3.4

///^{b} YOU CAN VERIFY THAT

$$(\nabla \times \vec{H})_\rho = (\nabla \times H)_\phi = 0$$

$$(\nabla \times \vec{H})_\rho = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\phi}{\partial z}\right)$$

$$(\nabla \times \vec{H})_\varphi = \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)$$

USE $\vec{H} = H_\varphi \vec{\varphi}$.

NOTE: IN THE EXAM, IF THIS ~~type~~ TYPE OF QUESTION IS ASKED, FORMULA FOR $\nabla \times H$ IN CYLINDRICAL CO-ORDINATES WILL BE PROVIDED.

NEXT CONSIDER THE REGION INSIDE THE CONDUCTOR $(0<\rho<a)$. IN THIS REGION, $\vec{J} = J_0 \vec{z}$.

$$H_\varphi = \frac{I\rho}{2\pi a^2} \quad (\text{FROM } ②)$$

$$(\nabla \times H)_z = \frac{1}{\rho}\left[\frac{\partial(\rho H_\varphi)}{\partial \rho}\right]$$

$$= \frac{1}{\rho}\frac{I}{2\pi a^2}\cdot 2\rho$$

$$= \frac{I}{\pi a^2} = J_0$$

YOU CAN VERIFY THAT $(\nabla \times H)_\rho = (\nabla \times H)_\varphi = 0$

$$\therefore \quad \nabla \times \vec{H} = \vec{J} \qquad \text{INSIDE THE CONDUCTOR}$$

4\. IN A METALLIC CONDUCTOR AT 60 Hz, $\mu_r = 1$, $\epsilon_r = 1$,

$\sigma = 4 \times 10^7$ S/m & $\vec{J} = 3 \sin(2\pi f t - 100z)\,\vec{x}$ MA/m². (MEGA)

FIND

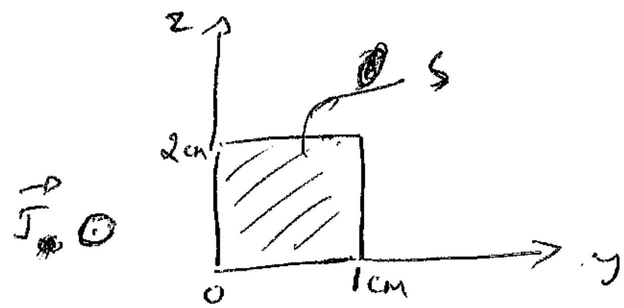
(i) DISPLACEMENT CURRENT DENSITY

(ii) ELECTRIC FLUX

(iii) DISPLACEMENT CURRENT

(iv) CONDUCTION CURRENT

THE CONDUCTOR HAS A RECTANGULAR CROSS-SECTION AS SHOWN BELOW



(i)

$$\vec{J} = \sigma \vec{E}$$

$$\therefore \vec{E} = \frac{\vec{J}}{\sigma} = \frac{3\times10^6 \sin(2\pi \times 60 \times t - 100z)\,\vec{x}}{4\times10^7} \quad V/m$$

$$E_x = 0.075 \sin(2\pi \times 60 \times t - 100z) \quad V/m$$

$$\vec{D} = \epsilon \vec{E} = 8.854\times10^{-12} \times 0.075 \sin(2\pi\times60\times t - 100z)\,\vec{x} \text{ C/m}^2$$

$$= (6.64\times10^{-13}) \sin(2\pi\times60\times t - 100z)\,\vec{x} \text{ C/m}^2$$

$\longrightarrow$ (1)

(4.2)

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = (2\pi \times 60 \times 6.64 \times 10^{-13}) \cos(2\pi \times 60 \times t - 100z)\, \vec{x} \ A/m^2$$

$$= 2.5 \times 10^{-10} \cos(2\pi \times 60 \times t - 100z)\, \vec{x} \ A/m^2$$

(ii) Electric flux

$$\Psi = \iint_S \vec{D} \cdot d\vec{s}$$

$$\vec{D} = D_x \vec{x} \ ; \quad d\vec{s} = dy\, dz\, \vec{x}$$ (NORMAL VECTOR TO THE SURFACE IN X-DIRECTION)

$$\therefore \Psi = \int_0^{2\times10^{-2}} \int_0^{1\times10^{-2}} D_x \, dy\, dz$$

DOES NOT DEPEND ON Y. SEE (1)

$$= \int_0^{2\times10^{-2}} D_x\, dz \int_0^{1\times10^{-2}} dy$$

$$= 6.64 \times 10^{-13} \left[\int_0^{2\times10^{-2}} \sin(2\pi \times 60 \times t - 100z)\, dz \right] \times 1 \times 10^{-2} \quad C$$

~~= 6.64 × 10^-13 cos(2π × 60~~

LET $u = 2\pi \times 60 \times t - 100z$; $\quad z = 0, \ u = 2\pi \times 60 \times t$

$du = -100\, dz$; $\quad z = 2\times10^{-2}, \ u = 2\pi \times 60 \times t - 2$

$$\therefore -1 \times \int_{2\pi\times60\times t}^{2\pi\times60\times t - 2} \frac{\sin(u)}{100}\, du = \frac{\cos u}{100} \Bigg|_{2\pi\times60\times t}^{2\pi\times60\times t - 2}$$

4.3

$$= \frac{1}{100} \cos(2\pi \times 60t - 2) - \cos(2\pi\, 60t)$$

$$\therefore \psi = 6.64 \times 10^{-17} \left[\cos(2\pi \times 60t - 2) - \cos(2\pi\, 60t)\right]$$

$$= -1.328 \times 10^{-17} \cdot \sin(2\pi \times 60t - 1)\sin(-1) \; C$$

$$= 1.117 \times 10^{-17} \sin(2\pi \times 60t - 1)$$

C

(iii) Displacement current

$$I_d = \frac{d}{dt}\iint_S \vec{D}\cdot d\vec{s} = \frac{d\psi}{dt}$$

$$= 1.117 \times 10^{-17} \times 2\pi \times 60 \cos(2\pi \times 60t - 1) \; A$$

$$= 4.212 \times 10^{-15} \cos(2\pi \times 60 \times t - 1) \; A$$

$\rightarrow$ 2

(iv) CONDUCTION CURRENT

$$I = \iint_S \vec{J}\cdot d\vec{s}$$

WE CAN INTEGRATE $\vec{J}$ OVER THE CROSS-SECTION OF THE CONDUCTOR. INSTEAD, LET US DO A SHORT CUT.

$$\vec{J} = \sigma \vec{E} = \frac{\sigma}{\epsilon}\vec{D}$$

$$\therefore I = \iint_S \vec{J}\cdot d\vec{s} = \frac{\sigma}{\epsilon}\iint_S \vec{D}\cdot d\vec{s}$$

(4.4)

$$I = \frac{\sigma}{\epsilon} \cdot \Psi \quad \text{Electric flux}$$

$$= \frac{4 \times 10^{7}}{8.854 \times 10^{-12}} \times 1.117 \times 10^{-17} \sin(2\pi \times 60t - 1) \text{ A}$$

$$= 5.04 \times 10^{[illegible]} \sin(2\pi \times 60t - 1) \text{ A} \quad \text{—— (3)}$$

COMPARING EQS. (2) & (3), WE SEE THAT THE ~~DISPLACEMEN~~ DISPLACEMENT CURRENT AMPLITUDE (~~4...~~ 4.2 fA) IS MUCH SMALLER THAN ~~...~~ THAT OF CONDUCTION CURRENT (~~...~~ 50.4 A)

THAT IS WHY THE DISPLACEMENT CURRENT WAS NEVER OBSERVED BEFORE MAXWELL INTRODUCED IN 1860S.

NOTE THE $\pi/2$ PHASE DIFFERENCE BETWEEN I & I_d.

($\because$ CONDUCTION CURRENT IS ASSOCIATED WITH RESISTANCE & DISPLACEMENT CURRENT IS ASSOCIATED WITH CAPACITANCE OF THE CONDUCTOR, WHICH IS VERY SMALL)

5. THE EYE IS MOST SENSITIVE TO LIGHT HAVING A WAVELENGTH IN FREESPACE, OF 0.55 MM WHICH IS THE GREEN-YELLOW REGION OF THE EM SPECTRUM.

(i) WHAT IS THE FREQUENCY OF THIS LIGHT?

(ii) WHAT IS THE WAVE NUMBER IN FREE SPACE?

(iii) IF THIS LIGHT TRAVELS IN WATER, WHICH HAS A RELATIVE PERMITTIVITY, $\epsilon_r = 1.75$, FIND WAVELENGTH, SPEED OF LIGHT & WAVE NUMBER IN WATER.

(iv) IF THE PEAK ELECTRIC FIELD INTENSITY OF THE LIGHT WAVE INCIDENT ON WATER IS 10^{-6} V/m & CALCULATE THE PEAK AMPLITUDE AFTER PROPAGATING A DISTANCE OF 2 m. ASSUME THE ATTENUATION COEFFICIENT α OF WATER TO BE $10\ m^{-1}$.

SOLUTION:

(i) $c = 3\times10^{8}\ m/s$; $\lambda_o = 0.55\times10^{-6}\ m$

$$f = \frac{c}{\lambda_o} = \frac{3\times10^{8}}{0.55\times10^{-6}} = 5.455\times10^{14}\ Hz$$

(ii) $$\beta_o = \frac{2\pi}{\lambda_o} = \frac{2\pi}{0.55\times10^{-6}} = 1.1423\times10^{6}\ rad/m$$

(iii) SPEED OF LIGHT IN WATER

$$u = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^{8}}{\sqrt{1.75}} = 2.26\times10^{8}\ m/s$$

(5.2)

(iii)

REFRACTIVE INDEX, $n = \sqrt{\epsilon_r} = 1.32$

WAVELENGTH IN WATER, $\lambda_m = \dfrac{\lambda_0}{n} = \dfrac{0.55 \times 10^{-6}}{1.32}\ m$

$= 4.16 \times 10^{-7}\ m$

NOTE THAT WAVELENGTH ~~of~~ IN A MEDIUM IS ALWAYS LESS THAN THAT IN FREESPACE.

WAVE NUMBER IN A MEDIUM, $\beta_m = \dfrac{2\pi}{\lambda_m} = \dfrac{2\pi}{\lambda_0} \cdot n$

$= \beta_0\, n$

$= \cancel{5.455 \times 10^{14}}\ 1.1423 \times 10^{6} \times 1.32\ rad/m$

$\cancel{= 7.2 \times 10^{14}\ rad/m}$

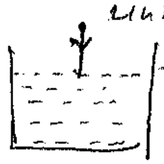
$= 1.51 \times 10^{6}\ rad/m$

(iv) $E_x = E_{x0}\, e^{-\alpha z} \cos(\omega t - \beta z)$

α = ATTENUATION COEFFICIENT $= 10\ m^{-1}$

E_{x0} = PEAK AMPLITUDE OF THE INCIDENT LIGHTWAVE

$= 10^{-6}\ V/m$



$z = 2\ m$

THE PEAK AMPLITUDE OF THE ELECTRIC FIELD AFTER PROPAGATING ~~A DIST~~ 2 m IN WATER

$E_{x0}' = E_{x0}\, e^{-\alpha z} = 10^{-6} e^{-20}\ V/m = 2.06 \times 10^{-15}\ V/m$