# WORKED EXAMPLES

EXAMPLE #1. CONSIDER A LOSSLESS COAXIAL CABLE OF LENGTH 20 m. THE CHARACTERISTIC IMPEDANCE OF THE LINE IS 75 Ω & THE SPEED ON THE LINE IS $2.2\times10^{8}$ m/s. THIS CABLE IS USED TO CONNECT AN ANTENNA & A TV RECEIVER. THE ANTENNA MAY BE REPRESENTED BY ITS THEVENIN EQUIVALENT $Z = 75\,\Omega$ IN SERIES WITH $V_{sv} = 5$ mV. THE IMPEDANCE OF THE RECEIVER IS 75 Ω, i.e. IT IS MATCHED TO THE LINE. CALCULATE THE

(i) VOLTAGE AT THE INPUT TO THE LINE & AT THE LOAD

(ii) POWER DELIVERED TO THE INPUT OF THE LINE & TO THE LOAD.

~~SOLUTION:~~

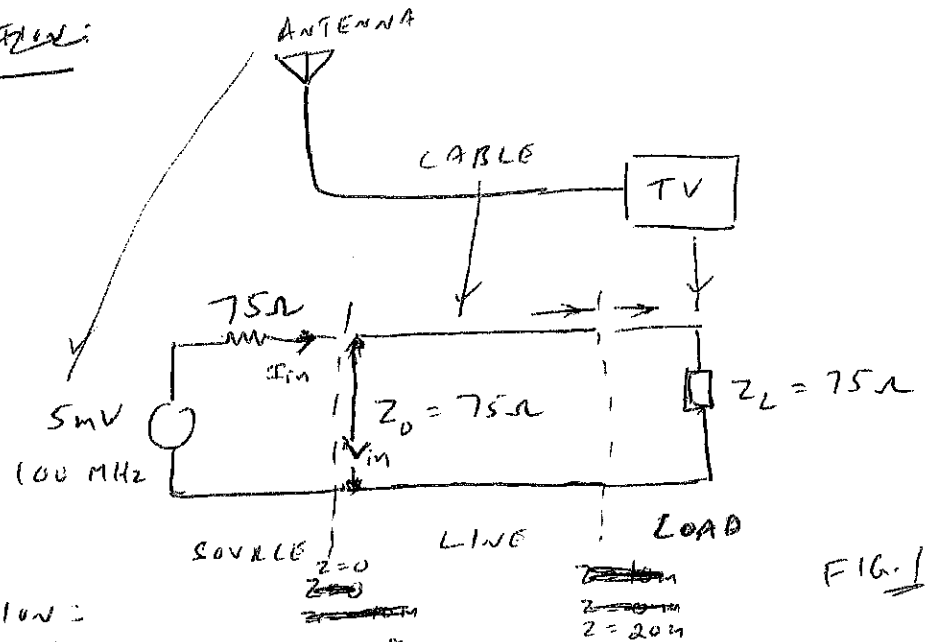



FIG. 1

SOLUTION:

$$v = 2.2\times10^{8}\ \text{m/s}$$

$$f = 100\times10^{6}\ \text{Hz}$$

$$\lambda = v/f = \frac{2.2\times10^{8}}{10^{8}}\ \text{m} = 2.2\ \text{m}.$$

PROP. CONSTANT, $\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{2.2}$ rad/m

$= 2.856$ rad/m

$l = 20$ m

Total phase shift accumulated over a length of 10 m

$= \phi = \beta l = 57.1190$ rad.

SINCE THE LINE IS MATCHED TO THE LOAD, LOAD CAN BE TRANSFERRED TO THE SOURCES TERMINALS BY REMOVING THE LINE COMPLETELY, i.e. $(Z_{in} = Z_L)$

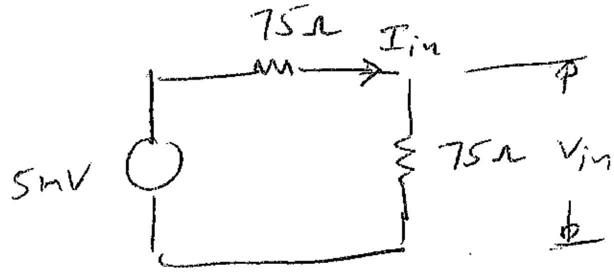



FIG. 2

So, $V_{in} = \frac{75}{150} \times 5\,mV = 2.5\,mV$

$v_{in} = Re\{V_{in} e^{j\omega t}\} = 2.5 \times 10^{-3} \cos(2\pi 10^{8} t)$ V

SINCE THERE IS NO REFLECTION, VOLTAGE & CURRENT EQUATIONS BECOME

$V_s(z) = V_0^+ e^{-j\beta z}$ $(\because$ LOSSLESS $\Rightarrow \alpha = 0)$

$I_s(z) = \frac{V_0^+}{Z_0} e^{-j\beta z}$

At $z=0$, $V_s(0) = V_0^+ = V_{in}$

$$I_s(0) = \frac{V_0^+}{Z_0} = I_{in} = \frac{2.5\times10^{-3}}{75}\,A = 3.33\times10^{-5}\,A$$

At $z = 20$ m, $V_s(20m) = V_L = V_{in}\,e^{-j\beta l} = V_{in}\,e^{-j\varphi}$

$$v_L = Re\{V_L e^{j\omega t}\} = V_{in}\cos(\omega t - \varphi)$$

$$= 2.5\times10^{-3}\cos(2\pi\,10^8 t - 57.1190)$$

$$I_s(10m) = I_L = I_{in}\,e^{-j\phi}$$

$$i_L = Re\{I_L\,e^{j\omega t}\}$$

$$= I_{in}\cos(\omega t - \varphi)$$

$$= 3.33\times10^{-5}\cos(2\pi\,10^8 t - 57.1190)\,A$$

POWER DELIVERED TO THE INPUT OF LINE $= P_{in} = \frac{1}{2}Re\{V_{in}\,I_{in}^*\}$

$$= \frac{1}{2}Re\{2.5\times10^{-3}\times3.33\times10^{-5}\}$$

$$= \cancel{8.33\times10^{-8}}$$

$$4.16\times10^{-8}\,W$$

ALTERNATIVELY, FROM FIG. 2, THE POWER DELIVERED TO $75\,\Omega$ RESISTOR IS,

$$P_{in} = \frac{|V_{in}|^2}{2R_0} = \frac{(2.5\times10^{-3})^2}{2\times75} = 4.16\times10^{-8}\,W$$

$R_0 \rightarrow$ Real part of $Z_0$

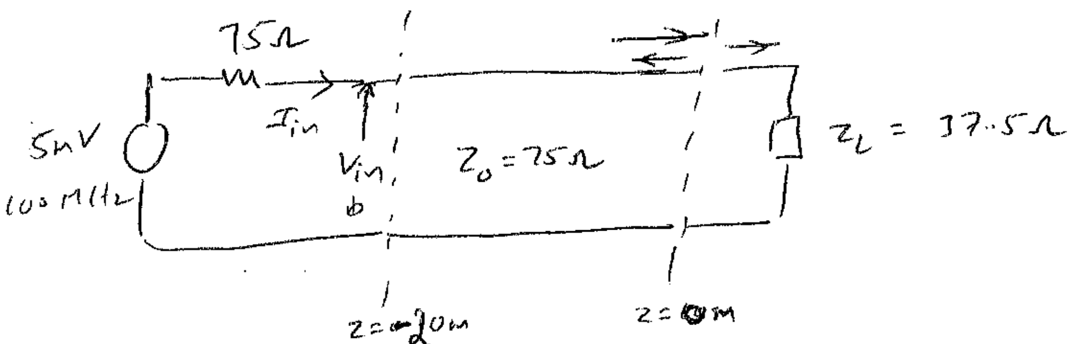
NOTE: ~~YOU COULD~~ TO CALCULATE THE POWER, YOU COULD USE ANY OF THE FOLLOWING FORMULAS:

$$\boxed{P = \frac{1}{2} Re\{V I^*\} = \frac{|V|^2}{2Re(Z)} = \frac{1}{2} |I|^2 Re(Z)}$$

POWER DELIVERED TO THE LOAD IS ALSO $4.16 \times 10^{-8}$ W SINCE THERE IS NO LOSS IN THE LINE.

<u>EXAMPLE 2:</u> CONNECT A SECOND TV RECEIVER, IDENTICAL TO THE FIRST ONE, ACROSS THE LINE IN PARALLEL WITH THE FIRST ONE. REPEAT EXAMPLE #1.

<u>SOLUTION:</u> THE NET LOAD IMPEDANCE IS $75\Omega \,\|\, 75\Omega = 37.5\Omega$




<u>FIG. 3</u>

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{37.5 - 75}{37.5 + 75} = -\frac{1}{3}.$$

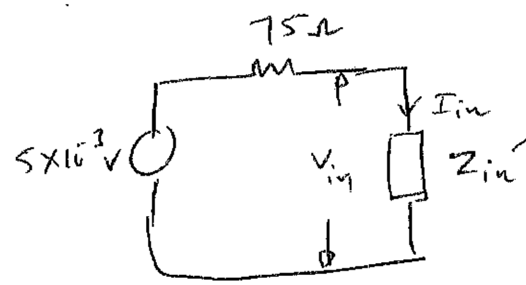
THE WAVE IMPEDANCE AT THE LINE INPUT (INPUT IMPEDANCE),

$$Z_{in} = Z_0 \left[ \frac{Z_L \cos\beta l + j Z_0 \sin\beta l}{Z_0 \cos\beta l + j Z_L \sin\beta l} \right] = 75 \left[ \cancel{37.5 \cos(57.12) + j75 \times \sin} \right.$$

$$= 75 \left[ \frac{37.5 \cos(57.12) + j\, 75 \sin(57.12)}{75 \cos(57.12) + j\, 37.5 \sin(57.12)} \right] \Omega$$

$$Z_{in} = 58.14\angle 0.59\ \Omega = 48.02 + j\,32.76\ \Omega$$

THE EQUIVALENT CIRCUIT OF FIG. 3 IS




NET IMPEDANCE DUE TO THE LINE & LOAD

$$I_{in} = \frac{5\times10^{-3}}{75 + 48.02 + j\,32.76}\ \text{A}$$

$$= \cancel{3.92\angle{-0.26}}\ 3.92\times10^{-5}\angle{-0.26}\ \text{A}$$

$$V_{in} = I_{in}Z_{in} = 2.28\times10^{-3}\angle 0.34\ \text{V}$$

POWER DELIVERED TO THE LINE IS

$$P_{in} = \frac{1}{2}|I_{in}|^2\,\mathrm{Re}\{Z_{in}\} = \frac{1}{2}\left(3.92\times10^{-5}\right)^2\times 48.02\ \text{W}$$

$$= 3.7\times10^{-8}\ \text{W}$$

THE POWER DELIVERED TO THE NET LOAD OF TWO RECEIVERS IN PARALLEL ($Z_L = 37.5\,\Omega$) IS ALSO $3.7\times10^{-8}$ W ($\because$ LOSSLESS TR. LINE)

SINCE THERE ARE TWO RECEIVERS IN PARALLEL, THE POWER DELIVERED TO EACH RECEIVER IS $3.7\times10^{-8}/2$ W $= 1.85\times10^{-8}$ W.

IN EXAMPLE #1, THE POWER DELIVERED TO THE LOAD IS $4.16\times10^{-8}$ W.
IN EXAMPLE #2, SINCE THERE ARE TWO RECEIVERS, WE MAY EXPECT THAT THE POWER DELIVERED TO EACH RECEIVER IS $2.08\times10^{-8}$ W (UNDER IDEAL CONDITIONS); BUT THE

ACTUAL POWER DELIVERED IS ONLY $1.85 \times 10^{-8}$ W BECAUSE OF THE REFLECTION (i.e. LOAD & LINE ARE NOT MATCHED)

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

AT THE LOAD, $z = 0$,

$$V_s(z=0) = V_L = V_0^+ e^0 + V_0^- \cdot e^0 = V_0^+ + V_0^-$$

$$= V_0^+ (1 + V_0^-/V_0^+) = V_0^+ (1 + \Gamma)$$

$$= \frac{2}{3} V_0^+$$

AT THE SOURCE, $z = -l$,

$$V_s(z=-l) = V_{in} = V_0^+ e^{j\beta l} + V_0^- e^{-j\beta l}$$

$$= V_0^+ \left[ e^{j\beta l} + \Gamma e^{-j\beta l} \right]$$

$$= V_0^+ \left[ 1 \angle 57.11 + 1/3 \angle -57.11 \right]$$

$$= V_0^+ \times 0.913 \angle 0.909$$

$$\therefore \quad V_L = 2/3\ V_0^+ = \frac{2}{3} \times \frac{V_{in}}{0.913 \angle 0.909}\ V = 1.66 \times 10^{-3} \angle -6.568\ V$$

NOTE: $|V_L| = 1.66\ \text{mV} < |V_{in}|$.

EXAMPLE #3: REPEAT EXAMPLE #1 IF THE SECOND RECEIVER IS CONNECTED IN SERIES WITH THE FIRST ONE.

SOLUTION: THE NET LOAD IMPEDANCE IS $2 \times 75\,\Omega = 150\,\Omega$

$$Z_{in} = Z_0 \left[ \frac{Z_L \cos(\beta l) + j Z_0 \sin(\beta l)}{Z_0 \cos(\beta l) + j Z_L \sin(\beta l)} \right]$$

$$= 75 \left[ \frac{150 \cos(57.12) + j 75 \sin(57.12)}{75 \cos(57.12) + j 150 \sin(57.12)} \right] \Omega$$

$$= 79.12 - j 54.52\ \Omega$$

$$I_{in} = \frac{5\times10^{-3}}{75 + 79.92 - j54.52} \text{ A}$$

$$= 3.044\times10^{-5} \angle 0.3384$$

POWER DELIVERED TO THE INPUT OF THE LINE IS (= POWER DELIVERED TO THE NET LOAD)

$$P_{in} = \frac{1}{2} |I_{in}|^2 Re(Z_{in})$$

$$= \frac{1}{2} (3.04\times10^{-5})^2 \times 79.92 \text{ W}$$

$$= 3.7\times10^{-8} \text{W}$$

POWER DELIVERED TO EACH RECEIVER IS $1.85\times10^{-8}$W.

NOTE: CONNECTING THE SECOND RECEIVER IN PARALLEL OR SERIES LEADS TO LINE-LOAD MISMATCH. IF A QUARTER WAVE TR. LINE IS INTRODUCED BETWEEN THE COAXIAL LINE & THE RECEIVERS, THE POWER TRANSFER CAN BE MAXIMIZED. HOWEVER, THIS IMPEDANCE-MATCHING IS LIMITED TO 100MHz FREQUENCY OR A NARROW BAND OF FREQUENCIES CENTERED AROUND 100MHz

EXAMPLE #4. REPEAT EXAMPLE #1 IF THE LOSSLESS CABLE IS REPLACED WITH A LOSSY CABLE WITH $R = 0.15\ \Omega/m$, $G = 12\ \mu S/m$, $L = 0.4\ \mu H/m$ & $C = 71\ pF/m$. ASSUME THAT THE LOAD IS MATCHED TO THE LINE SO THAT THERE IS NO REFLECTED WAVE.

SOLUTION:

~~PART~~

$$f = 100 \times 10^6 \ Hz$$

$$\omega = 2\pi f = 6.28 \times 10^8 \ rad/s$$

$$Z = 0.15 + j\, 6.28 \times 10^8 \times 0.4 \times 10^{-6} \ \Omega/m$$

$$Y = 12 \times 10^{-6} + j\, 6.28 \times 10^8 \times 71 \times 10^{-12} \ S/m$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = 75.058 - j\, 0.0123 \ \Omega$$

$$\gamma = \sqrt{ZY} = \underbrace{0.0014}_{\alpha} + j\, \underbrace{3.3484}_{\beta} \ m^{-1}$$

$$\alpha = 0.0014 \ Np/m \text{ or } 1/m$$

$$\beta = 3.3484 \ rad/m \text{ or } 1/m$$

$$v = \frac{\omega}{\beta} = 1.87 \times 10^8 \ m/s$$

( or USE $\beta = \frac{2\pi}{\lambda}$ & $v = \lambda f$ )

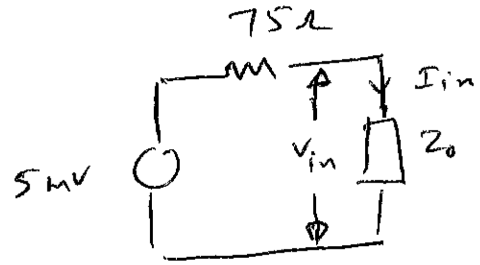
VOLTAGE & CURRENT PHASORS ARE

$$V_s(z) = V_0^+ e^{-(\alpha + j\beta)z}$$

$$I_s(z) = \frac{V_0^+}{Z_0} \cdot e^{-(\alpha + j\beta)z}$$

$$V_s(0) = V_0^+ = V_{in}$$

$$I_s(0) = \frac{V_0^+}{Z_0} = I_{in}$$




$$V_{in} = \frac{5\text{ mV} \times Z_0}{Z_0 + 75} = 2.5 \times 10^{-3} - j2.04 \times 10^{-7} \text{ V}$$

$$I_{in} = 3.332 \times 10^{-5} + 2.73 \times 10^{-9} \text{ A}$$

$$P_{in} = \frac{1}{2} |I_{in}|^2 \operatorname{Re}\{Z_0\} = 4.16 \times 10^{-8} \text{ W}$$

$$P_{out} = P_{in} e^{-2\alpha l}$$

~~OUTPUT~~ POWER DISSIPATED IN THE LOAD

$$= 4.16 \times 10^{-8} \times e^{-2 \times 0.0014 \times 20}$$

$$= 3.93 \times 10^{-8} \text{ W}$$

$$\text{loss (dB)} = -10 \log_{10} \frac{P_{out}}{P_{in}} = 0.25 \text{ dB}$$

$V_s(z = 20m) = V_L$ = LOAD VOLTAGE

$= V_0^+ \cdot e^{-\gamma l}$ ; $\gamma = \alpha + j\beta$

$|V_L| = |V_0^+| e^{-\alpha l}$

$= |V_{in}| \cdot e^{-0.0014 \times 20}$ V

$= 2.42 \times 10^{-3}$ V

$V_L = |V_L| . e^{-j\phi_L}$

$\phi_L = \beta l = 57.11$ rad.

$\therefore V_L = 2.42 \times 10^{-3} \angle -57.11$ V

EXAMPLE #5 A SINUSOIDAL WAVE ON A TRANSMISSION LINE IS SPECIFIED BY VOLTAGE & CURRENT IN PHASOR FORM:

$$V_s(z) = V_0 e^{\alpha z} e^{j\beta z} \quad \& \quad I_s(z) = I_0 e^{\alpha z} e^{j\beta z} e^{j\varphi}$$

WHERE $I_0$ & $V_0$ ARE BOTH REAL & POSITIVE. (a) IN WHICH DIRECTION DOES THIS WAVE PROPAGATE & WHY? (b) IT IS FOUND THAT $\alpha = 0$, $Z_0 = 50\,\Omega$ & THE WAVE VELOCITY IS $2.5 \times 10^8$ m/s, WITH $\omega = 10^8\, s^{-1}$. EVALUATE $R, G, L, C, \lambda$ & $\varphi$.

SOLUTION: (a)

$$V_s(z) = V_0 e^{(\alpha + j\beta)z}$$

$$v(t) = Re\{V_s(z) \cdot e^{j\omega t}\} = e^{\alpha z} Re\{e^{j(\omega t + \beta z)}\}$$

$$= V_0 e^{\alpha z} \cos(\omega t + \beta z) \rightarrow (1)$$

$$i(t) = Re\{I_s(z) e^{j\omega t}\}$$

$$= I_0 e^{\alpha z} \cos(\omega t + \beta z + \varphi)$$

AT $t = 0$, & $z = 0$, $v(t) = V_0$, i.e. THE PEAK OCCURS AT $z = 0$.

AT A LATER TIME, SAY $\Delta t > 0$, THE PEAK OCCURS AT

$$\omega \Delta t + \beta \Delta z = 0 \quad (\because \cos(0) = 1)$$

SINCE, $\omega$ & $\beta$ ARE BOTH POSITIVE, $\Delta z$ HAS TO BE

THEREFORE, THE PEAK MOVES IN THE NEGATIVE
NEGATIVE. ~~THE PEAK MOVES IN~~

~~A~~ Z DIRECTION & WE SAY THAT THE WAVE PROPAGATES IN

-Z DIRECTION.

(b) $\alpha = 0 \Rightarrow R = G = 0$;

$$Z_0 = \sqrt{\frac{L}{C}} = 50\ \Omega$$

$$v = \frac{1}{\sqrt{LC}} = 2.5\times10^8\ \text{m/s}$$

$$Z_0 v = \frac{1}{C} \Rightarrow C = \frac{1}{2.5\times10^8\times50}\ \text{F}$$

$$= 80\ \text{pF/m}$$

$$\frac{Z_0}{v} = \sqrt{\frac{L}{C}}\cdot\sqrt{LC} = L = \frac{50}{2.5\times10^8}\ \text{H}$$

$$= 0.2\ \mu\text{H/m}$$

$$\omega = 2\pi f = 10^8\ \text{s}^{-1}.$$

$$\lambda = \frac{v}{f} = \frac{2.5\times10^8\times2\pi}{\omega}\ \text{m} = 15.7\ \text{m}$$

FOR A BACKWARD PROP. WAVE:

$$V_s(z) = V_0^- e^{(\alpha+j\beta)z}$$

$$I_s(z) = -\frac{V_0^-}{Z_0} e^{(\alpha+j\beta)z}$$

$$\therefore \quad V_0^- = V_0; \quad I_0 e^{j\varphi} = \frac{-V_0}{Z_0}$$

$I_0$ is +ve $\Rightarrow$ $\phi = \pi$.

$$I_0 = \frac{V_0}{Z_0}.$$