Problem 2 With reference to the sliding bar shown in Figure 2, let d=7cm, **B**=0.5$\vec{\mathbf{z}}$ T, and **v**=0.21 e^{20y} $\vec{\mathbf{y}}$ m/s. Let y=0 at t=0. Find

(a) $v(t=0)$;

(b) $y(t=0.1)$;

(c) $v(t=0.1)$;

(d) V_{12} at t=0.1

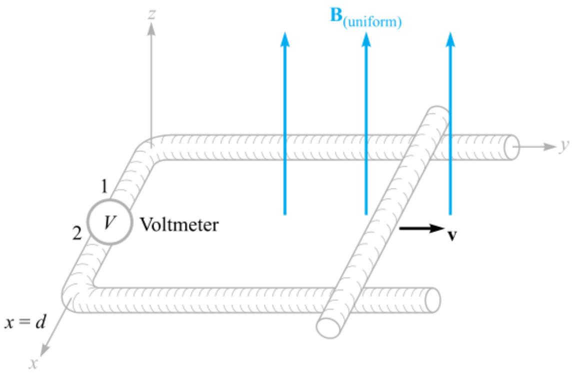

Figure 9.1 An example illustrating the application of Faraday's law to the case of a constant magnetic flux density B and a moving path. The shorting bar moves to the right with a velocity **v**, and the circuit is completed through the two rails and an extremely small high-resistance voltmeter. The voltmeter reading is $V_{12} = -Bvd$.

Solution:

(a)

$$v(t=0) = 0.1e^{20\times 0} = 0.1 \quad {}^{m}\!/_{s}$$

(b)

$$v = 0.21e^{20y} = \frac{dy}{dt} \Rightarrow e^{-20y}dy = 0.21dt \Rightarrow \int_0^y e^{-20y}dy = \int_0^t 0.21dt \Rightarrow \left.\frac{e^{-20y}}{-20}\right|_0^y = 0.21t\Big|_0^t$$

$$\Rightarrow e^{-20y} - 1 = -4.2t \Rightarrow y = -\frac{1}{20}\ln(1-4.2t)$$

$$y(t=0.1) = -0.05\times\ln(1-4.2\times 0.1) = 0.0272m$$

(c)

$$v(t)=\frac{dy}{dt}=-\frac{1}{20}\frac{d}{dt}\ln(1-4.2t)=\frac{0.21}{(1-4.2t)}$$

$$v(t=0.1)=\frac{0.21}{(1-4.2\times 0.1)}=0.362 \quad {}^{m}\!/_{s}$$

(d)

$$emf=-\frac{d\Phi}{dt}=-\frac{d}{dt}\int \vec{B}.d\vec{s}=-\frac{d}{dt}\int_0^d\int_0^y B_0\vec{z}.dxdy\vec{z}=-B_0\frac{dy}{dt}d=-B_0*v(t)*d$$

$$emf\,(t=0.1)=-B_0*v(t=0.1)*d=-0.5\times 0.362\times 0.05=-9.05mV$$

According to Lenz law, the current is induced in clockwise direction so that the magnetic flux due to induced current opposes the incident flux. Therefore, the current direction in the voltmeter is from terminal 2 to terminal 1. So,

$$V_{12}(t=0.1)=-9.05mV$$

(e)
The closed loop integral should be broken into several segments. The stationary arms do not contribute since v = 0 and hence the integration is done only for the sliding bar. Integrating in a counterclockwise direction (i.e. keeping the positive side of the surface on our left), we find

$$V_{12}=\oint_c \vec{v}\times\vec{B}.dl=\oint_c v\vec{y}\times B\vec{z}.d\vec{l}=\oint vB(\vec{y}\times\vec{z}).d\vec{l}$$

$$=\int_y^0 vB\vec{x}.dy\vec{y}+\int_0^d vB\vec{x}.dx\vec{x}+\int_0^y vB\vec{x}.dy\vec{y}+\int_d^0 vB\vec{x}.dx\vec{x}$$

$$=0+\int_d^0\frac{0.21}{1-4.2t}*0.5dx+0+0=-\frac{5.25}{1-4.2t}mV$$

$$V_{12}(t=0.1)=-9.05mV$$