Quiz 2

A sinusoidal voltage source of frequency 1 GHz drives the series combination of an impedance, $Z_g = 50 - j14.8941\Omega$, and a lossless transmission line of length L and a load impedance, $Z_L = 25\Omega$. The speed of the voltage wave in the transmission line is 2×10^8 m/s and the line characteristic impedance is 50 Ω. Determine the shortest line length L that will result in the voltage source driving a total impedance (= $Z_g + Z_{in}$) of 78.0846 Ω.

Note: If you get a complex number with its real part less than 10^{-2}, ignore the real part and proceed with the imaginary part.

Wave impedance,

$$\mathrm{Z}_w(z) = Z_0 \frac{Z_L - jZ_0 \tan(\beta z)}{Z_0 - jZ_L \tan(\beta z)}$$

$$\beta = \frac{2\pi}{\lambda} = \text{wave number}$$

speed, $v = \lambda f$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = 10\pi \text{ rad.}$$

$$Z_{in} = Z_0 \frac{Z_L - jZ_0 \tan(\beta l)}{Z_0 - jZ_L \tan(\beta l)}$$

$$Z_T = Z_g + Z_{in} = Z_g + Z_0 \frac{Z_L - jZ_0 \tan(\beta l)}{Z_0 - jZ_L \tan(\beta l)}$$

$$\frac{Z_T}{Z_0}\left(Z_0 - jZ_L \tan(\beta l)\right) = \frac{Z_g}{Z_0}\left(Z_0 - jZ_L \tan(\beta l)\right) + Z_L - jZ_0 \tan(\beta l)$$

$$Z_T - j\frac{Z_T Z_L}{Z_0}\tan(\beta l) = Z_g - j\frac{Z_g Z_L}{Z_0}\tan(\beta l) + Z_L - jZ_0 \tan(\beta l)$$

$$Z_T - Z_g - Z_L = j(\frac{Z_T Z_L}{Z_0} - \frac{Z_g Z_L}{Z_0} - Z_0)\tan(\beta l)$$

$$78.0846 - (50 - j14.894) - 25 = j(39.04 - 25 + j7.447 - 50)\tan(\beta l)$$

$$\tan(\beta l) = \frac{3.0846 + j14.894}{-7.447 - j35.96} = -0.414 - j1.473*10^{-6}$$

The Imaginary part is ignored as $1.473*10^{-6} \ll 10^{-2}$

$$\tan(10\pi l) = -0.414 \rightarrow l_{\min} = 0.0125m$$

Note

If the formula $Z_w(z) = Z_0 \frac{Z_L - jZ_0 \tan(\beta z)}{Z_0 - jZ_L \tan(\beta z)}$ with negative sign is used then z is a positive number and show the length.

For the original formula $Z_w(z) = Z_0 \frac{Z_L + jZ_0 \tan(\beta z)}{Z_0 + jZ_L \tan(\beta z)}$ with positive sign z is the point with reference to the z=0 mostly at load end. Therefore z can be negative.