Tutorial 1

Problem 1: Within a certain region, $\varepsilon = 10^{-11}$ F/m and $\mu = 10^{-5}$ H/m. If $B_x = 2\times10^{-4}\cos10^5 t\sin10^{-3}y$ T:

(a) use $\nabla\times\mathbf{H} = \varepsilon\frac{\partial\mathbf{E}}{\partial t}$ to find $\mathbf{E}$;

(b) find the total magnetic flux passing through the surface $x = 0$, $0 < y < 40$ m, $0 < z < 2$ m, at $t = 1\mu s$;

(c) find the value of the closed line integral of $\mathbf{E}$ around the perimeter of the given surface.

Solution:

(a) $H_x = \frac{B_x}{\mu} = \frac{2\times10^{-4}\cos10^5 t\sin10^{-3}y}{10^{-5}} = 20\cos10^5 t\sin10^{-3}y \quad A/m$

$$\nabla\times\mathbf{H} = -\frac{\partial H_x}{\partial y}\vec{\mathbf{z}} + \frac{\partial H_x}{\partial z}\vec{\mathbf{y}} = -0.02\cos10^5 t\cos10^{-3}y\,\vec{\mathbf{z}} \quad A/m^2$$

$$\frac{\partial\mathbf{E}}{\partial t} = \frac{\nabla\times\mathbf{H}}{\varepsilon} = \frac{-0.02\cos10^5 t\cos10^{-3}y\vec{\mathbf{z}}}{10^{-11}} = -2\times10^9\cos10^5 t\cos10^{-3}y\vec{\mathbf{z}} \quad V/ms$$

$$\mathbf{E} = \int\frac{\partial\mathbf{E}}{\partial t}dt = -2\times10^4\sin10^5 t\cos10^{-3}y\vec{\mathbf{z}} \quad V/m$$

(b) We assume that the integral surface direction (direction of $d\mathbf{S}$)is $+\vec{x}$, same as the flux density.

$$\Phi = \iint_S \mathbf{B}\cdot d\mathbf{S} = \int_0^{40}\int_0^2 B_x dzdy = \int_0^2 dz\int_0^{40} 2\times10^{-4}\cos10^5 t\sin10^{-3}ydy$$

$$= 2\times2\times10^{-4}\cos10^5 t\times\left(-\frac{\cos10^{-3}y}{10^{-3}}\right)\Bigg|_0^{40} = 3.1996\times10^{-4}\cos10^5 t \quad Wb$$

$$\Phi(t = 1\mu s) = 3.1996\times10^{-4}\cos(10^5\times10^{-6}) = 3.18\times10^{-4} \quad Wb$$

(c) The line integral is taken along the counter clockwise direction (right hand rule based on the surface direction).

$$\oint_L \mathbf{E}\cdot d\mathbf{l} = \int_0^2 E_z(y = 40)dz + \int_2^0 E_z(y = 0)dz$$

$$= \int_0^2 (-2\times10^4\sin10^5 t\cos(0.04) + 2\times10^4\sin10^5 t)dz$$

$$= 2\times2\times10^4\sin10^5 t(-\cos(0.04) + 1)$$

$$= 2\times2\times10^4\sin(10^5\times10^{-6})\times(1 - \cos(0.04)) = 3.19 \quad V$$

Or

$$\oint_L \mathbf{E}\cdot d\mathbf{l} = emf = -\frac{d\Phi}{dt}$$

$$= 3.1996\times10^{-4}\times10^{5}\times\sin(10^{5}t) = 3.1996\times10\times\sin(10^{5}\times10^{-6}) = 3.19 \qquad V$$

Note that in the case of electrostatics, line integral about the closed loop is zero. However, in the case of time-vary field, the system is not conservative due to the presence of external magnetic source.

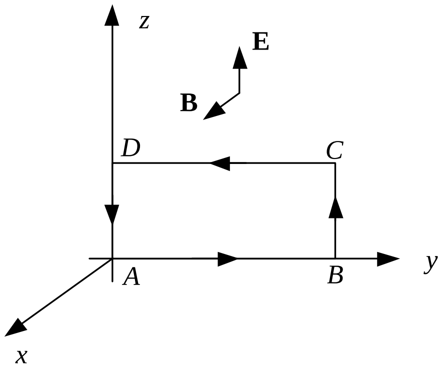

Figure 1

Problem 2 With reference to the sliding bar shown in Figure 2, let d=7cm, **B**=0.3 $\vec{\mathbf{z}}$ T, and **v**=0.1 $\vec{\mathbf{y}}$ e^{20y} m/s. Let y=0 at t=0. Find

(a) $v(t=0)$;

(b) $y(t=0.1)$;

(c) $v(t=0.1)$;

(d) V_{12} at t=0.1

Solution:

(a) $v(t=0) = 0.1e^{20\times0} = 0.1 \qquad {}^{m}\!/_{s}$

(b)

$$v = 0.1e^{20y} = \frac{dy}{dt} \Rightarrow e^{-20y}dy = 0.1dt \Rightarrow \int_0^y e^{-20y}dy = \int_0^t 0.1dt \Rightarrow \left.\frac{e^{-20y}}{-20}\right|_0^y = 0.1t\Big|_0^t$$

$$\Rightarrow e^{-20y} - 1 = -2t \Rightarrow y = -\frac{1}{20}\ln(1-2t)$$

$$y(t=0.1) = -0.05\times\ln(1-2\times0.1) = 0.0112m$$

(c) $v = \dfrac{dy}{dt} = -\dfrac{1}{20}\dfrac{d}{dt}\ln(1-2t) = \dfrac{1}{10(1-2t)}$

$$v(t=0.1)=\frac{1}{10(1-2\times 0.1)}=0.125 \quad m/s$$

(d) $V_{12}=emf=-\frac{d\Phi}{dt}=-B_0\frac{dy}{dt}d=-B_0vd=-0.3\times 0.125\times 0.07=-0.002625V$

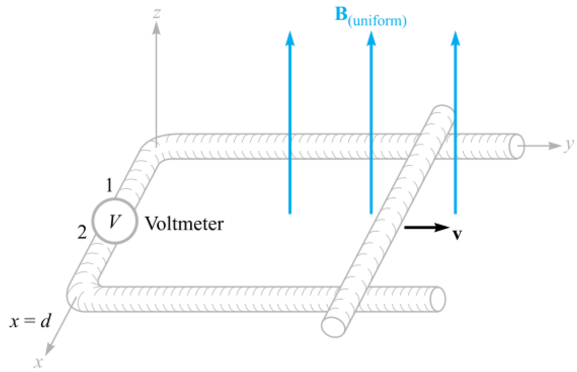

Figure 9.1 An example illustrating the application of Faraday's law to the case of a constant magnetic flux density B and a moving path. The shorting bar moves to the right with a velocity v, and the circuit is completed through the two rails and an extremely small high-resistance voltmeter. The voltmeter reading is $V_{12}=-Bvd$.

Figure 2

Problem 3 A rectangular loop of wire containing a high-resistance voltmeter has corners initially at (a/2,b/2,0), (-a/2,b/2,0), (-a/2,-b/2,0), and (a/2,-b/2,0). The loop begins to rotate about the x axis at constant angular velocity ω, with the first-named corner moving in the $+\vec{\mathbf{z}}$ direction at t=0. Assume a uniform magnetic flux density $\mathbf{B}=B_0\vec{\mathbf{z}}$. Determine the induced emf in the rotating loop and specify the direction of the current.

Solution:

Assume the direction of the loop wire's surface is $+\vec{\mathbf{z}}$ at t=0, then the angle between surface direction vector and the flux density should be $\theta=\omega t$, then

$$\Phi=\int_S \mathbf{B}\cdot d\mathbf{S}=B_0 S\cos(\theta)=B_0 ab\cos(\omega t)\Rightarrow \quad emf=-\frac{d\Phi}{dt}=B_0 ab\omega\sin(\omega t)\quad \text{V}$$

Decide the direction of current with Lenz's law: if an induced current flows, its direction is always such that it will oppose the change which produced it. In Figure 4:

1) When θ changes from 0 to $\pi/2$, original flux through the loop decreases, this change of flux will generate a current, and the current will generate a flux to oppose the decreasing, so the induced flux must have the +z direction component. From the right hand rule, the current should be A->B->C->D.
2) When θ changes from $\pi/2$ to π, original flux through the loop increases, this change of flux will generate a current, and the current will generate a flux to oppose the increasing, so the induced flux must have the -z direction component. From the right hand rule, the current should be A->B->C->D.

3) When θ changes from π to $3\pi/2$, original flux through the loop decreases, this change of flux will generate a current, and the current will generate a flux to oppose the decreasing, so the induced flux must have the +z component. From the right hand rule, the current should be A->D->C->B.
4) When θ changes from $3\pi/2$ to π, original flux through the loop increases, this change of flux will generate a current, and the current will generate a flux to oppose the increasing, so the induced flux must have the -z component. From the right hand rule, the current should be A->D->C->B.

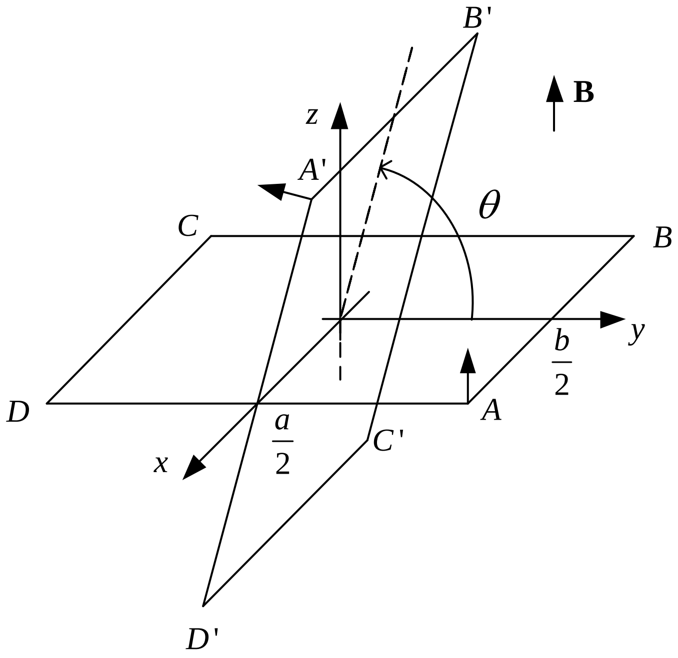

Figure 3

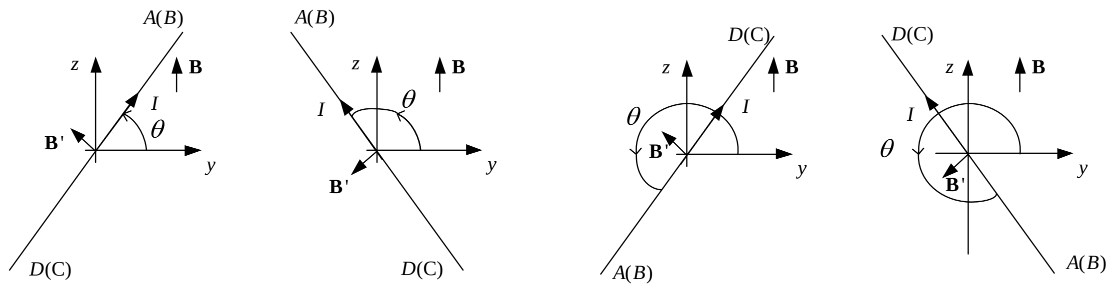

Figure 4