Tutorial 6

Problem 1. Slotted line measurements yield a VSWR of 5, a 15-cm spacing between successive voltage maxima, and the first maximum at a distance of 7.5cm in front of the load. Determine the load impedance, assuming a 50-Ω impedance for the slotted line.

Solution:

Assume $\Gamma = |\Gamma| e^{j\phi}$

$$V_s(z) = (V_0^+ e^{-j\beta z} + \Gamma V_0^+ e^{j\beta z}) = V_0^+ e^{-j\beta z}(1 + |\Gamma| e^{j(2\beta z + \phi)})$$

$$V_{\max} = 1 + |\Gamma|,\ 2\beta z + \phi = -2m\pi \Rightarrow z_{\max} = -\frac{\phi}{4\pi}\lambda - \frac{m}{2}\lambda, m = 0,1,2\cdots$$

$$V_{\min} = |1 - |\Gamma||,\ 2\beta z + \phi = \pi - 2m\pi \Rightarrow z_{\min} = -\frac{\phi}{4\pi}\lambda + \frac{\lambda}{4} - \frac{m}{2}\lambda, m = 0,1,2\cdots$$

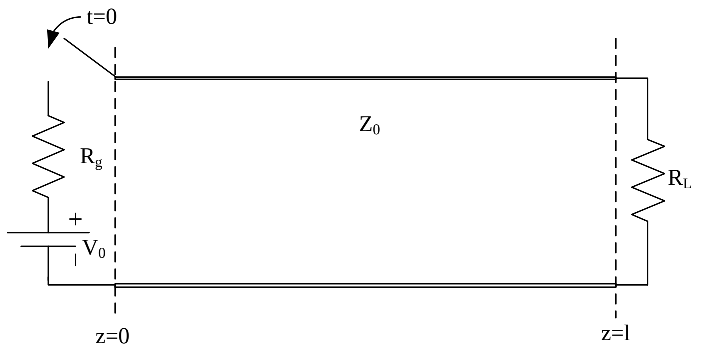
$$s = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \Rightarrow |\Gamma| = \frac{s-1}{s+1} = \frac{2}{3}$$

$$\Delta z_{\max} = z_{m,\max} - z_{m+1,\max} = \frac{\lambda}{2} = 15\text{cm} \Rightarrow \lambda = 30\text{cm}$$

$$z_{1,\max} = -\frac{\phi}{4\pi}\lambda = -7.5\text{cm} \Rightarrow \phi = \pi \qquad \Gamma = |\Gamma| e^{j\phi} = -\frac{2}{3}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow Z_L = 10\Omega$$

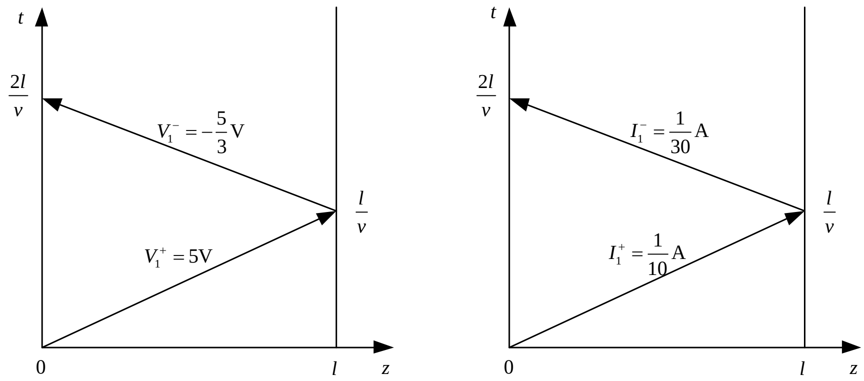
Problem 2. In the figure below, $R_g = Z_0 = 50\Omega$, $R_L = 25\Omega$, and the battery voltage is $V_0 = 10V$. The switch is closed at time t=0. Determine the voltage at the load resistor and the current in the battery as functions of time.



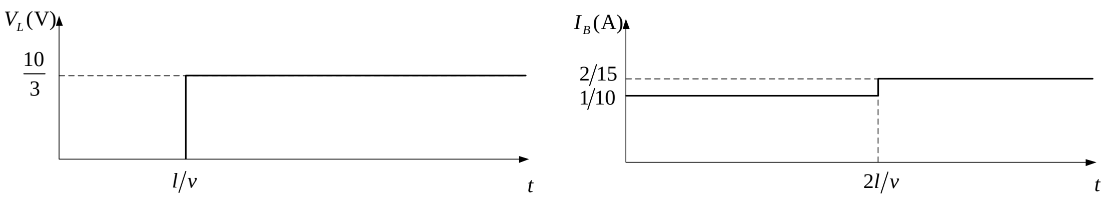
Solution:

At t=0 and z=0, $V_1^+ = \dfrac{V_0 Z_0}{Z_0 + Z_g} = 5\text{V}$. After $t = l/v$, the wave reaches R_L. Because the reflection coefficient is $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = -\dfrac{1}{3}$, the reflected voltage: $V_1^- = \Gamma_L V_1^+ = -\dfrac{5}{3}\text{V}$. This wave returns to the battery when $t = 2l/v$, and then it encounters another reflection with reflection coefficient $\Gamma_g = \dfrac{Z_g - Z_0}{Z_g + Z_0} = 0$. Thus, no further waves appear; steady state is reached.

For the currents, $I_1^+ = \dfrac{V_1^+}{Z_0} = \dfrac{1}{10}\text{A}$, and $I_1^- = -\dfrac{V_1^-}{Z_0} = \dfrac{1}{30}\text{A}$. The voltage and current reflection diagram can be constructed as below:



The voltage at the load as a function of time is now found by summing the voltages along the vertical line at the load position. And the same way to achieve the current in the battery.



Problem 3. In the transmission line system as shown in the same figure as in problem 1, Z_0=50Ω, and R_L=R_g=25Ω. The switch is closed at t=0 and is opened again at time $t = l/4v$, thus creating a rectangular voltage pulse in the line. Construct an appropriate voltage reflection diagram for this case and use it to make a plot of the voltage at the load resistor as a function of time for $0 < t < 8l/v$ (note that the effect of opening the switch is to initiate a second voltage wave, whose value is such that it leaves a net current of zero in its wake).

Solution:

At t=0 and z=0, the switch is closed and the first wave is generated with $V_1^+ = \dfrac{V_0 Z_0}{Z_0 + R_g} = \dfrac{2}{3}V_0$. After

$t = l/v$, the wave reaches R_L and is reflected with reflection coefficient: $\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = -\frac{1}{3}$, so the reflected voltage $V_1^- = \Gamma_L V_1^+ = -\frac{2}{9}V_0$. This wave returns to the battery at $t = 2l/v$, at which time the switch is opened again, so the reflection coefficient encountered is $\Gamma_g = 1$ due to open circuit and $V_2^+ = V_1^- = -\frac{2}{9}V_0$. As it proceeds, the voltages after successive reflections before $8l/v$ are

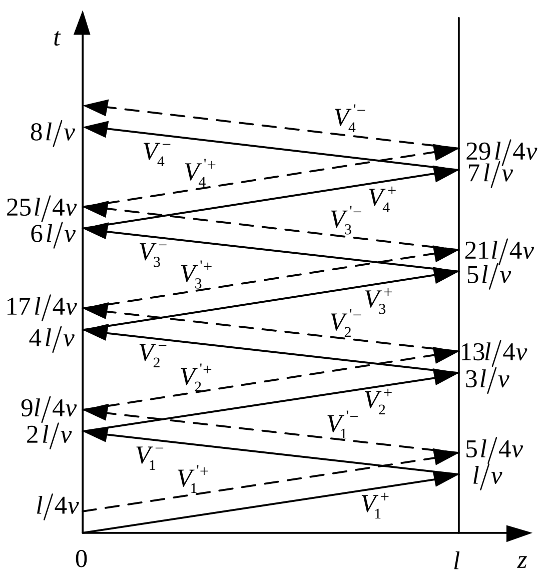
$V_2^- = \Gamma_L V_2^+ = \frac{2}{27}V_0 = V_3^+, V_3^- = \Gamma_L V_3^+ = -\frac{2}{81}V_0 = V_4^+, V_4^- = \Gamma_L V_4^+ = \frac{2}{243}V_0$.

At $t = l/4v$, the switch is opened again, and the effect of opening the switch is to initiate a second voltage wave, whose value is such that it leaves a net current of zero in its wake (no current at the left end of the transmission line). Assuming the second wave's voltage is $V_1^{'+}$, and it is forwardly propagating, so the current associated is $\frac{V_1^{'+}}{Z_0}$. At such time, the first wave V_1^+ still exists and the current associated is $\frac{V_1^+}{Z_0}$. As the net current is the summation of the two currents from first and second wave, and should be 0, so $\frac{V_1^{'+}}{Z_0} + \frac{V_1^+}{Z_0} = 0 \Rightarrow V_1^{'+} = -V_1^+ = -\frac{2}{3}V_0$.

After l/v, which makes $t = 5l/4v$, the second wave reaches R_L, suffering the same coefficient Γ_L, leading to a reflected voltage $V_1^{'-} = \Gamma_L V_1^{'+} = -\Gamma_L V_1^+ = -V_1^- = \frac{2}{9}V_0$. Following the same procedures as for the first wave, we can find the reflected voltages generated by the second wave which are sharing the same values as the ones generated by the first wave but with opposite signs as $V_2^{'+} = -V_2^+ = \frac{2}{9}V_0$,

$V_2^{'-} = -V_2^- = -\frac{2}{27}V_0, V_3^{'+} = -V_3^+ = -\frac{2}{27}V, V_3^{'-} = -V_3^- = \frac{2}{81}V_0, V_4^{'+} = -V_4^+ = \frac{2}{81}V_0, V_4^{'-} = -V_4^- - \frac{2}{243}V_0$.

With the aforementioned voltages, we can construct the voltage reflection diagram as below



So at the load position, summing the voltage values along the vertical line, we can get the voltages for load at different time:

$$t < l/v, V_L = 0$$

$$l/v < t < 5l/4v, V_L = V_1^+ + V_1^- = \frac{4}{9}V_0$$

$$5l/4v < t < 3l/v, V_L = V_1^+ + V_1^- + V_1^{'+} + V_1^{'-} = 0$$

$$3l/v < t < 13l/4v, V_L = V_1^+ + V_1^- + V_1^{'+} + V_1^{'-} + V_2^+ + V_2^- = -\frac{4}{27}V_0$$

$$13l/4v < t < 5l/v, V_L = V_1^+ + V_1^- + V_1^{'+} + V_1^{'-} + V_2^+ + V_2^- + V_2^{'+} + V_2^{'-} = 0$$

$$5l/v < t < 21l/4v, V_L = V_1^+ + V_1^- + V_1^{'+} + V_1^{'-} + V_2^+ + V_2^- + V_2^{'+} + V_2^{'-} + V_3^+ + V_3^- = \frac{4}{81}V_0$$

$$21l/4v < t < 7l/v, V_L = V_1^+ + V_1^- + V_1^{'+} + V_1^{'-} + V_2^+ + V_2^- + V_2^{'+} + V_2^{'-} + V_3^+ + V_3^- + V_3^{'+} + V_3^{'-} = 0$$

$$7l/v < t < 29l/4v, V_L = V_1^+ + V_1^- + V_1^{'+} + V_1^{'-} + V_2^+ + V_2^- + V_2^{'+} + V_2^{'-} + V_3^+ + V_3^- + V_3^{'+} + V_3^{'-} + V_4^+ + V_4^- = -\frac{4}{243}V_0$$

$$29l/4v < t < 8l/v, V_L = V_1^+ + V_1^- + V_1^{'+} + V_1^{'-} + V_2^+ + V_2^- + V_2^{'+} + V_2^{'-} + V_3^+ + V_3^- + V_3^{'+} + V_3^{'-} + V_4^+ + V_4^- + V_4^{'+} + V_4^{'-} = 0$$

Represent the above results in the voltage-time plot and the result is shown as below.