Tutorial 3

Problem 1: The electric field amplitude of a uniform plane wave propagating in the $\vec{\mathbf{z}}$ direction in the free-space is 250V/m. If $\mathbf{E} = E_x \vec{\mathbf{x}}$ and ω =1.00Mrad/s, find:

(a) The frequency;
(b) The wavelength;
(c) The period
(d) The wavenumber
(e) The impedance
(f) The amplitude of H_y
(g) The real instantaneous expression of H_y
(h) Repeat (a)~(g) for the same EM wave in glass with refractive index 1.5.

Solutions:

(a) $\omega = 2\pi f \Rightarrow f = \dfrac{\omega}{2\pi} = 1.59\times10^5 Hz = 159kHz$

(b) $f\lambda_0 = c \Rightarrow \lambda_0 = \dfrac{c}{f} = 1.88\times10^3 m = 1.88km$

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3\times10^8 \, m/s$$

(c) $T = \dfrac{1}{f} = 6.28\times10^{-6} s = 6.28\mu s$

(d) $\beta_0 = \dfrac{2\pi}{\lambda_0} = \dfrac{\omega}{c} = 0.033 rad\,/\,\mathrm{m}$

(e) $\eta_0 = \dfrac{E_x}{H_y} = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = \sqrt{\dfrac{4\pi\times10^{-7}}{8.857\times10^{-12}}} = 376.7\Omega$

(f) $\eta_0 = \dfrac{E_x}{H_y} \Rightarrow \hat{H}_y = \dfrac{\hat{E}_x}{\eta_0} = 0.664 \, A/m$

(g) $H_y = 0.664\cos(10^6 t - 0.033z) \, A/m$

(h) $f = 159kHz$, $T = 6.28\mu s$ (not change)

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n} = 2\times10^8 \, m/s$$

$$f\lambda_m = v \Rightarrow \lambda_m = \frac{v}{f} = \frac{\lambda_0}{n} = 1.26\times 10^3 m = 1.26km$$

$$\beta_m = \frac{2\pi}{\lambda_m} = \frac{\omega}{v} = n\beta_0 = 0.05rad / \mathrm{m}$$

$$\eta_m = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0 n^2}} = \frac{\eta_0}{n} = 251.1\Omega$$

$$\hat{H}_y = \frac{\hat{E}_x}{\eta_m} = 0.996\, A/m$$

$$H_y = 0.996\cos(10^6 t - 0.05z)\, A/m$$