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Tutorial 4

1) Express $E_y(z,t) = 100\cos(10^8 t - 0.5z + 30^\circ)$ V/m as a phasor.

Remember: If $E_y(z,t) = \mathrm{Re}[E_0 e^{j\omega t} e^{-jk_0 z} e^{j\varphi_1}] \Rightarrow$ phasor expression = $E_{ys}(z) = E_0 e^{-jk_0 z} e^{j\varphi_1}$

$$E_y(z,t) = \mathrm{Re}(100e^{j(10^8 t - 0.5z + 30^\circ)}) \Rightarrow E_{ys}(z) = 100e^{-j0.5z + j30^\circ}$$

2) Given the complex amplitude of the electric field of a uniform plane wave, $\mathbf{E}_0 = 100\mathbf{a}_x + 20\angle 30^\circ\,\mathbf{a}_y$ V/m, construct the phasor and real instantaneous fields if the wave is known to propagate in the forward z direction in free space and has frequency of 10 MHz.

$\mathbf{E}_s(z) = [100\,\mathbf{a}_x + 20e^{j30^\circ}\,\mathbf{a}_y]\,e^{-jk0z}$

$k_0 = \omega/c = 2\pi \times 10 \times 10^6 / 3 \times 10^8 = 0.21$ rad/m

$\mathbf{E}(z,t) = \mathrm{Re}\,[\,100\,e^{-j0.21z} e^{j2\pi\times 10^7 t}\,\mathbf{a}_x + 20\,e^{-j30^\circ} e^{-j0.21z} e^{j2\pi\times 10^7 t}\,\mathbf{a}_y]$

$= \mathrm{Re}\,[100\,e^{j(2\pi\times 10^7 t - 0.21z)}\,\mathbf{a}_x + 20\,e^{j(2\pi\times 10^7 t - 0.21z + 30^\circ)}\,\mathbf{a}_y]$

$= 100\cos(2\pi\times 10^7 t - 0.21z)\,\mathbf{a}_x + 20\cos(2\pi\times 10^7 t - 0.21z + 30^\circ)\,\mathbf{a}_y$

3) Let $\mu = 3\times 10^{-5}$ H/m, $\varepsilon = 1.2\times 10^{-10}$ F/m, and $\sigma = 0$ everywhere. If $\mathbf{H} = 2\cos(10^{10}t - \beta x)\mathbf{a}_z$ A/m, use Maxwell's equations to obtain expressions for **B**, **D**, **E** and β.

$\mathbf{B} = \mu\,\mathbf{H} = 6\times 10^{-5}\cos(10^{10}t - \beta x)\mathbf{a}_z$ T

$\beta = \omega\sqrt{\mu\varepsilon}$

$= 10^{10}\sqrt{3\times 10^{-5}\times 1.2\times 10^{-10}} = 600$

Another method for calculation of β can be from $\beta = \frac{2\pi}{\lambda_m} = \frac{2\pi\sqrt{\mu_r \varepsilon_r}}{\lambda_0}$,

$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial \boldsymbol{t}}$, σ = 0 ⇒ **J** = σ**E** = 0 ⇒ $\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial \boldsymbol{t}} \Rightarrow \frac{\partial H_z}{\partial x}\vec{a}_y = \frac{\partial \vec{D}}{\partial t}$

$2\beta \sin(10^{10}t - \beta x)\vec{a}_y = \frac{\partial \vec{D}}{\partial t} \Rightarrow \int 2\beta \sin(10^{10}t - \beta x) dt \vec{a}_y = \vec{D}$

D = $\vec{D} = -2\times 10^{-10}\beta \cos(10^{10}t - \beta x)\vec{a}_y$ C/m² = $-12\times 10^{-8}\cos(10^{10}t - 1.34\times 10^{-3}x)\vec{a}_y$ C/m²

E = **D**/ε = $-2\times 10^{-10}\beta\cos(10^{10}t - \beta x)\vec{a}_y / (1.2\times 10^{-10}) = -0.6\beta\cos(10^{10}t - \beta x)\vec{a}_y$ V/m =

$-3600\cos(10^{10}t - 1.34\times 10^{-3}x)\vec{a}_y$ V/m

4) A 150 MHz uniform plane wave in free space is described by **H**s = (4 + j10)(2$\mathbf{a}_x$ + j$\mathbf{a}_y$)$e^{-j\beta z}$ A/m.(a) find numerical values for *ω*, *λ*, and *β*. Find *H(z,t)* at *t* = 1.5 ns, *z* = 20 cm. (c) what is $|E|_{\max}$?

(a) $\omega = 2\pi f = 2\pi \times 150\times 10^6 = 9.42\times 10^8$ rad/s

$\lambda = c/f = 3\times 10^8 / 150\times 10^6 = 2$ m

$\beta = 2\pi/\lambda = 3.14$ rad/m

(b) **H**s = (4 + j10)(2$\mathbf{a}_x$ + j$\mathbf{a}_y$)$e^{-j\beta z}$ A/m =

$12.7e^{j68.2^\circ}(2\vec{a}_x + e^{j90^\circ}\vec{a}_y)e^{-j\beta z} = (25.4e^{j68.2^\circ}\vec{a}_x + 12.7e^{j158.2^\circ}\vec{a}_y)e^{-j\beta z}$

***H**(z,t)* = Re [$25.4 e^{j68.2^\circ} e^{-j3.14z} e^{j9.42\times 10^8 t}$ $\mathbf{a}_x$ + 12.7 $e^{j158.2^\circ} e^{-j3.14z} e^{j9.42\times 10^8 t}$ $\mathbf{a}_y$]

***H**(z,t)* = Re [$25.4 e^{j(9.42\times 10^8 t - 3.14z + 68.2^\circ)}$ $\mathbf{a}_x$ + 12.7 $e^{j(9.42\times 10^8 t - 3.14z + 158.2^\circ)}$ $\mathbf{a}_y$]

$= 25.4\cos(9.42\times 10^8 t - 3.14z + 68.2^\circ)$ $\mathbf{a}_x$ $+ 12.7\cos(9.42\times 10^8 t - 3.14z + 158.2^\circ)$ $\mathbf{a}_y$ A/m at t = 1.5 ns, z = 20 cm

H*(20cm,1.5ns)*= 25.4 cos(68.98) $\mathbf{a}_x$ +12.7 cos(158.98) $\mathbf{a}_y$ = 9.11 $\mathbf{a}_x$ -11.85 $\mathbf{a}_y$ A/m

(c)

Using

$$\boldsymbol{H}(z,t) = \frac{1}{\eta}\hat{\mathbf{z}} \times \boldsymbol{E}(z,t) \quad \Rightarrow \quad \boldsymbol{E}(z,t) = \eta \boldsymbol{H}(z,t) \times \hat{\mathbf{z}}$$

H*(20cm,1.5ns)*= 9.11 $\mathbf{a}_x$ -11.85 $\mathbf{a}_y$ A/m

E*(20cm,1.5ns)*=η_0(9.11 $\mathbf{a}_x \times \mathbf{a}_z$ -11.85 $\mathbf{a}_y \times \mathbf{a}_z$) V/m

E*(20cm,1.5ns)*=120π(9.11 (-$\mathbf{a}_y$) -11.85 $\mathbf{a}_x$) V/m

$$\begin{aligned} E_y &= -120\pi \times (9.11) = -343.4 \\ E_x &= 120\pi \times (-11.85) = -446.7 \end{aligned} \quad \text{V/m}$$

$$|E|_{\max} = \sqrt{E_x^2 + E_y^2} = 563.4 \text{ V/m}$$