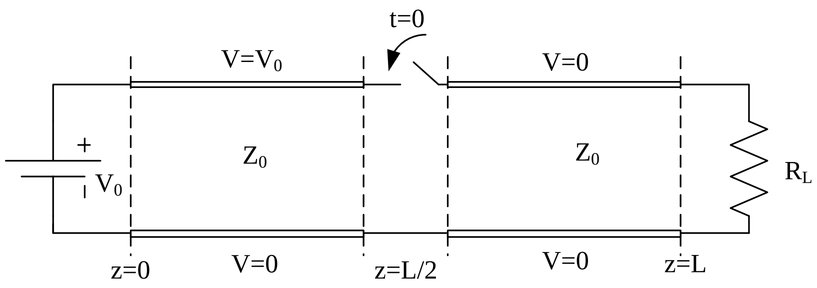
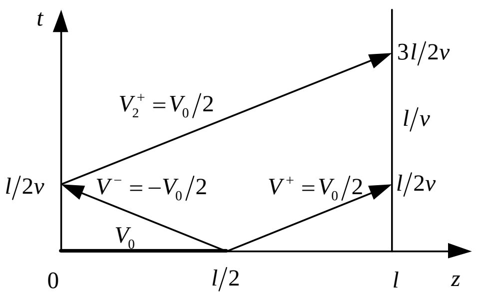
Problem 1. In the transmission line of the figure below, the switch is located midway down the line and is closed at t=0. Construct a voltage reflection diagram for this case, where $R_L=Z_0$. Plot the load resistor voltage as a function of time.



Solution:

At the mid-point, two waves will be excited – one propagates to the right (forward) and the other to the left (backward). Due to the existence of these two waves, the voltage will be continuous at the middle point, and if we view the connected switch as a node and apply the KCL law, the currents flowing in equals the current flowing out, so $V_0+V_1^-=V_1^+, 0=\frac{V_1^-}{Z_0}+\frac{V_1^+}{Z_0}\Rightarrow V_1^+=\frac{V_0}{2}, V_1^-=-\frac{V_0}{2}$.

After $t=l/2v$, forward wave will reach R_L, and because the reflection coefficient is 0 (impedance matched), no reflected wave appears. At the same time, the backward wave will reach the source, and because the reflection coefficient is -1 (source impedance is 0), the reflected wave's voltage is $V_2^+=-V_1^-$, which will be totally absorbed after another l/v. The corresponding voltage reflection diagram is as below:



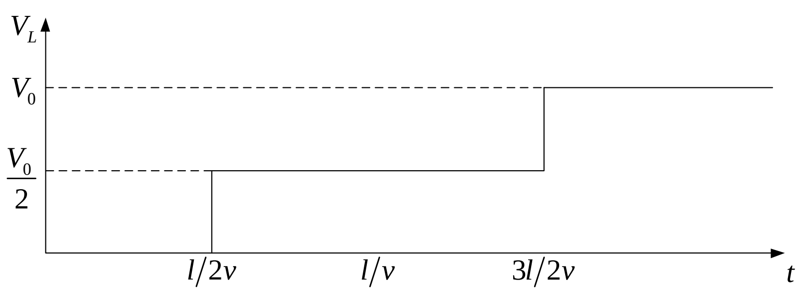
Summing the voltage value at $z=l$, we can find V_L as a function in terms of t, which is

$0<t<l/2v,\ V_L=0$

$l/2v<t<3l/2v,\ V_L=V_0/2$

$t>3l/2v,\ V_L=V_0$

and the load voltage plot should be:



Problem 2. In a medium characterized by intrinsic impedance $\eta = |\eta| e^{j\phi}$, a linearly polarized plane wave propagates, with magnetic field given as $\mathbf{H}_s = (H_{0y}\vec{\mathbf{y}} + H_{0z}\vec{\mathbf{z}})e^{-\alpha x}e^{-j\beta x}$. Find

(a) $\mathbf{E}_s$

(b) $\mathbf{E}(x,t)$

(c) $\mathbf{H}(x,t)$

Solution:

(a) $\mathbf{H}_s = (H_{0y}\vec{\mathbf{y}} + H_{0z}\vec{\mathbf{z}})e^{-\alpha x}e^{-j\beta x} = (H_{0y}\vec{\mathbf{y}} + H_{0z}\vec{\mathbf{z}})e^{-j(\beta - j\alpha)x} \Rightarrow$

$$k = \beta - j\alpha, H_{ys} = H_{0y}e^{-jkx}, H_{zs} = H_{0z}e^{-jkx}$$

$$\nabla \times \mathbf{H}_s = j\omega\varepsilon \mathbf{E}_s$$

Proof:

$$\begin{aligned}
&\mathbf{E} = \mathrm{Re}\{\mathbf{E}_s e^{j\omega t}\}, \mathbf{H} = \mathrm{Re}\{\mathbf{H}_s e^{j\omega t}\} \\
&\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} \Rightarrow \mathrm{Re}\{\nabla \times \mathbf{H}_s e^{j\omega t}\} = \mathrm{Re}\{\varepsilon \frac{\partial \mathbf{E}_s e^{j\omega t}}{\partial t}\} \\
&\Rightarrow \mathrm{Re}\{\nabla \times \mathbf{H}_s e^{j\omega t}\} = \mathrm{Re}\{ j\omega\varepsilon \mathbf{E}_s e^{j\omega t}\} \\
&\Rightarrow \mathrm{Re}\{(\nabla \times \mathbf{H}_s - j\omega\varepsilon \mathbf{E}_s) e^{j\omega t}\} = 0 \\
&\Rightarrow \nabla \times \mathbf{H}_s = j\omega\varepsilon \mathbf{E}_s
\end{aligned}$$

$$\nabla \times \mathbf{H}_s = -\frac{\partial H_{zs}}{\partial x}\hat{\mathbf{y}} + \frac{\partial H_{ys}}{\partial x}\hat{\mathbf{z}} = -jk(-H_{0z}\vec{\mathbf{y}} + H_{0y}\vec{\mathbf{z}})e^{-jkx}$$

$$\Rightarrow \mathbf{E}_s = \frac{\nabla \times \mathbf{H}_s}{j\omega\varepsilon} = \frac{-k}{\omega\varepsilon}(-H_{0z}\vec{\mathbf{y}} + H_{0y}\vec{\mathbf{z}})e^{-jkx}$$

$$k = \frac{\omega}{v}, v = \frac{1}{\sqrt{\mu\varepsilon}} \Rightarrow \frac{k}{\omega\varepsilon} = \frac{1}{v\varepsilon} = \frac{\sqrt{\mu\varepsilon}}{\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$$

$$\Rightarrow \mathbf{E}_s = -\eta(-H_{0z}\vec{\mathbf{y}} + H_{0y}\vec{\mathbf{z}})e^{-jkx} = |\eta|(H_{0z}\vec{\mathbf{y}} - H_{0y}\vec{\mathbf{z}})e^{-\alpha x - j\beta x + j\phi}$$

(b)

$$\begin{aligned}
\mathbf{E}(x,t) &= \mathrm{Re}\{\mathbf{E}_s e^{j\omega t}\} = \mathrm{Re}\{|\eta|(H_{0z}\vec{\mathbf{y}} - H_{0y}\vec{\mathbf{z}})e^{-\alpha x - j\beta x + j\phi + j\omega t}\} \\
&= |\eta|(H_{0z}\vec{\mathbf{y}} - H_{0y}\vec{\mathbf{z}})e^{-\alpha x}\cos(\omega t - \beta x + \phi)
\end{aligned}$$

(c)

$$\mathbf{H}(x,t) = \mathrm{Re}\{\mathbf{H}_s e^{j\omega t}\} = \mathrm{Re}\{(H_{0y}\vec{\mathbf{y}} + H_{0z}\vec{\mathbf{z}})e^{-\alpha x - j\beta x + j\omega t}\}$$
$$= (H_{0y}\vec{\mathbf{y}} + H_{0z}\vec{\mathbf{z}})e^{-\alpha x}\cos(\omega t - \beta x)$$

Problem 3. Let $jk = 0.2 + j1.5 m^{-1}$ and $\eta = 450 + j60\Omega$ for a uniform plane propagating in the $\hat{\mathbf{z}}$ direction. If $\omega = 300\, Mrad/s$, find (a) μ (b) ε' (c) ε'' for the medium.

Solution:

$$\varepsilon = \varepsilon' - j\varepsilon'' \quad \Rightarrow k = \frac{\omega}{v} = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')}$$

$$jk = 0.2 + j1.5 \Rightarrow k = -0.2j + 1.5$$

$$\Rightarrow \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = -0.2j + 1.5$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon' - j\varepsilon''}} = 450 + j60$$

$$\mu = \sqrt{\mu(\varepsilon' - j\varepsilon'')\frac{\mu}{\varepsilon' - j\varepsilon''}} = \frac{-0.2j + 1.5}{\omega}(450 + j60) = 2.29 \times 10^{-6}\, H/m$$

$$\varepsilon' - j\varepsilon'' = \frac{(-0.2j + 1.5)^2}{\mu\omega^2} = 1.072 \times 10^{-11} - 2.911 \times 10^{-12}\, j\, F/m$$

$$\Rightarrow \varepsilon' = 1.072 \times 10^{-11}\, F/m, \varepsilon'' = 2.911 \times 10^{-12}\, F/m$$