Problem 1. A certain nonmagnetic material has the material constants $\varepsilon_r' = 2$ and $\varepsilon''/\varepsilon' = 4\times10^{-4}$ at $\omega = 1.5\, Grad/s$. Find the distance a uniform plane wave can propagate through the material before

(a) It is attenuated by 1Np;

(b) the phase shift 360º

Solution

(a) $\varepsilon_r'' = 4\times10^{-4}\varepsilon_r' = 8\times10^{-4}$

$$k = \frac{\omega}{v} = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0(\varepsilon_r' - j\varepsilon_r'')} = \frac{\omega}{c}\sqrt{(\varepsilon_r' - j\varepsilon_r'')}$$
$$= \frac{1.5\times10^9}{3\times10^8}\sqrt{(2-8\times10^{-4}j)} = 7.0711 - 1.4142\times10^{-3}j\, rad/m$$
$$= \beta - j\alpha$$

$$\alpha L = 1.4142\times10^{-3}L = 1 \Rightarrow L = 707.1m$$

(b) $\beta L = 7.0711L = 2\pi \Rightarrow L = 0.8159m$

Problem 2. A 10GHz radar signal may be represented as a uniform plane wave in a sufficiently small region. Calculate the wavelength in centimeters and the attenuation in nepers per meter if the wave is propagation in a nonmagnetic material for which

(a) $\varepsilon_r' = 1$ and ε_r'' =0

(b) $\varepsilon_r' = 1.04$ and $\varepsilon_r'' = 9\times10^{-4}$

(c) $\varepsilon_r' = 2.5$ and ε_r'' =7.2

Solution

(a)

$$k = \frac{\omega}{v} = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0(\varepsilon_r' - j\varepsilon_r'')} = \frac{2\pi f}{c}\sqrt{(\varepsilon_r' - j\varepsilon_r'')}$$
$$= \frac{2\times3.14\times10\times10^9}{3\times10^8}\sqrt{(1-j0)} = 209.4395\, rad/m$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = 0.03m = 3cm \quad , \quad \alpha = 0$$

(b)

$$k = \frac{2\pi f}{c}\sqrt{(\varepsilon_r' - j\varepsilon_r'')} = \frac{2\pi\times10\times10^9}{3\times10^8}\sqrt{(1.04 - j9\times10^{-4})}$$
$$= \frac{200\pi}{3}(1.0198 - 4.4126\times10^{-4}j)\, rad/m = \beta - j\alpha$$

$$\beta = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{\beta} = 0.0294m = 2.94cm$$

$$\alpha = \frac{200\pi}{3} 4.4126 \times 10^{-4} = 0.092417 Np/m$$

(c)

$$k = \frac{2\pi f}{c}\sqrt{(\varepsilon_r' - j\varepsilon_r'')} = \frac{2\pi \times 10 \times 10^9}{3 \times 10^8}\sqrt{(2.5 - j7.2)}$$
$$= \frac{200\pi}{3}(2.24963 - 1.60026j) \, {}^{rad}\!/_{m} = \beta - j\alpha$$

$$\beta = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{\beta} = 0.00885m = 0.885cm$$

$$\alpha = \frac{200\pi}{3} 1.60026 = 335.16 Np/m$$

Problem 3. Consider an incident wave of 1MHz propagating in the see water with σ=4S/m, and ε_r'=81. Find the skin depth, wavelength, and phase velocity.

Solution We first evaluate the loss tangent, using the given data:

$$\frac{\sigma}{\omega\varepsilon'} = \frac{4}{2\pi \times 10^6 \times 81 \times 8.85 \times 10^{-12}} = 8.9 \times 10^2 >> 1$$

Seawater is therefore a good conductor at 1MHz or frequencies lower than this.

The skin depth is $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 10^6 \times 4\pi \times 10^{-7} \times 4}} = 0.25\text{m}=25\text{cm}$

$$\delta = \frac{1}{\alpha} = \frac{1}{\beta} \Rightarrow \lambda = \frac{2\pi}{\beta} = 2\pi\delta = 1.6\text{m}$$

$$v_p = \frac{\omega}{\beta} = \omega\delta = 1.6 \times 10^6 \text{ m/s}$$