Tutorial 7

Problem 1. A 50-Ω lossless transmission line is terminated by a load impedance $Z_L = 50 - j75\Omega$. If the incident power is 100 mW, find the power dissipated by the load.

Solution: The reflection coefficient is

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j75 - 50}{50 - j75 + 50} = 0.36 - j0.48 = 0.6e^{-j0.93}$$

$$P_t = (1 - |\Gamma|^2)P_{in} = (1 - 0.6^2)100 = 64\text{mW}$$

Problem 2. The characteristics of a certain lossless transmission line are $Z_0 = 50\Omega$ and $\gamma = 0 + j0.2\pi\text{m}^{-1}$ at $f = 60\text{MHz}$

(a) Find L and C for the line;

(b) A load $Z_L = 60 + j80\Omega$ is located at $z = 0$. What is the shortest distance from the load to a point at which $Z_{in} = R_{in} + j0$?

Solution:

(a)

$$Z = R + j\omega L = j\omega L \quad Y = G + j\omega C = j\omega C$$

$$\gamma = \sqrt{ZY} = j\omega\sqrt{LC} \Rightarrow \sqrt{LC} = \frac{\gamma}{j\omega}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{L}{C}}$$

$$L = \sqrt{LC}\cdot\sqrt{\frac{L}{C}} = \frac{\gamma}{j\omega}Z_0 = \frac{0 + j0.2\pi}{j2\pi \times 60 \times 10^6} \times 50 = 8.333 \times 10^{-8}\,\text{H/m} = 83.33\text{nH/m}$$

$$\text{C} = \frac{\sqrt{LC}}{\sqrt{L/C}} = \frac{\gamma}{j\omega Z_0} = \frac{0 + j0.2\pi}{j2\pi \times 60 \times 10^6 \times 50} = 3.333 \times 10^{-11}\,\text{F/m} = 33.33\text{pF/m}$$

(b)

$$Z_{in} = Z_0 \frac{Z_L + jZ_0\tan(\beta z)}{Z_0 + jZ_L\tan(\beta z)} = 50\frac{(60 + j80) + j50\tan(\beta z)}{50 + j(60 + j80)\tan(\beta z)}$$

$$= 50\frac{60 + j(80 + 50\tan(\beta z))}{50 - 80\tan(\beta z) + j60\tan(\beta z)}$$

$$\mathrm{Im}(Z_{in}) = 50\frac{(80+50\tan(\beta z))(50-80\tan(\beta z))-60\times 60\tan(\beta z)}{(50-80\tan(\beta z))^2+(60\tan(\beta z))^2}=0$$

$$\Rightarrow \tan^2(\beta z)+1.875\tan(\beta z)-1=0$$

$$\Rightarrow \tan(\beta z)=\frac{-1.875\pm\sqrt{1.875^2+4}}{2}=0.4332 \text{ or } -2.3082$$

$$\Rightarrow \beta z \pm n\pi = 0.4088 \text{ or -1.1620}$$

$$\Rightarrow z = 0.65 \pm 5n \text{ or -18494} \pm 5n$$

$$z_{\min} = -1.8494\text{m } (z_{\min} < 0)$$

Problem 3. A sinusoidal voltage source drives the series combination of an impedance, $Z_g = 50 - j50\Omega$, and a lossless transmission line of length *L*, shorted at the load end. The line characteristic impedance is 50 Ω, and wavelength λ is measured on the line. (a) Determine, in terms of wavelength, the shortest line length that will result in the voltage source driving a total impedance of 50 Ω. (b) Will other line lengths meet the requirements of part (a)? If so, what are they?

Solution:

$$Z_{tot} = Z_g + Z_{in} = 50 - 50j + Z_{in} = 50 \Rightarrow Z_{in} = 50j\Omega$$

$$Z_L = 0, Z_0 = 50\Omega \Rightarrow$$

$$Z_{in} = Z_0\frac{Z_L\cos(\beta l)+jZ_0\sin(\beta l)}{Z_0\cos(\beta l)+jZ_L\sin(\beta l)} = j50\tan(\beta l) = j50 \Rightarrow \tan(\beta l)=1$$

$$\beta l = \frac{2\pi}{\lambda}l = \frac{\pi}{4}+n\pi, n=0,1,2\cdots \Rightarrow l = \frac{\lambda}{8}+n\frac{\lambda}{2}, n=0,1,2\cdots$$

$$l_{\min} = \frac{\lambda}{8}$$