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Elec Eng 3FK4

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DAY CLASS
DURATION OF EXAMINATION: 2.5 Hours
MCMASTER UNIVERSITY FINAL EXAMINATION

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THIS EXAMINATION PAPER INCLUDES 7 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Aids allowed: Use of Casio FX-991 calculator only is allowed.
No books or papers of any kind.

Answer: Answer all questions.
Answer questions in Answer Booklet/s provided.

Marking: Marks are shown at the end of each question.

This paper must be returned with your answers.

START OF EXAM QUESTIONS

1. Find the amplitude of the displacement current density:
   (a) adjacent to an automobile antenna where the magnetic field intensity of an FM signal is $H_x = 0.25\cos[3.12(3\times10^8 t - y)]\,\mathrm{A/m}$;
   (b) in the air space at a point within a large power distribution transformer where $\mathbf{B} = 1.2\cos[1.257\times10^{-6}(3\times10^8 t - x)]\vec{\mathbf{y}}\,\mathrm{T}$;
   (c) within a large, oil-filled power capacitor where $\varepsilon_r = 5$ and $\mathbf{E} = 0.5\cos[1.257\times10^{-6}(3\times10^8 t - z\sqrt{5})]\vec{\mathbf{x}}\,\mathrm{MV/m}$;
   (d) in a metallic conductor at 60Hz, if $\varepsilon = \varepsilon_0$, $\mu = \mu_0$, $\sigma = 5.8\times10^7\,\mathrm{S/m}$, and conduction current density
   $\mathbf{J} = 2\sin(377t - 117.1z)\vec{\mathbf{x}}\,\mathrm{MA/m^2}$.
   Comment on the magnitudes of conduction current density and displacement current density in this example.

6 marks

2. With reference to the sliding bar shown in Figure 1, let the length of the sliding bar d=7cm, magnetic flux density **B**=0.3 $\vec{\mathbf{z}}$ T, and velocity of the sliding bar **v**=0.1 e^{20y} $\vec{\mathbf{y}}$ m/s. Let y=0 m at t=0 s. Find
   (a) $\mathbf{v}(t=0s)$;
   (b) $y(t=0.1s)$;
   (c) $\mathbf{v}(t=0.1s)$;
   (d) Voltage V_{12} at t=0.1s.

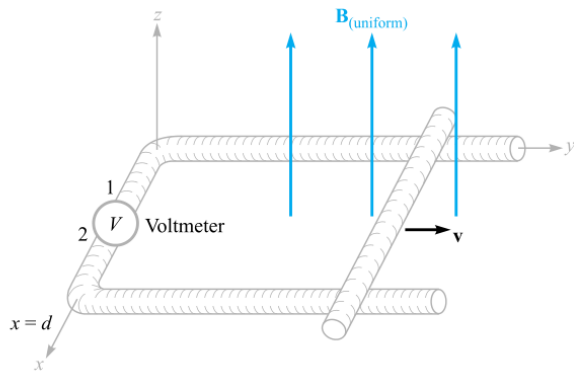

Figure 9.1 An example illustrating the application of Faraday's law to the case of a constant magnetic flux density B and a moving path. The shorting bar moves to the right with a velocity **v**, and the circuit is completed through the two rails and an extremely small high-resistance voltmeter. The voltmeter reading is $V_{12} = -Bvd$.

Fig. 1

4 marks

3. Draw an equivalent circuit of a transmission line and show that

$$\frac{dV_s}{dz} = -(R + j\omega L)I_s \ ,$$

$$\frac{dI_s}{dz} = -(G + j\omega C)V_s$$

5.5 marks

4. Explain the meaning of impedance matching. Suppose you need to connect two dissimilar lines of characteristic impedances Z_{01} and Z_{02}. You need to insert a short transmission line so that there is no reflected wave going towards the source. Find the length and characteristic impedance of the short line. Assume that the line 2 (with the impedance Z_{02}) is connected to a load impedance that is matched to the line 2.

5.5 *marks*

5. In Figure 2, characteristic impedance $Z_0 = 50\ \Omega$; $R_g = R_L = Z_0/3$. The battery voltage is V_0=10V. The switch is closed at time t=0. The length of the transmission line is l and the speed of the voltage wave is v. Draw the voltage and current reflection diagrams. Determine the line voltage and the line current at

$z = 3l/4$ as functions of time (include terms up to V_2^- (I_2^-)). Plot the line voltage and line current at $z = 3l/4$ as functions of time. What are the steady state load voltage and load current?

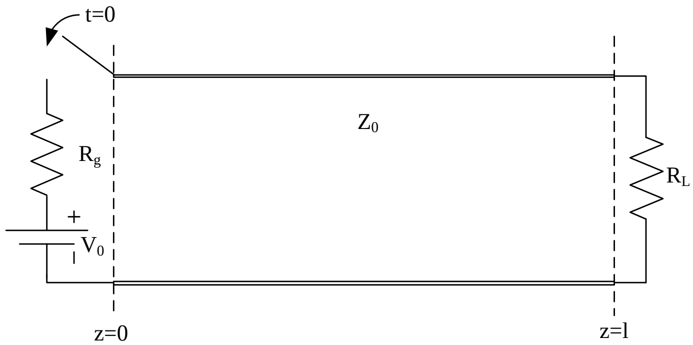

Fig. 2

6 *marks*

6. In a good conductor, the electric field amplitude decreases exponentially with distance, as given by

$$E_x = A\exp(-\alpha z)\cos(\omega t - \beta z),$$
$$jk = \alpha + j\beta,$$
$$k^2 = -j\omega\mu(\sigma + j\omega\varepsilon).$$

Show that

$$\alpha = \beta = \sqrt{\pi f \mu\sigma}$$

and derive an expression for skin depth, δ.

5.5 *marks*

7. Explain the meaning of:

(i) Voltage standing wave ratio (VSWR). How would you calculate the characteristic impedance of a transmission line using the measured VSWR and the location of first maximum/minimum?

(ii) Speed and propagation constant (= wave number) of a plane wave (solution of Maxwell's equations)

(iii) Perfect dielectric, lossy dielectric, good conductor and perfect conductor.

7.5 marks

END OF EXAM QUESTIONS

Useful information on the following four pages

Useful information

Speed of light in vacuum   c = 2.99793×10^{8} m/s

ε_0 = free space permeability = $8.854\times10^{-12} H/m$

μ_0 = free space permeability = $4\pi\times10^{-7} F/m$

Faraday's Law:

Emf = - $\frac{d\psi}{dt}$ where ψ = magnetic flux.

$$\psi = \int_S \vec{B}.d\vec{S} \qquad \text{Emf} = \oint_L \vec{E}.d\vec{L}$$

Faraday's law in point form:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\vec{x} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\vec{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\vec{z}$$

Ampere's Law (Magnetostatics):

Integral form:

$$\oint_L \vec{H}.d\vec{L} = I$$

Differential form:

$$\nabla\times\vec{H} = \vec{J}$$

$$I = \int_s \vec{J}.\vec{dS}$$

Ampere-Maxwell Law

Integral form:

$$\oint_L \vec{H}.d\vec{L} = I + I_d$$

Differential form:

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

I_d =displacement current

Maxwell's equations:

Differential form:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

$$\nabla.\vec{D} = \rho$$

$$\nabla.\vec{B} = 0$$

Integral form:

$$\oint_L \vec{E}.d\vec{L} = -\int_s \frac{\partial\vec{B}}{\partial t}.d\vec{S}$$

$$\oint_L \vec{H}.d\vec{L} = I + I_d$$

$$\oint_s \vec{D}.\vec{ds} = q$$

$$\oint_s \vec{B}.\vec{ds} = 0$$

$$\vec{B} = \mu\vec{H} \qquad \vec{D} = \varepsilon\vec{E} \qquad \vec{J} = \sigma\vec{E}$$

Permittivity, $\varepsilon = \varepsilon_0\varepsilon_r$

Permeability, $\mu = \mu_0\mu_r$

Wave equation:

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0$$

Speed of EM wave in a medium, $v = \frac{1}{\sqrt{\mu\varepsilon}}$

$$v = \frac{c}{n}, \quad n = \sqrt{\varepsilon_r \mu_r}$$

Phasor Form:

$$E_x = \mathrm{Re}\{E_{xs} \exp(j\omega t)\}$$

$$H_y = \mathrm{Re}\{H_{ys} \exp(j\omega t)\}$$

E_{xs} and H_{ys} are phasors corresponding to E_x and H_y, respectively.

$$\frac{d^2 E_{xs}}{dz^2} = -k^2 E_{xs}$$

$$k^2 = -j\omega\mu(\sigma + j\omega\varepsilon)E_{xs}$$

Propagation in dielectrics and conductors:

The plane wave in a perfect dielectric medium is given by

$$E_x = E_{x0} \cos(\omega t - \beta z)$$

$$H_y = H_{y0} \cos(\omega t - \beta z)$$

$$\frac{E_{x0}}{H_{y0}} = \eta = \left(\frac{\mu}{\varepsilon}\right)^{1/2}$$

$$\frac{\omega}{\beta} = v = \text{speed of EM wave}$$

$$\beta = \frac{2\pi}{\lambda} = \text{wave number}$$

$$v = \lambda f$$

$$\lambda_m = \lambda_0 / n$$

λ_0 is the wavelength in free space, λ_m = wavelength in a medium

The plane wave in a lossy dielectric medium is

$$E_x = E_{x0} \exp(-\alpha z)\cos(\omega t - \beta z)$$

$$H_y = H_{y0} \exp(-\alpha z)\cos(\omega t - \beta z)$$

$$\alpha = \mathrm{Re}(jk) = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right)^{1/2}$$

$$\beta = \mathrm{Im}(jk) = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right)^{1/2}$$

$$\frac{E_{x0}}{H_{y0}} = \eta = \left(\frac{\mu}{\varepsilon' - j\varepsilon''} \right)^{1/2}$$

The plane wave in a good conductor is

$$E_x = E_{x0} \exp(-\alpha z) \cos(\omega t - \beta z)$$

$$H_y = H_{y0} \exp(-\alpha z) \cos(\omega t - \beta z)$$

$$\alpha = \beta = \sqrt{\pi f \mu \sigma}$$

Skin depth

$$\delta = 1 / \alpha$$

Loss tangent

$$\tan\theta = \frac{\sigma}{\omega \varepsilon'}$$

Transmission Lines:

$$\frac{d^2 V_s}{dz^2} = \gamma^2 V_s$$

$$\gamma^2 = ZY$$

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

For the loss less case, speed v and characteristic impedance Z_0 are

$$v = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}}$$

In general,

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

Power loss (dB) = 8.69 αz

Reflection coefficient and VSWR

$$\Gamma = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad s = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Wave impedance,

$$Z_w(z) = Z_0 \frac{Z_L - jZ_0 \tan(\beta z)}{Z_0 - jZ_L \tan(\beta z)}$$

Plane wave reflection

Reflection coefficient

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Transmission coefficient

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

The wave impedance

$$\eta_w(z) = \eta_2 \frac{\eta_3 - j\eta_2 \tan(\beta_2 z)}{\eta_2 - j\eta_3 \tan(\beta_2 z)}$$

Trigonometric identities

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$
$$\exp(jA) = \cos A + j \sin A$$

THE END