1. Suppose an ac source is connected to a capacitor through conductors. Explain the meaning of the conduction current in the conductor and the meaning of the displacement current between the capacitor plates, which is filled with a dielectric material. Can there be displacement current in the conductor? Provide explanation.

5 *marks*

The conduction current is the current due to moving charges, $I = \frac{dQ}{dt}$. The displacement current is an "equivalent" current that is related to the rate of change of electric flux, i.e., $I_d = \frac{d\Phi_E}{dt}$.

From one of Maxwell's equations, we have

$$\nabla \times H = J + \frac{\partial D}{\partial t},$$

where the first term on the right hand side is the conduction current density and the second term is the displacement current density.

$$\int (\nabla \times \vec{H}).d\vec{s} = \int \vec{J}.d\vec{s} + \int \left(\frac{\partial \vec{D}}{\partial t}\right).d\vec{s}$$

Applying Stokes Theorem

$$\int_s (\nabla \times \vec{H}).d\vec{s} = \oint_L H.dl = I + I_d$$

Where

$I = \int \vec{J}.d\vec{s}$ is the conduction current and

$I_d = \int \left(\frac{\partial \vec{D}}{\partial t}\right).d\vec{s}$ is the displacement current

Between the capacitor plates, there is no conduction current if the plates are separated by a perfect insulator. This is because there are no free charge carriers in a perfect insulator. Practically the real insulator consists of a very few charge carriers and therefore a very small leakage current passes in the capacitor depending on the conductivity of the insulator.

When a time varying voltage (AC) is applied on the capacitor, there exists a displacement current between the capacitor plates since the electric flux density changes as a function of time. The electric flux density, D= εE where epsilon is the dielectric constant and E is the electric field intensity. If the applied voltage varies sinusoidally with time, electric field intensity E (which is proportional to V) also varies sinuoidally with time, leading to time-changing D.

Since the electric field intensity changes with time in the conductor as well, there is a small displacement current in the conductor, too. However, the magnitude of the displacement current is much smaller than the conduction current.

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$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} = -\mu \frac{\partial \underline{H}}{\partial t} \rightarrow ① \quad , \text{as } \underline{B} = \mu \underline{H}$$

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t} = \sigma \underline{E} + \epsilon \frac{\partial \underline{E}}{\partial t} \rightarrow ② \quad , \text{as } \underline{J} = \sigma \underline{E} , \underline{D} = \epsilon \underline{E}$$

since, $\underline{E} = E_x \vec{x}$, $\underline{H} = H_y \vec{y}$

subst. in RHS of ① :-

$$-\mu \frac{\partial \underline{H}}{\partial t} = -\mu \frac{\partial H_y}{\partial t} \vec{y}$$

So, we need, only, y-component of LHS of ① :-

$$(\underline{\nabla} \times \underline{E})_y = \begin{vmatrix} \vec{x} & \vec{y} & \vec{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \frac{\partial E_x}{\partial z} \vec{y}$$

$$\therefore \quad \frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \rightarrow ③$$

subst. in RHS of ② :-

$$\sigma E_x \vec{x} + \epsilon \frac{\partial E_x}{\partial t} \vec{x}$$

So, we need x-component of LHS of ② :-

$$(\nabla \times \underline{H})_x = \begin{vmatrix} \vec{x} & \vec{y} & \vec{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_y & 0 \end{vmatrix} = -\frac{\partial H_y}{\partial z} \vec{x}$$

$$\therefore \quad -\frac{\partial H_y}{\partial z} = \sigma E_x + \epsilon \frac{\partial E_x}{\partial t} \rightarrow ④$$

Assume, $E_x = Re\{E_{xs} e^{j\omega t}\}$, $H_y = Re\{H_{ys} e^{j\omega t}\}$

subst. in ③ & ④ :-

$$\frac{\partial E_{xs}}{\partial z} = -j\omega\mu H_{ys} \rightarrow ⑤$$

$$-\frac{\partial H_{ys}}{\partial z} = \sigma E_{xs} + j\omega\epsilon E_{xs} \rightarrow ⑥$$

diff. ⑤ w.r.t. z :-

$$\frac{\partial^2 E_{xs}}{\partial z^2} = -j\omega\mu \frac{\partial H_{ys}}{\partial z} \rightarrow ⑦$$

Subst. from ⑥ in ⑦ :-

$$\frac{\partial^2 E_{xs}}{\partial z^2} = j\omega\mu(\sigma E_{xs} + j\omega\epsilon E_{xs})$$

$$\therefore \quad \boxed{\frac{\partial^2 E_{xs}}{\partial z^2} = j\omega\mu(\sigma + j\omega\epsilon) E_{xs}} \quad \#$$

(a) 3’

$$\mathbf{H}=\frac{\mathbf{B}}{\mu}=\frac{B_x}{\mu}\vec{\mathbf{x}}=\frac{3\times10^{-4}\cos10^5t\sin10^{-3}y}{10^{-5}}\vec{\mathbf{x}}=30\cos10^5t\sin10^{-3}y\vec{\mathbf{x}}\frac{A}{m}$$ (0.5’)

$$\nabla\times\mathbf{H}=-\hat{\mathbf{z}}\frac{\partial H_x}{\partial y}=-0.03\cos10^5t\cos10^{-3}y\hat{\mathbf{z}}\frac{A}{m^2}$$ (0.5’)

$$\frac{d\mathbf{E}}{dt}=\frac{\nabla\times\mathbf{H}}{\varepsilon}=\frac{-0.03\cos10^5t\cos10^{-3}y\hat{\mathbf{z}}}{2\times10^{-11}}=-1.5\times10^9\cos10^5t\cos10^{-3}y\hat{\mathbf{z}}\frac{V}{ms}$$ (0.5’)

$$\mathbf{E}=\int\frac{d\mathbf{E}}{dt}dt==-1.5\times10^4\sin10^5t\cos10^{-3}y\hat{\mathbf{z}}\frac{V}{m}$$ (0.5’)

Direction: 0.5, Unit: 0.5

(b) 2’

$$\Phi=\iint\mathbf{B}d\mathbf{S}=\iint B_x dydz=\int_0^5 dz\int_0^{30}3\times10^{-4}\cos10^5t\sin10^{-3}ydy=1.5\cos10^5t(1-\cos0.03)$$

(0.5’)

$$=1.5\cos0.1(1-\cos0.03)$$ (0.5’)

$$=6.72\times10^{-4}Wb$$ (0.5’)

Unit 0.5’.