EE-3FK4 Mid. Term Exam 2015

Duration of examination: 1 hour
Answer all the questions.
Total marks = 15

1. Suppose an ac source is connected to a capacitor through conductors. Explain the meaning of the conduction current in the conductor and the meaning of the displacement current between the capacitor plates, which is filled with a dielectric material. Can there be displacement current in the conductor? Provide explanation.
   5 *marks*
2. Starting from Maxwell's equations and assuming that $\vec{E} = E_x \vec{x}$ and $\vec{H} = H_y \vec{y}$, prove that
   $$\frac{d^2 E_{xs}}{\partial z^2} = j\omega\mu(\sigma + j\omega\varepsilon)E_{xs},$$
   where
   $$E_x = \mathrm{Re}[E_{xs}\exp(j\omega t)]$$
   5 *marks*
3. Within a certain region, $\varepsilon = 2\times10^{-11}$ F/m and $\mu = 10^{-5}$ H/m. If
   $$B_x = 3\times10^{-4}\cos 10^5 t \sin 10^{-3} y \text{ T:}$$
   (a) use $\nabla\times\mathbf{H} = \varepsilon\dfrac{\partial \mathbf{E}}{\partial t}$ to find $\mathbf{E}$;
   (b) find the total magnetic flux passing through the surface $x = 0$, $0 < y < 30$ m, $0 < z < 5$m, at $t = 1\mu s$.
   5 *marks*

Useful Information:

Faraday's Law:

$$\text{Emf} = -\frac{d\psi}{dt}$$

where ψ = magnetic flux.

$$\psi = \int_S \vec{B}.d\vec{S}$$

$$\text{Emf} = \oint_L \vec{E}.d\vec{L}$$

Faraday's law in the point form:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\vec{x} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\vec{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\vec{z}$$

Ampere's Law (Magnetostatics):

Integral form:

$$\oint_L \overrightarrow{H}.d\overrightarrow{L} = I$$

Differential form:

$$\nabla \times \overrightarrow{H} = \overrightarrow{J}$$

$$I = \int_s \overrightarrow{J}.\overrightarrow{dS}$$

Ampere-Maxwell Law

Integral form:

$$\oint_L \overrightarrow{H}.d\overrightarrow{L} = I + I_d$$

I_d =displacement current

Differential form:

$$\nabla \times \overrightarrow{H} = \overrightarrow{J} + \frac{\partial \overrightarrow{D}}{\partial t}$$

Maxwell's equations:

Differential form:

$$\nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t}$$

$$\nabla \times \overrightarrow{H} = \overrightarrow{J} + \frac{\partial \overrightarrow{D}}{\partial t}$$

$$\nabla.\overrightarrow{D} = \rho$$

$$\nabla.\overrightarrow{B} = 0$$

Integral form:

$$\oint_L \overrightarrow{E}.d\overrightarrow{L} = -\int_s \frac{\partial \overrightarrow{B}}{\partial t}.d\overrightarrow{S}$$

$$\oint_L \overrightarrow{H}.d\overrightarrow{L} = I + I_d$$

$$\oint_s \overrightarrow{D}.\overrightarrow{ds} = q$$

$$\oint_s \overrightarrow{B}.\overrightarrow{ds} = 0$$

$\vec{B} = \mu \vec{H}$ $\quad \vec{D} = \varepsilon \vec{E}$ $\quad \vec{J} = \sigma \vec{E}$

Permittivity, $\varepsilon = \varepsilon_0 \varepsilon_r$

$\varepsilon_0 =$ free space permeability = $8.854 \times 10^{-12} H/m$

Permeability, $\mu = \mu_0 \mu_r$

$\mu_0 =$ free space permeability = $4\pi \times 10^{-7} F/m$

Wave equation:

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0$$

Speed of EM wave in a medium, $v = \dfrac{1}{\sqrt{\mu\varepsilon}}$

$v = \dfrac{c}{n}$, $n = \sqrt{\varepsilon_r \mu_r}$

Speed of light in vacuum c 2.99793×10^8 m/s

The plane wave in a perfect dielectric medium is given by

$E_x = E_{x0} \cos(\omega t - \beta z)]$

$H_y = H_{y0} \cos(\omega t - \beta z)]$

$$\frac{E_{x0}}{H_{y0}} = \eta = \left(\frac{\mu}{\varepsilon}\right)^{1/2}$$

$\dfrac{\omega}{\beta} = v =$ speed of EM wave

$\beta = \dfrac{2\pi}{\lambda} =$ wave number

$v = \lambda f$

$\lambda_m = \lambda_0 / n$

λ_0 is the wavelength in free space, $\lambda_m =$ wavelength in a medium

Phasor Form:

$E_x = \mathrm{Re}\{E_{xs} \exp(j\omega t)\}$

$H_y = \mathrm{Re}\{H_{ys} \exp(j\omega t)\}$

E_{xs} and H_{ys} are phasors corresponding to E_x and H_y, respectively.

Trigonometric identities

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\exp(jA) = \cos A + j \sin A$