FIND THE FOURIER TRANSFORM OF $A \operatorname{sinc}(at)\sin(2\pi f_0 t)$.

ASSUME $a > 0$ and $a \ll f_0$. SKETCH THE MAGNITUDE & PHASE SPECTRA

SOLUTION:

$$\operatorname{rect}(t) \rightleftharpoons \operatorname{sinc}(f)$$

DUALITY: $$\operatorname{sinc}(t) \rightleftharpoons \operatorname{rect}(f)$$

SCALING: $$\operatorname{sinc}(at) \rightleftharpoons \frac{1}{a}\operatorname{rect}(f/a)$$

$$A\operatorname{sinc}(at) \rightleftharpoons \frac{A}{a}\operatorname{rect}(f/a)$$

Let $$g(t) = A\operatorname{sinc}(at)$$

$$G(f) = \frac{A}{a}\operatorname{rect}(f/a) \longrightarrow (*)$$

LET $$g_1(t) = g(t)\sin(2\pi f_0 t)$$

$$= g(t)\left[\frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j}\right]$$

FREQ. SHIFTING $$g(t)e^{j2\pi f_0 t} \rightleftharpoons G(f - f_0)$$

$$g(t)e^{-j2\pi f_0 t} \rightleftharpoons G(f + f_0)$$

$$\therefore\; g(t)\left[\frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j}\right] \rightleftharpoons \frac{G(f - f_0) - G(f + f_0)}{2j}$$

USING EQ. (*)

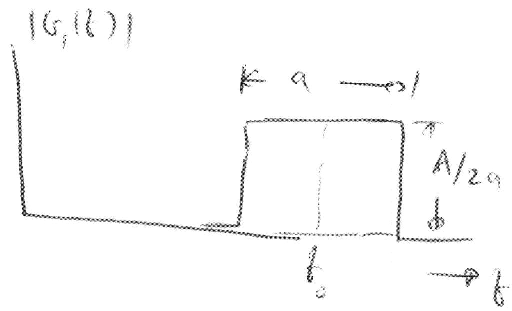
ie. $$g_1(t) \rightleftharpoons \frac{A}{2aj}\left\{ \mathrm{rect}\left(\frac{f-f_0}{a}\right) - \mathrm{rect}\left(\frac{f+f_0}{a}\right)\right\}$$
$$= G_1(f)$$

$\therefore$ $\cancel{G_1(f) =}$

WHEN $f > 0$

$$G_1(f) = \frac{A}{2aj}\,\mathrm{rect}\left(\frac{f-f_0}{a}\right) \qquad \left(\because\ \mathrm{rect}\left(\frac{f+f_0}{a}\right) \text{ is CENTERED AT } f=-f_0 \text{ \& } a \ll f_0\right)$$

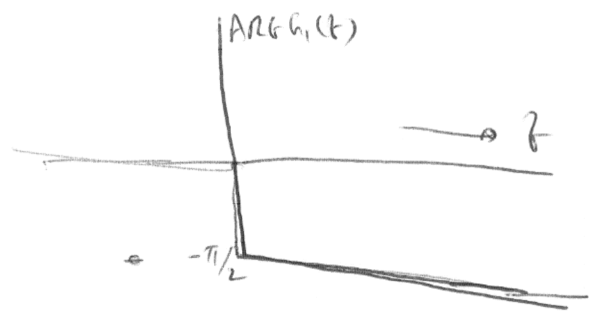
POSITIVE SPECTRUM



$$|G_1(f)| = \frac{A}{2a}\,\mathrm{rect}\left(\frac{f-f_0}{a}\right)$$

$$\frac{1}{j} = -j = e^{-j\pi/2}$$

$$\therefore \angle G_1(f) = -\pi/2 \quad \text{when } f > 0$$



WHEN $f < 0$,

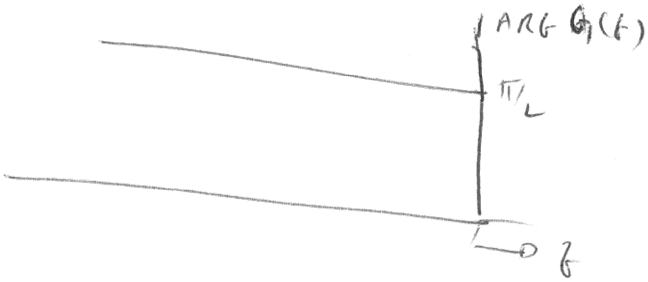
$$G_1(f) = \frac{-A}{2aj} \operatorname{rect}(f + f_0)$$

$$|G_1(f)| = \frac{A}{2a} \operatorname{rect}(f + f_0)$$

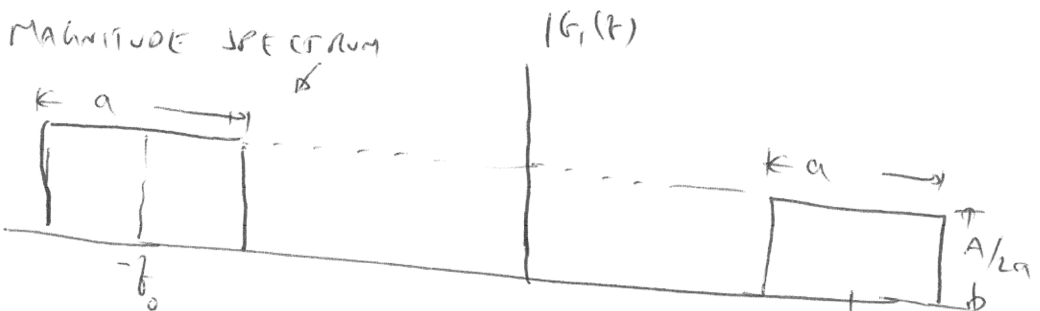
NEGATIVE SPECTRUM

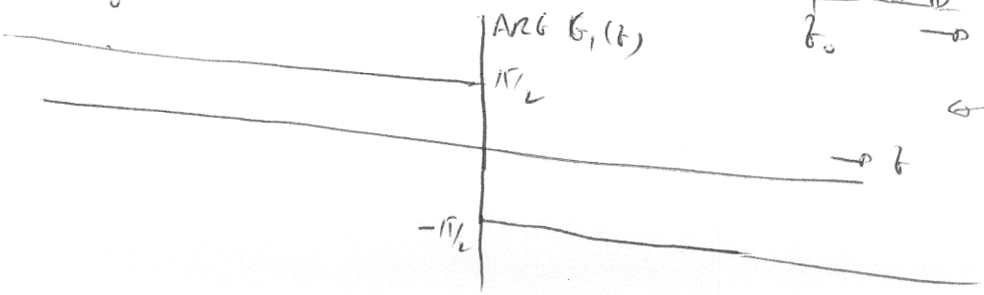
$$\frac{-1}{j} = j = e^{j\pi/2}$$

$$\therefore \angle G_1(f) = \pi/2 \quad \text{WHEN } f < 0$$



MAGNITUDE SPECTRUM





← PHASE SPECTRUM

NOTE THAT MAGNITUDE SPECTRUM IS SYMMETRIC & PHASE SPECTRUM IS ANTI-SYMMETRIC SINCE $g_i(t)$ IS REAL.