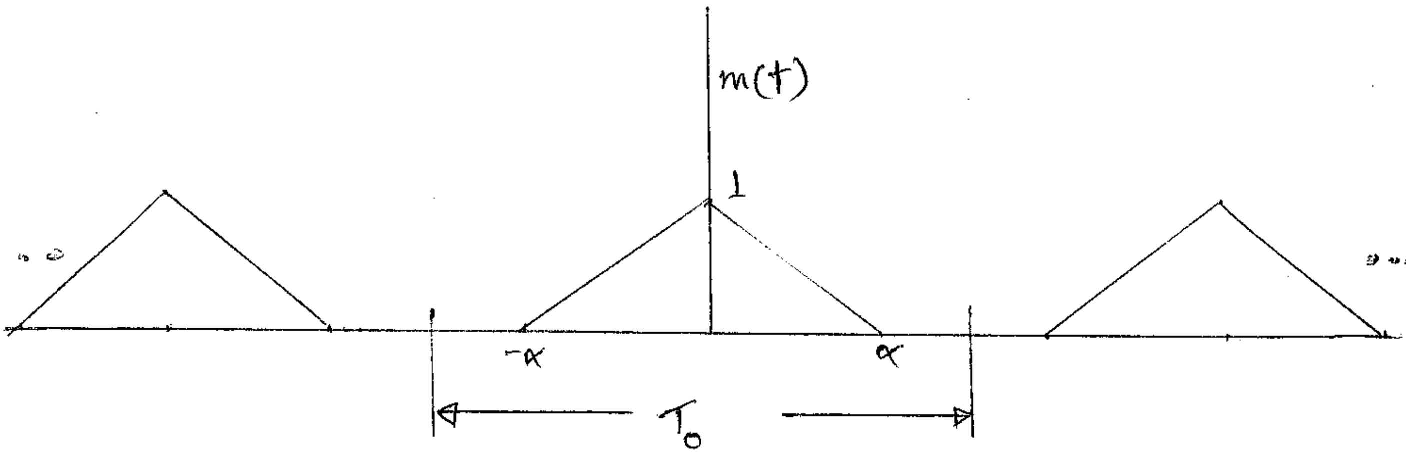
Consider the message signal $m(t)$:



a) Sketch the AM wave

b) Sketch the Spectrum

Assume 100% Modulation. & $A_c = 1$.

Solution :

$$\cancel{Max}\ MAX|k_a\, m(t)| \times 100 = 100$$

$$MAX\ m(t) = 1 \; ; \quad \therefore\ k_a = 1 \; ;$$

The AM wave is given by

$$s(t) = [1 + m(t)] \cdot \cos(2\pi f_c t)$$

Approach 1:

Let us define

$$\text{triang}(t) = 1 - |t| \quad \text{for } |t| < 1$$
$$= 0 \quad \text{elsewhere}$$

The triangular pulse in the figure can be written as

$$g(t) = \text{triang}(t/\alpha)$$

and the message signal $m(t)$ can be written as

$$m(t) = \sum_{n=-\infty}^{\infty} g(t - nT_0)$$

Since $\text{triang}(t) = \text{rect}(t) * \text{rect}(t)$

Using the convolution theorum, we have

$$\text{triang}(t) \rightleftharpoons \text{sinc}^2(f)$$

Using the scaling property,

$$g(t) = \text{triang}(t/\alpha) \rightleftharpoons \alpha \, \text{sinc}^2(f\alpha)$$

Using the time-shifting property

$$g(t - nT_0) \rightleftharpoons \alpha \, \text{sinc}^2(f\alpha) \cdot e^{-j2\pi nT_0 f}$$

$$\therefore m(t) = \sum_{n=-\infty}^{\infty} g(t - nT_0) \rightleftharpoons \alpha \, \text{sinc}^2(f\alpha) \cdot \sum_{n=-\infty}^{\infty} e^{-j2\pi nT_0 f}$$

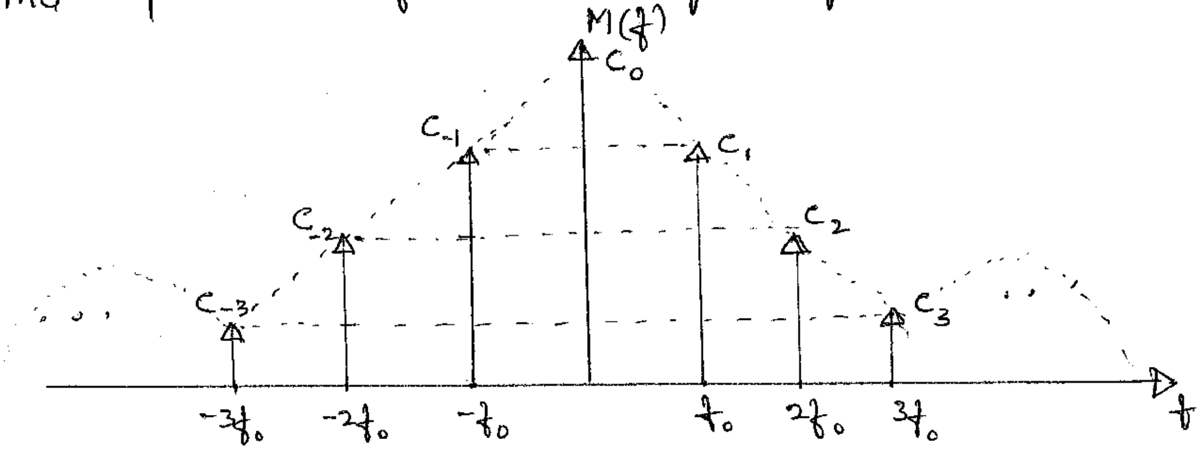
Let $T_0 = 1/f_0$.

Now we use the identity,

$$\sum_{n=-\infty}^{\infty} \delta(f - nf_0) = \frac{1}{f_0} \sum_{n=-\infty}^{\infty} e^{-j2\pi nf/f_0}$$

$$m(t) \rightleftharpoons \alpha f_0 \operatorname{sinc}^2(f\alpha) \cdot \sum_{n=-\infty}^{\infty} \delta(f - nf_0)$$

The spectrum of the message signal is as follows.



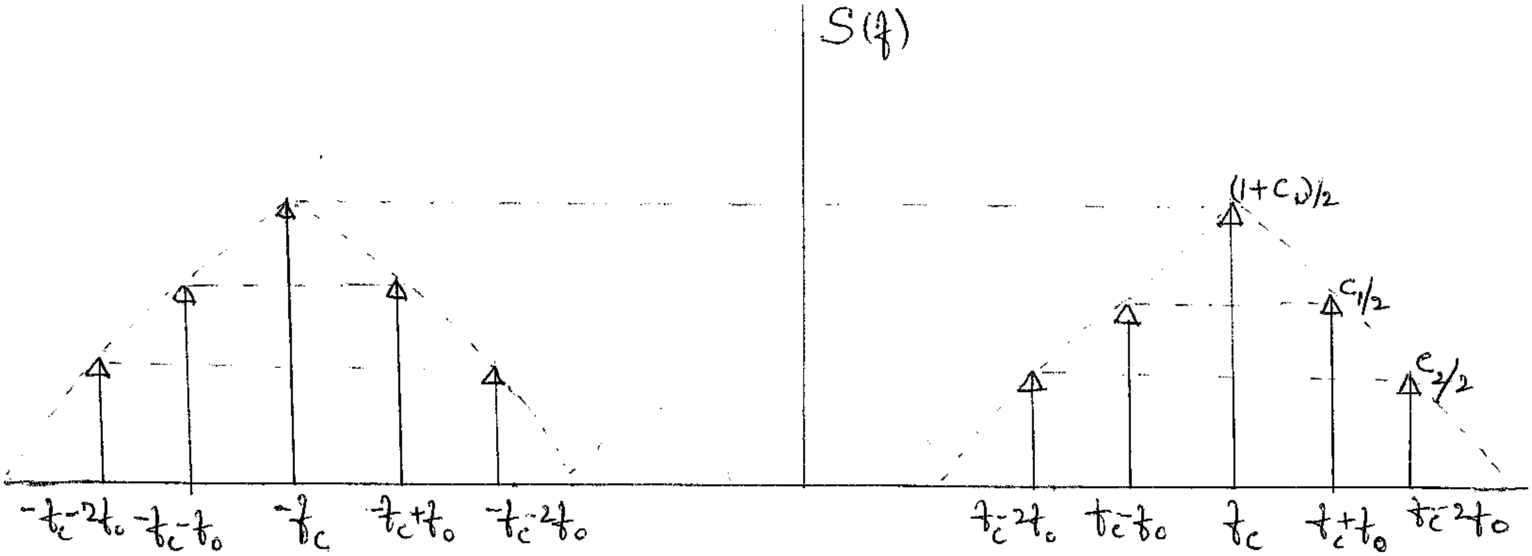
$$c_n = \alpha f_0 \operatorname{sinc}^2(nf_0\alpha);$$

$$\therefore M(f) = \sum_{n=-\infty}^{\infty} c_n \cdot \delta(f - nf_0)$$

The AM wave can be written as

$$s(t) = [1 + m(t)] \cdot \cos(2\pi f_c t)$$

$$S(f) \rightleftharpoons \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{2}[M(f - f_c) + M(f + f_c)]$$


S(f)
(1+c0)/2
c1/2
c2/2
-fc-2f0
-fc-f0
-fc
-fc+f0
-fc-2f0
fc-2f0
fc-f0
fc
fc+f0
fc-2f0

Approach 2 :

The message signal m(t) is a periodic signal. Therefore, let us represent it as a fourier series.

$$m(t) = \sum_{n=-\infty}^{\infty} c_n \cdot e^{j2\pi n t/T_0}$$

$$c_n \rightleftharpoons c_n \cdot \delta(f)$$

$$c_n \cdot e^{j2\pi n t/T_0} \rightleftharpoons c_n \cdot \delta(f - n/T_0)$$

$$\therefore \; m(t) \rightleftharpoons \sum_{n=-\infty}^{\infty} c_n \, \delta(f - nf_0)$$

To evaluate c_n, we note that triangular wave can be imagined as the convolution of two rectangular waves, i.e. $g_p(t) * g_p(t) = \alpha\, m(t)$.

If the fourier coefficient of the rectangular wave is G_n, the fourier coefficient of the triangular wave $\alpha . m(t)$ is $G_n^2 \cdot T_0$ *.

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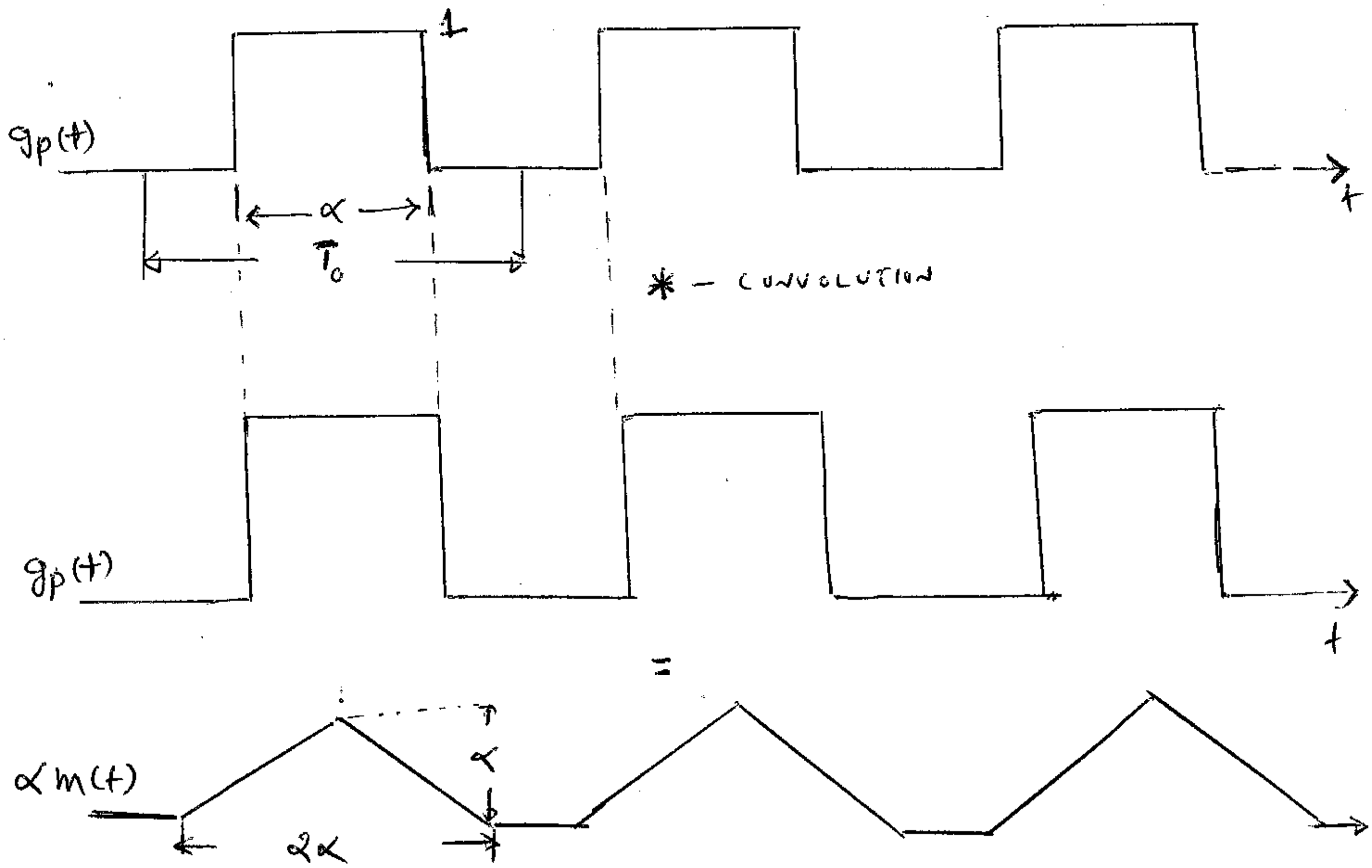
* As $T_0 \to \infty$, $T_0 G_n \to G(f)$, the fourier transform. The multiplication in freq. domain corresponds to convolution in time domain, i.e. $G^2(f) = \alpha M(f)$ or $T_0^2 G_n^2 = c_n \cdot T_0 \cdot \alpha$.



$$\therefore \quad C_n = \frac{G_n^2 \cdot T_0}{\alpha}$$

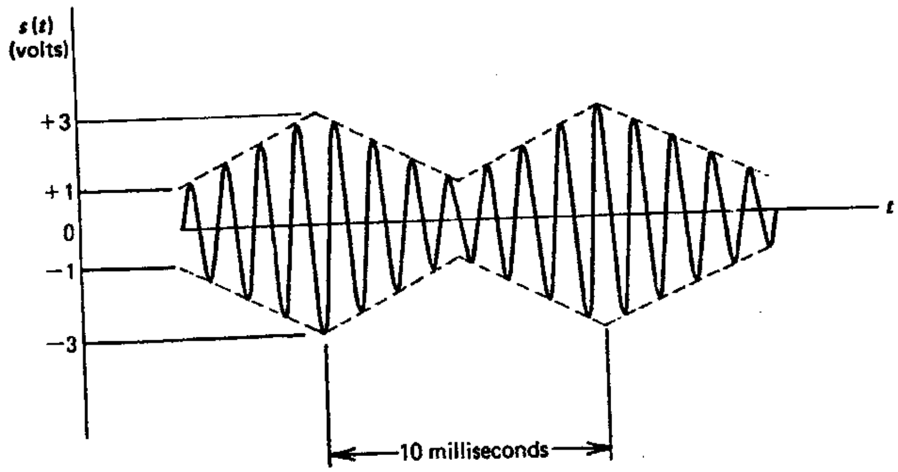
Since. $G_n = \dfrac{\alpha}{T_0} \cdot \mathrm{sinc}^2\left(\dfrac{n\alpha}{T_0}\right)$

$$C_n = \alpha \cdot f_0 \cdot \mathrm{sinc}^2(n \cdot \alpha \cdot f_0)$$

$$M(f) = \sum_{n=-\infty}^{\infty} \alpha f_0 \cdot \mathrm{sinc}^2(n\alpha f_0) \cdot \delta(f - nf_0)$$

Problem 6, P. 386 text

Consider the AM wave shown below. This modulated wave is applied to an envelope detector with zero source resistance and a load resistance of $250\,\Omega$. The carrier freq. $f_c = 40\,kHz$. Suggest a suitable value for the capacitor C so that the distortion is negligible for frequencies up to and including the 11th harmonic of the modulating wave.



Problem 6

By expanding the triangular envelope of the AM wave in a Fourier series, we find that the components of the envelope beyond the 11th harmonic become increasingly insignificant. This means that for negligible distortion, the envelope detector must be able to reproduce faithfully the sinusoidal components of the envelope up to the 11th harmonic. The frequency of this component is 1100 Hz. For the evenlope detector to function properly, we must therefore choose

$$R_\ell C \ll \frac{1}{W}$$

where $R_\ell = 250\ \Omega$

$W = 1100$ Hz

That is, C must be small compared to 4 μF. Thus, the value C = 0.4 μF would be acceptable.