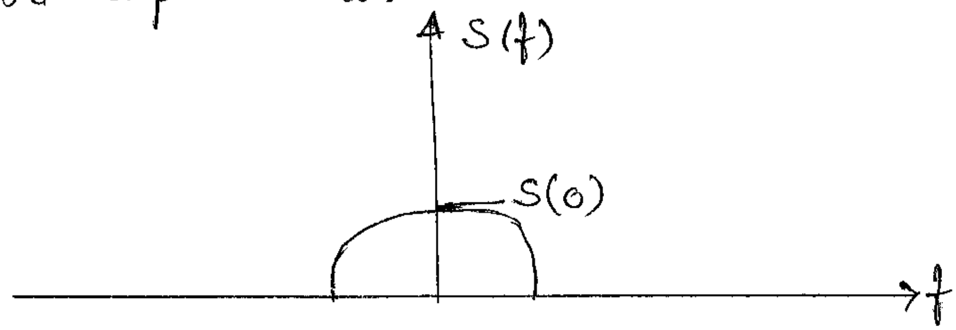
Frequency or Channel Selection

Here, we look at how any radio or TV, or any receiver, selects one channel or band of interest from a large member of frequency-multiplexed signals. First we must look at the effect of multiplying a signal $s(t)$ by a sinusoid $c(t) = \cos 2\pi f_0 t$.

1. $s(t)$ is a baseband Signal

If $s(t)$ is baseband, its freq. domain rep'n may be depicted as:



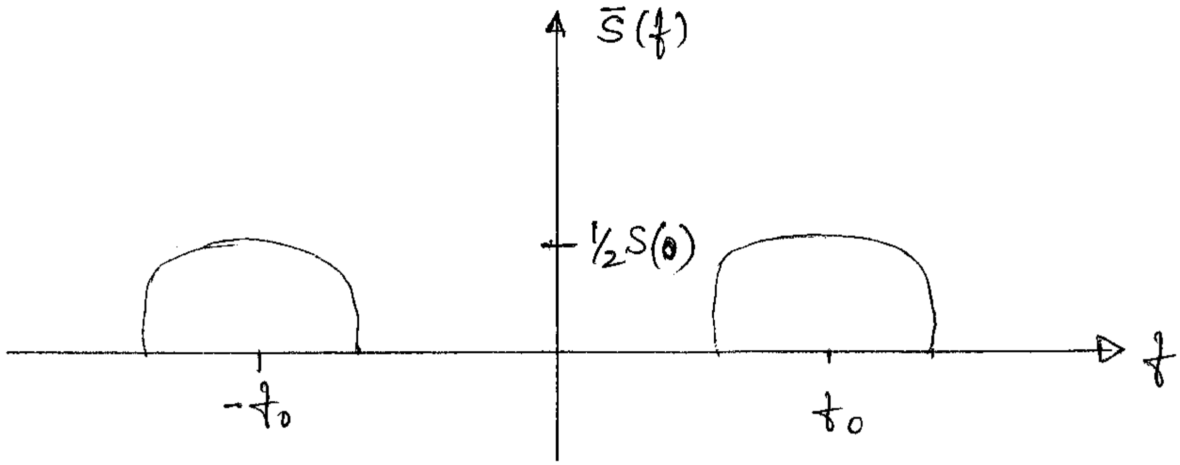
Now we form $\tilde{s}(t)$ by mult. by a sinusoid:

$$\tilde{s}(t) = s(t) \cdot \cos 2\pi f_0 t$$

$$= \tfrac{1}{2} s(t) \left[\exp(j 2\pi f_0 t) + \exp(-j 2\pi f_0 t) \right]$$

$$\rightleftharpoons \tfrac{1}{2} S(f - f_0) + \tfrac{1}{2} S(f + f_0)$$

Where in the last line we have used the "mult by complex exponential" property of the F.T.



The above shows the frequency-domain rep'n of $\bar{S}(f) = F[s(t)]$. Note this important effect: multiplying a signal $s(t)$ by $\cos(2\pi f_0 t)$ shifts the spectrum upwards and downwards by an amount f_0.

Further, each spectral portion is ½ the level of the baseband version.

2. $s(t)$ is a frequency-translated signal

Now suppose $s(t)$ is not a baseband signal, but one centred at f_0, as $\bar{s}(t)$ in Fig. 1. Now what happens when this signal $\bar{s}(t)$ is again multiplied by a sinusoid $\cos(2\pi f_0 t)$?

Let us define $\hat{s}(t) = \bar{s}(t) \cdot \cos(2\pi f_0 t)$.

$$\hat{s}(t) = \bar{s}(t) \cos(2\pi f_0 t)$$

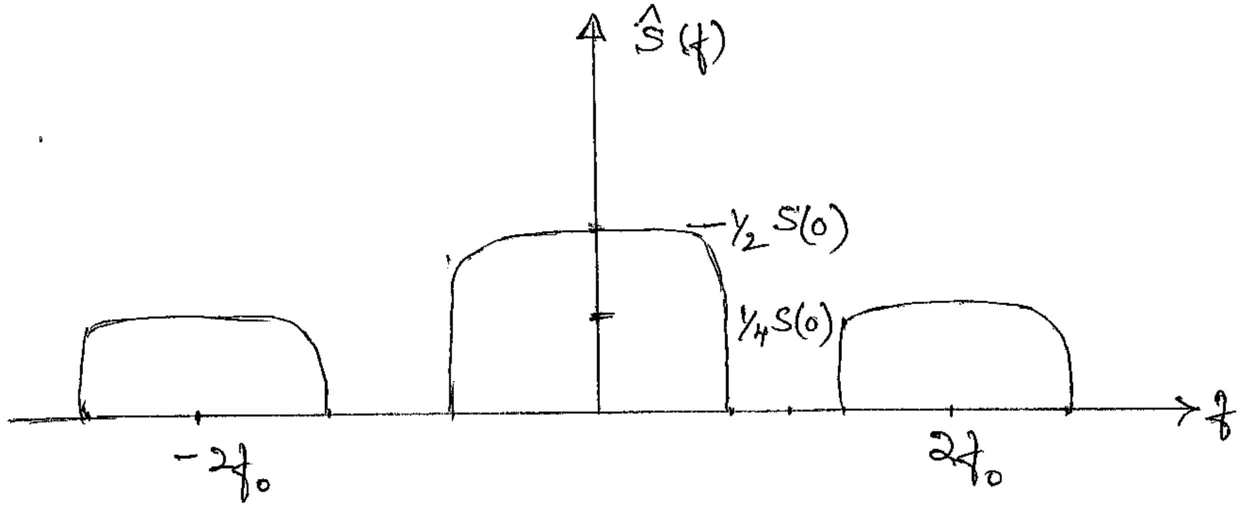
$$= s(t) \cos^2(2\pi f_0 t)$$

$$= \frac{s(t)}{2}\left[1 + \cos(4\pi f_0 t)\right]$$

$$= \frac{s(t)}{2} + \frac{s(t)}{2} \cdot \cos(4\pi f_0 t)$$

$$\therefore \hat{s}(t) \rightleftharpoons \tfrac{1}{2} S(f) + \tfrac{1}{4}\left[S(f + 2f_0) + S(f - 2f_0)\right]$$

The spectrum $\hat{S}(f)$ of $\hat{s}(t)$ is thus as follows:



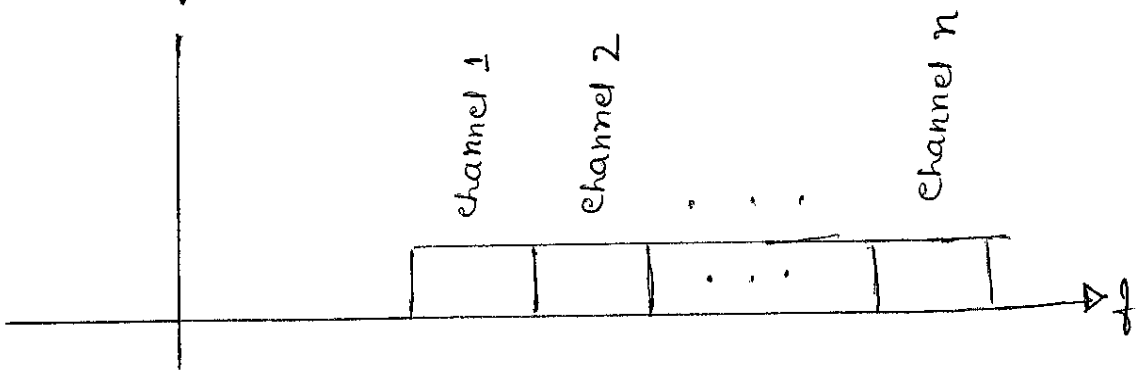
- This spectrum may be interpreted as each portion of $\bar{S}(f)$ in Fig. 1 also being shifted upwards and downwards by f_0 Hz. The two resulting baseband portions add: the baseband amplitude is twice that of the component centred at $2f_0$.
- Note that the original spectrum $\hat{S}(f)$ may be recovered (to within a mult. constant) from $\bar{S}(f)$ by low-pass filtering, to remove the portion centred at $2f_0$ Hz.

* Thus, multiplying a signal $s(t)$ by a sinusoid
* $\cos 2\pi f_0 t$ shifts each portion of $S(f)$ upwards
* and downwards by f_0 Hz. (regardless of whether the signal is baseband or frequency translated.)

This has very significant applications as we see later:
- spectrum analyzer
- heterodyne tuning in any radio/TV
- DSB-SC modulation.

Selection of channel/band in a radio/TV

- In commercial radio/TV, we have many channels all frequency multiplexed together:



(only +ve half drawn for convenience).

The signal we receive from the antenna consists of all these channels mixed together. In the AM radio case, for example, how do we select out the one freq. band of interest to feed into our envelope detector? (If we don't select out our desired band, the envelope detector output will be the envelope of all bands together, which would yield a meaningless result.)

There are two ways:

1. <u>The straightforward approach</u>:

Use an RF (radio frequency; i.e., the freq. of the received signal) to select out the band of interest. The block diagram for an AM receiver is:

Fig. 4.

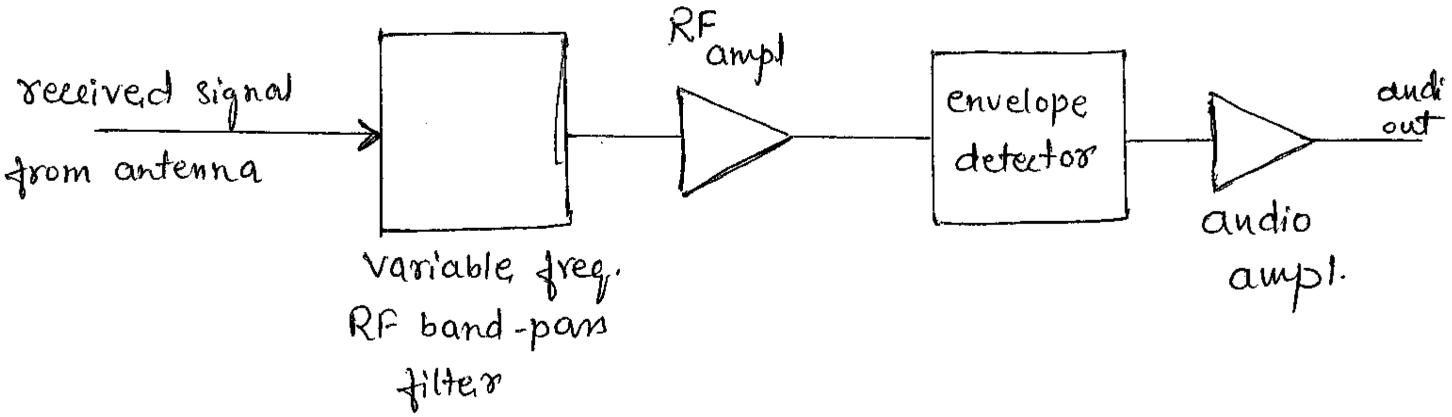
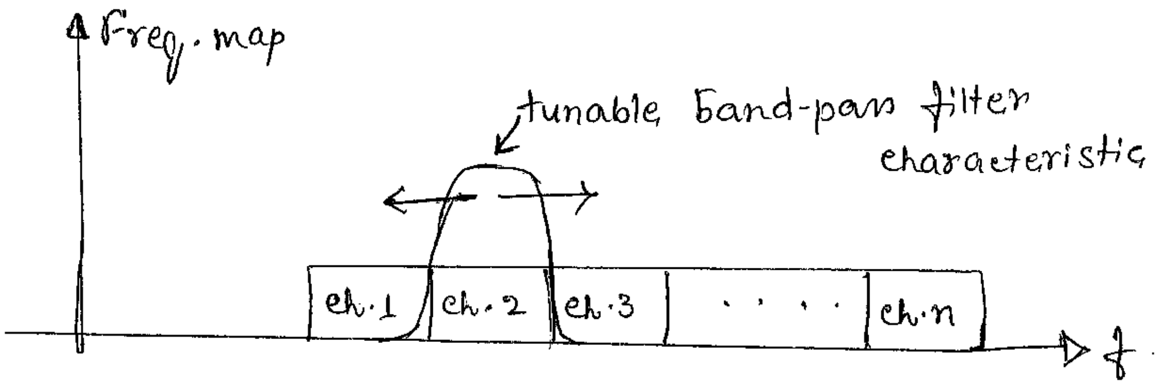


Fig. 5.



By "tuning" i.e. varying the centre freq. of the filter, we can select different channels.

The difficulty with this approach is that a good filter that can vary its centre freq. while still keeping its desirable band-pass filter characteristic (i.e. keeping lots of attenuation in the stop band) is very difficult and expensive to make.

We therefore consider another scheme:

2. Heterodyning

This is by far the smarter approach for selecting a channel. The block diagram for the heterodyne receiver is shown below:

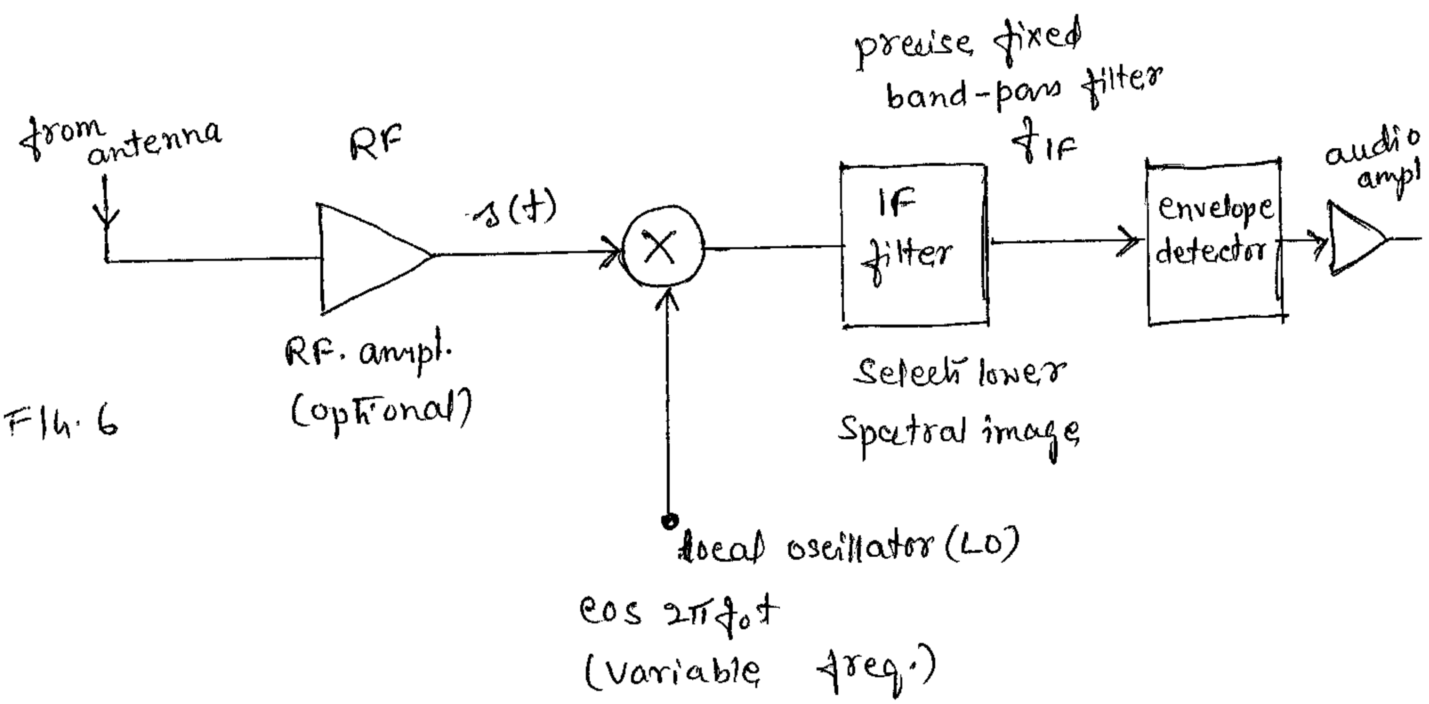
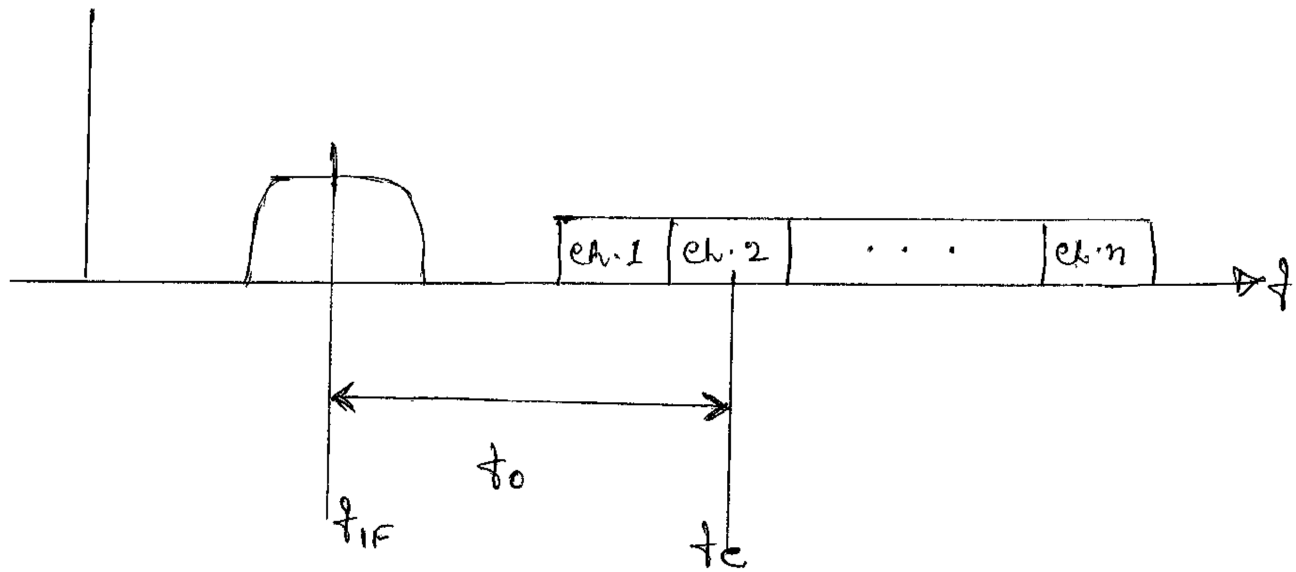


Fig. 6

The corresponding frequency "map" is as follows

Fig. 7



Note "IF" stands for intermediate freq.

With this scheme, the local oscillator freq. f_0 is selected so that $f_0 = f_c - f_{IF}$, where f_c is the centre freq. corresponding to ch. 2, and f_{IF} is the fixed IF freq.

Because the signal $s(t)$ in Fig. 6 is multiplied by $\cos 2\pi f_0 t$, the spectrum $S(f)$ is shifted upwards and downwards by the local oscillator freq. f_0. i.e.,

$$s'(t) = s(t) \cdot \cos(2\pi f_0 t) \rightleftharpoons \frac{1}{2}\left[S(f-f_0) + S(f+f_0)\right].$$

Therefore, if the spectrum of $S(t)$ is centered around f_c. The spectrum of $s'(t)$ is centered around $f_c - f_0$ and $f_c + f_0$. f_0 is selected so that the lower spectral image of ch. 2 resulting from this multiplication coincides with the IF filter centre freq. (i.e., the spectrum centered around $f_c - f_0 = f_{IF}$) Thus, only a frequency-shifted version of the ch. 2. signal passes through to the envelope detector, as desired.

Different channels are selected simply by varying the LO freq. f_0.

The advantage of this scheme is that it is very easy to build both a variable-freq. oscillator, and a fixed, sharply-tuned band-pass IF filter. This results in much better rejection of the unwanted channels.