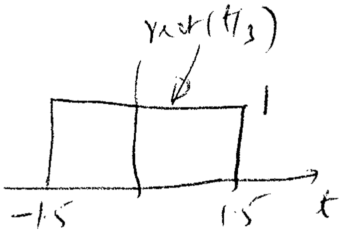
ANGLE MODULATION - TUTORIAL

1. SUPPOSE THE MESSAGE SIGNAL IS

$$m(t) = 2\,\text{rect}(t/3)$$

SKETCH THE FM & PM SIGNALS. ASSUME $A_c = 1V$ & $= 100$ KHz.
$k_f = 5$ KHz/V & $k_p = \pi/4$ RAD/V

FM:



$$f_i = f_c + k_f m(t)$$

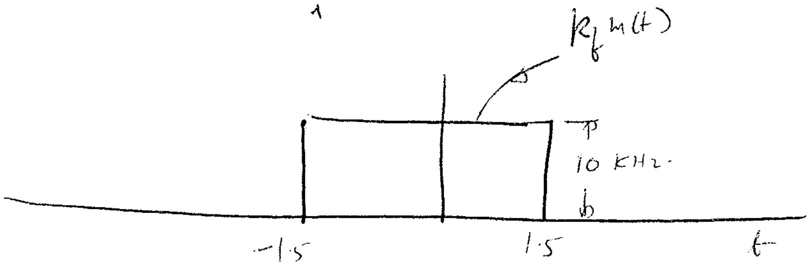
$$= f_c + 5\cdot 2\,\text{RECT}(t/3)\times 10^3$$

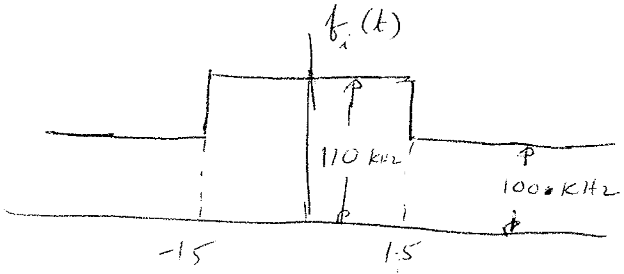
WHEN $|t| < 1.5$, $\text{RECT}(t/3) = 1$

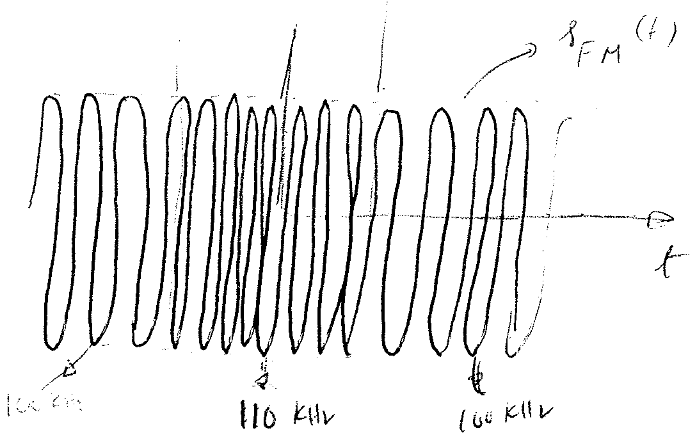
$$f_i = 100\times10^3 + 10\times10^3 = 110\times10^3 \text{ Hz}$$

WHEN $|t| \geq 1.5$, $\text{RECT}(t/3) = 0$

$$f_i = 100\times10^3 \text{ Hz.}$$







$$\frac{d\varphi}{dt} = 2\pi f_i = 2\pi\left[f_c + k_f\, m(t)\right]$$

$$\therefore \varphi(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt = 2\pi f_c t + 2\pi k_f\, 2 \int_0^t \text{rect}(t/3)\,dt$$

For ~~for~~ $0 < t < 1.5, \quad \text{rect}(t/3) = 1$

$$\varphi(t) = 2\pi f_c t + 4\pi k_f t$$

$$= 2\pi\left[100\times10^3 t + 10\times10^3 t\right] = 2\pi \times 110\times10^3 t$$

For $t \geq 1.5, \quad \text{rect}(t/3) = 0$

$$\varphi(t) = 2\pi f_c t + 4\pi k_f \cancel{\int_0^t 0\,dt} \left\{\int_0^{1.5} \text{RECT}(t/3)\,dt + \int_{1.5}^t 0\,dt\right\}$$

$$= 2\pi f_c t + 4\pi k_f \times 1.5$$

$$= 2\pi\left\{f_c t + 3k_f\right\} = 2\pi\left\{100\times10^3 t + 3\times5\times10^3\right\}$$

Similarly, For $-1.5 < t < 0, \quad \varphi(t) = 2\pi \times 110\times10^3 t$

For $t \leq -15, \quad \varphi(t) = 2\pi\left\{100\times10^3 t + 15\times10^3\right\}$

$$s_{FM}(t) = \cos(\varphi(t))$$

PM:

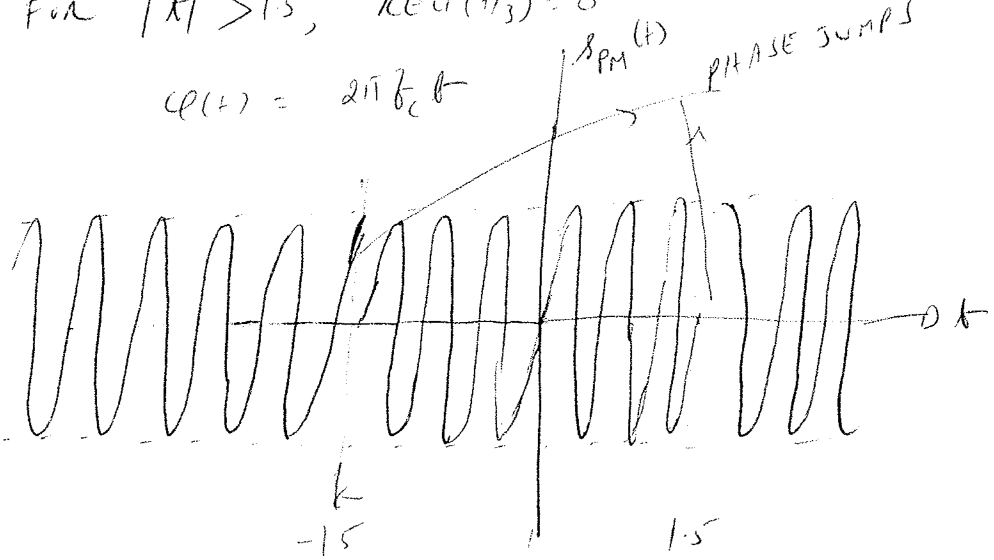
$$\varphi(t) = 2\pi f_c t + k_p m(t)$$

For $-1.5 < t < 1.5$, $\mathrm{RECT}(t/3) = 1$

$$\varphi(t) = 2\pi f_c t + k_p 2 = 2\pi f_c t + \pi/2$$

For $|t| > 1.5$, $\mathrm{RECT}(t/3) = 0$

$$\varphi(t) = 2\pi f_c t$$



$$s_{PM}(t) = \cos(\varphi(t))$$

2. AN FM WAVE $s_{FM}(t)$ IS MODULATED BY A SINUSOIDAL MESSAGE $m(t) = A_m \cos(2\pi f_m t)$. THE CARRIER FREQUENCY $f_c = 1\,MHz$ & THE CARRIER AMPLITUDE $A_c = 1V$. $A_m = 1V$, $f_m = 1\,kHz$ & MODULATION SENSITIVITY $k_f = 1000\,Hz/volt$.

(a) WRITE DOWN AN EXPRESSION FOR THE INSTANTANEOUS FREQUENCY

ⓑ WRITE DOWN A MATHEMATICAL EXPRESSION FOR s_{FM}

(c) SKETCH THE SPECTRUM OF $s_{FM}(t)$. INDICATE ALL APPROPRIATE NUMERICAL VALUES.

(d) ESTIMATE THE BANDWIDTH USING CARSON'S RULE.

(a)
$$f_i = f_c + k_f A_m \cos(2\pi f_m t)$$
$$= 1\times10^6 + 1000\times1 \cos(2\pi\times1\times10^3 t)$$

ⓑ
$$\frac{d\varphi}{dt} = 2\pi f_i$$
$$\varphi(t) = 2\pi f_c t + k_f A_m \sin(2\pi f_m t) \times \frac{2\pi}{2\pi f_m}$$
$$= 2\pi\times10^6 t + \frac{1000}{10^3} \sin(2\pi f_m t)$$
$$= 2\pi\times10^6 t + \sin(2\pi f_m t) \quad \text{———} \quad (*)$$

$$s_{FM}(t) = \cos(2\pi\times10^6 t + \beta \sin(2\pi\times10^3 t))$$

FROM (*), WE SEE THAT $\beta = 1 \left(\equiv \frac{k_f A_m}{f_m} \right)$

(c) $s_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} c_n \cos[2\pi(f_c + n f_m)t]$

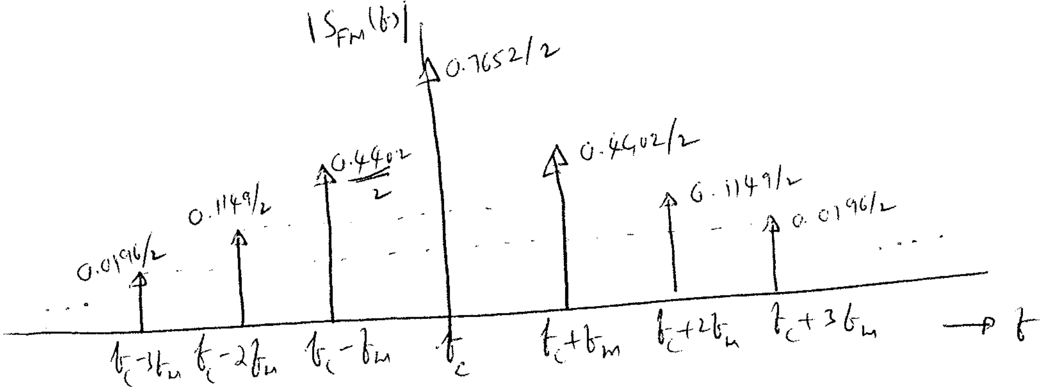
$c_n = J_n(\beta)$, $A_c = 1$

FROM THE TABLE, A3.1
$J_0(1) = 0.7652$
$J_1(1) = 0.4402$
$J_2(1) = 0.1149$
$J_3(1) = 0.0196$

PROPERTY 1 OF THE BESSEL FUNCTION $\rightarrow J_{-n}(\beta) = (-1)^n J_n(\beta)$

$\therefore J_{-1}(1) = -0.4402$, $|J_{-1}(1)| = |J_1(1)|$
$J_{-2}(1) = 0.1149$
$J_{-3}(1) = -0.0196$

$$S_{FM}(f) = \sum_{n=-\infty}^{\infty} c_n\left[\delta(f-(f_c+nf_m)) + \delta(f+f_c+nf_m)\right]/2$$



(ONLY POSITIVE SIDE OF THE SPECTRUM IS SHOWN HERE.

(d) CARSON'S RULE: BANDWIDTH $= 2(\Delta f + f_m) = B$

$\Delta f = k_f A_m = 1000$ Hz , $f_m = 1000$ Hz

$B = 4 \times 10^3$ Hz.

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APPENDIX 3

BESSEL FUNCTIONS

A3.1 Series Solution of Bessel's Equation

In its most basic form, *Bessel's equation of order n* is written as

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0 \tag{A3.1}$$

which is one of the most important of all variable-coefficient differential equations. For each order n, a solution of this equation is defined by the power series

$$J_n(x) = \sum_{m=0}^{\infty} \frac{(-1)^m\left(\frac{1}{2}x\right)^{n+2m}}{m!(n+m)!} \tag{A3.2}$$

The function $J_n(x)$ is called a *Bessel function of the first kind of order n*. Equation (A3.1) has two coefficient functions—namely, $1/x$ and $(1 - n^2/x^2)$. Hence, it has no finite singular points except the origin. It follows therefore that the series expansion of Eq. (A3.2) converges for all $x > 0$. Equation (A3.2) may thus be used to numerically calculate $J_n(x)$ for $n = 0, 1, 2, \ldots$. Table A3.1 presents values of $J_n(x)$ for different orders n and varying x. It is of interest to note that the graphs of $J_0(x)$ and $J_1(x)$ resemble the graphs of $\cos x$ and $\sin x$, respectively; see the graphs of Fig. 4.6 in Chapter 4.

TABLE A3.1 *Table of Bessel Functions*[a]

→ β

n ↓

$n \backslash x$	$J_n(x)$ 0.5	1	2	3	4	6	8	10	12
0	0.9385	0.7652	0.2239	−0.2601	−0.3971	0.1506	0.1717	−0.2459	0.0477
1	0.2423	0.4401	0.5767	0.3391	−0.0660	−0.2767	0.2346	0.0435	−0.2234
2	0.0306	0.1149	0.3528	0.4861	0.3641	−0.2429	−0.1130	0.2546	−0.0849
3	0.0026	0.0196	0.1289	0.3091	0.4302	0.1148	−0.2911	0.0584	0.1951
4	0.0002	0.0025	0.0340	0.1320	0.2811	0.3576	−0.1054	−0.2196	0.1825
5	—	0.0002	0.0070	0.0430	0.1321	0.3621	0.1858	−0.2341	−0.0735
6		—	0.0012	0.0114	0.0491	0.2458	0.3376	−0.0145	−0.2437
7			0.0002	0.0025	0.0152	0.1296	0.3206	0.2167	−0.1703
8			—	0.0005	0.0040	0.0565	0.2235	0.3179	0.0451
9				0.0001	0.0009	0.0212	0.1263	0.2919	0.2304
10				—	0.0002	0.0070	0.0608	0.2075	0.3005
11					—	0.0020	0.0256	0.1231	0.2704
12						0.0005	0.0096	0.0634	0.1953
13						0.0001	0.0033	0.0290	0.1201
14						—	0.0010	0.0120	0.0650

[a] For more extensive tables of Bessel functions, see Abramowitz and Stegun (1965, pp. 358–406).