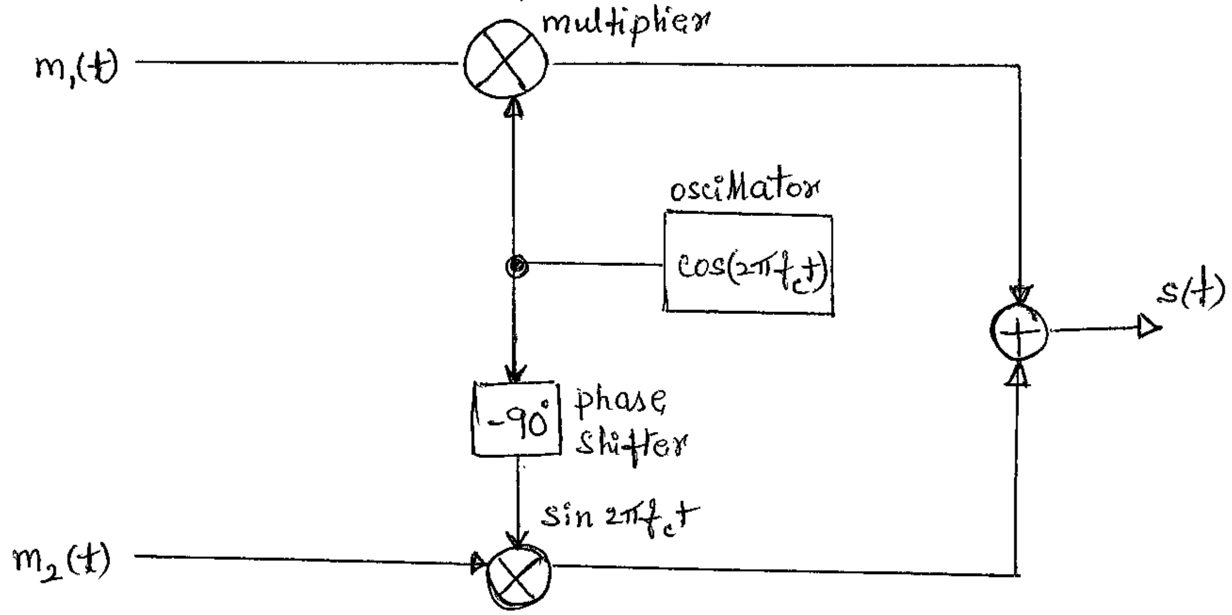
We have seen that DSB/SC is wasteful of bandwidth since it requires a bandwidth of 2W Hz to transmit a message of W Hz. SSB only requires W Hz. Is it possible to send two messages of bandwidth W in a bandwidth of 2W Hz, using a modified form of DSB/SC? The result is that the bandwidth efficiency is the same as SSB. If the scheme exists, draw the block diagram of the system. What happens if there is a phase error in the receiver local oscillator?

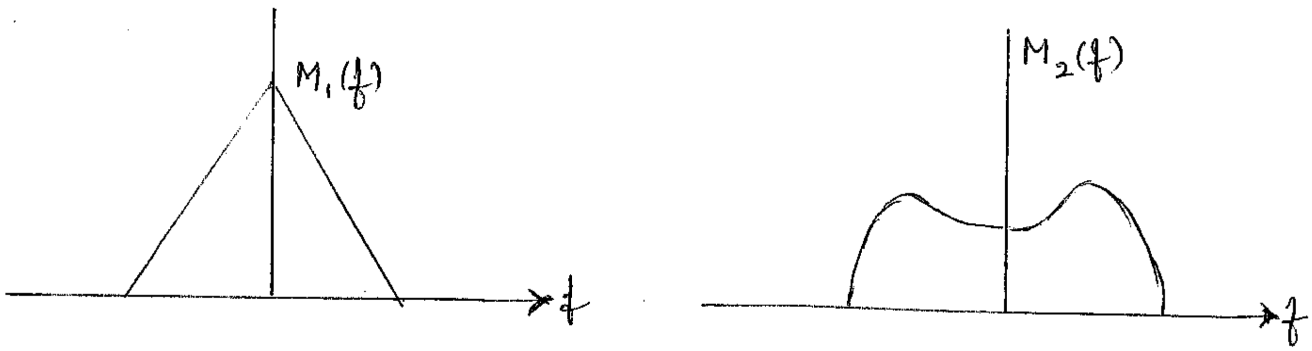
Consider the following scheme:



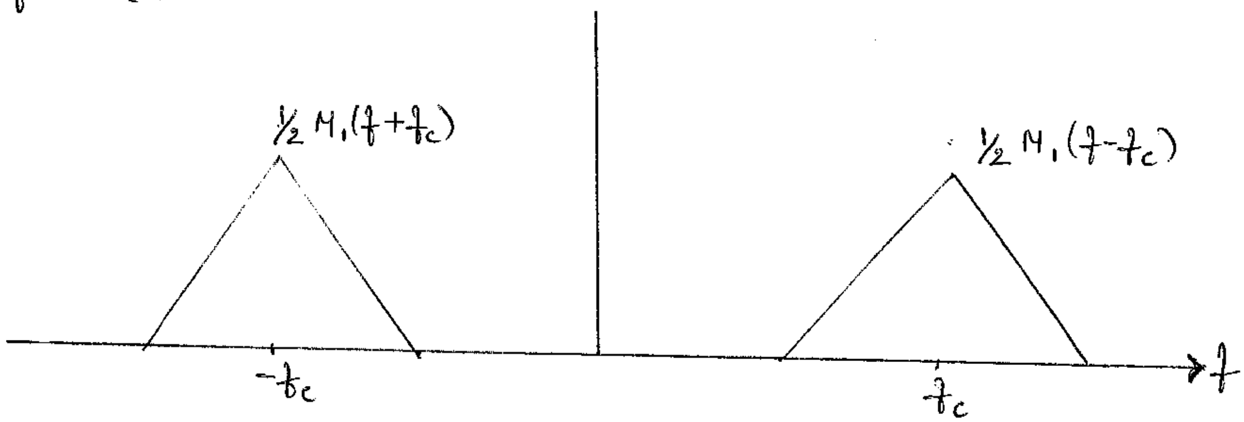
<u>Transmitter</u>: $s(t) = m_1(t)\cos 2\pi f_c t + m_2(t)\sin 2\pi f_c t$

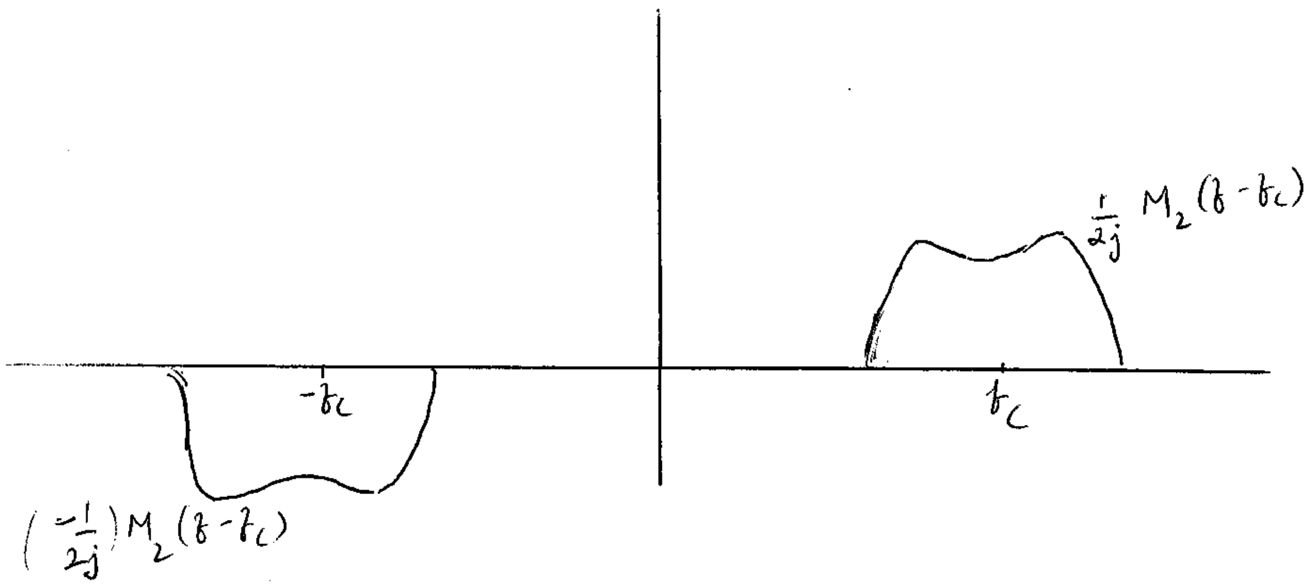
where $m_1(t)$ & $m_2(t)$ are message signals. The component of $s(t)$ corresponding to the carrier $\cos(2\pi f_c t)$ is called the in-phase component (that is $m_1(t)$) and that corresponding to the carrier $\sin 2\pi f_c t$ is called the quadrature component ($m_2(t)$). This is because the carrier $\sin 2\pi f_c t$ is -90° phase shifted with respect to $\cos 2\pi f_c t$. Since this scheme utilizes two carriers in quadrature, it is called quadrature carrier multiplexing or quadrature amplitude modulation (QAM).

Let $m_1(t)$ and $m_2(t)$ have the following spectra:

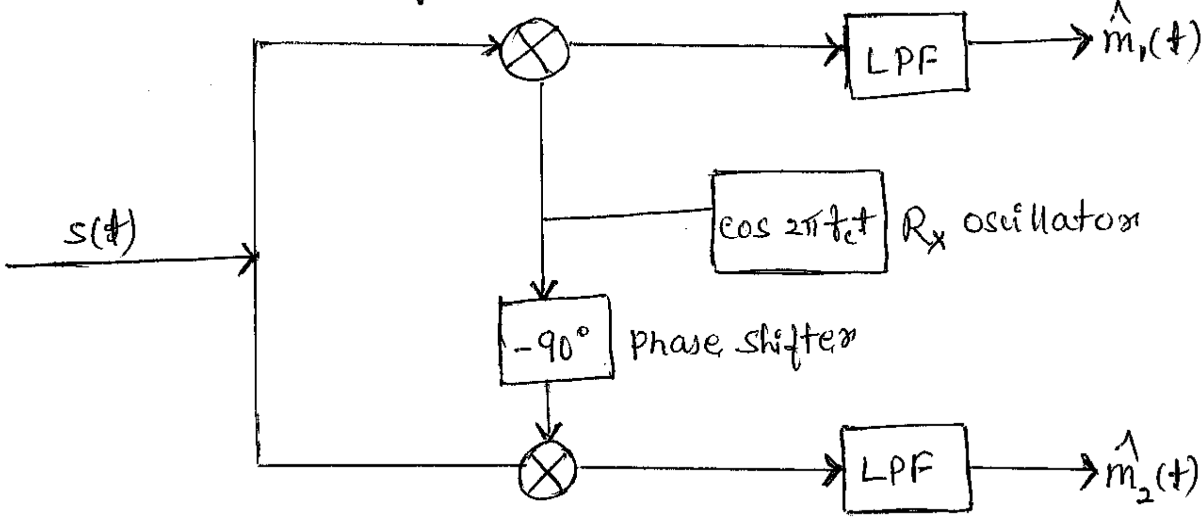


Then, the spectra of the first and second terms of $s(t)$ above are:





The receiver may be drawn as follows:



The question is: How is $\hat{m}_1(t)$ related to $m_1(t)$ and $\hat{m}_2(t)$ related to $m_2(t)$?

The output $\hat{m}_1(t)$ is:

$$\hat{m}_1(t) = s(t) \cos 2\pi f_c t$$

$$= \left[m_1(t) (\cos 2\pi f_c t) + m_2(t) \sin 2\pi f_c t\right] \cos 2\pi f_c t$$

Using the identity $\cos^2 A = \frac{(1 + \cos 2A)}{2}$

and $\sin 2A = 2 \sin A \cdot \cos A$

$$\hat{m}_1(t) = m_1(t) \left[\tfrac{1}{2} + \tfrac{1}{2} \cos 4\pi f_c t\right] + \frac{m_2(t)}{2} \sin 4\pi f_c t$$

The components at $2f_c$ are filtered out with the LPF's. Therefore, $\hat{m}_1(t) = \tfrac{1}{2} m_1(t)$.

$\therefore$ $m_1(t)$ can be recovered using this scheme.
(since $\hat{m}_1(t)$ is within a scale factor of $m_1(t)$).

Now. letus see about $\hat{m}_2(t)$:

$$\hat{m}_2(t) = s(t) \sin 2\pi f_c t$$

$$= [m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t] \sin 2\pi f_c t$$

Using the further identity

$$\sin^2 A = 1/2 - 1/2 \cos 2A$$

we get

$$\hat{m}_2(t) = m_2(t)\left[1/2 - 1/2 \cos 4\pi f_c t\right] + m_1(t) \sin 2\pi f_c t . \cos 2\pi f_c t .$$

$$= \frac{m_2(t)}{2} + \frac{m_1(t)}{2} \sin 4\pi f_c t - \frac{m_2(t)}{2} \cos 4\pi f_c t$$

After low-pass filtering, the components at $2f_c$ are eliminated. Therefore,

$$\hat{m}_2(t) = 1/2\, m_2(t)$$

Thus, $m_2(t)$ can be recovered as well.

Thus, the modified DSB/SC scheme can be used to transmit 2 messages of bandwidth W over a channel of 2W Hz bandwidth.

What happens when the receiver oscillator has a phase error?

In this case, the received signal $\hat{m}_1(t)$ is;

$$\hat{m}_1(t) = s(t) \cos(2\pi f_c t + \phi)$$

$$= \left[m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t\right] \cos(2\pi f_c t + \phi)$$

Using $\cos A \cos B = \frac{1}{2} \cos A-B + \frac{1}{2} \cos A+B$

$\cos A \sin B = \frac{1}{2} \sin B-A + \frac{1}{2} \sin A+B$

we get

$$\hat{m}_1(t) = \frac{1}{2} m_1(t) \cos\phi + \frac{1}{2} m_2(t) \sin\phi$$

$$+ \text{terms at } 2f_c$$

Therefore, after the lowpass filter, the output $\hat{m}_1(t)$ contains contributions from both $m_1(t)$ and $m_2(t)$. i.e., we get interference from the other channel. This is called the cross talk.

In a similar way,

$$\hat{m}_2(t) = -\frac{m_1}{2}(t) \sin\phi + \frac{m_2}{2}(t) \cos\phi.$$

$$+ \text{terms at } 2f_c.$$

$\therefore$ $\hat{m}_2(t)$ also contains unwanted components, from $m_1(t)$.