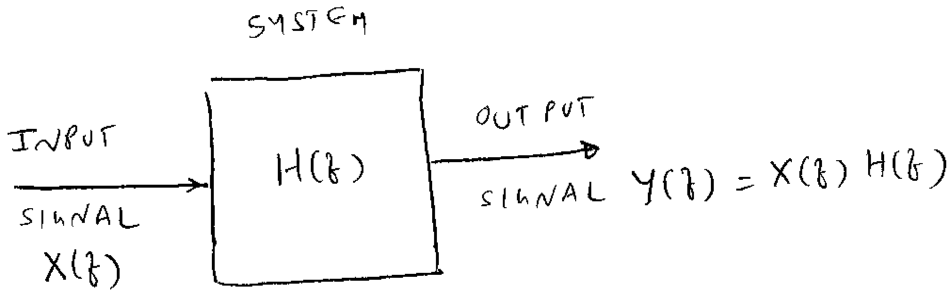
FILTERING:

WHEN AN INPUT SIGNAL PASSES THROUGH A SYSTEM, EACH FREQUENCY
COMPONENT OF THE INPUT SIGNAL MAY BE ALTERED BY THE SYSTEM.
THE SYSTEM CAN BE DESCRIBED BY A SET OF COMPLEX QUANTITIES
$H(f)$ AS SHOWN BELOW:



THE FREQUENCY COMPONENTS OF THE INPUT SIGNAL AT FREQUENCY 'f'
IS ATTENUATED/AMPLIFIED BY A FACTOR $|H(f)|$ AND THE SYSTEM
ADDS A PHASE EQUAL TO $\arg[H(f)]$ TO THE FREQ. COMPONENT OF
THE SIGNAL AT 'f'.

SUPPOSE THE INPUT SIGNAL SPECTRUM IS GIVEN BY $X(f)$.
THE OUTPUT SIGNAL SPECTRUM IS GIVEN BY

$$Y(f) = X(f)\,H(f) \qquad (1)$$

FROM THE CONVOLUTION THEOREM, WE KNOW THAT THE MULTIPLICATION
IN THE FREQUENCY DOMAIN IS THE CONVOLUTION IN TIME DOMAIN.

THEREFORE, IT FOLLOWS THAT

IF $x(t) \rightleftharpoons X(f)$

$y(t) \rightleftharpoons Y(f)$

$h(t) \rightleftharpoons H(f)$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \longrightarrow (2)$$

$H(f)$ IS CALLED THE TRANSFER FUNCTION OF THE SYSTEM. ITS INVERSE FOURIER TRANSFORM $h(t)$ IS CALLED THE IMPULSE RESPONSE.

TO UNDERSTAND WHY IT IS CALLED IMPULSE RESPONSE, CONSIDER THE INPUT WHICH IS AN IMPULSE $\delta(t)$. SINCE

$$\delta(t) \rightleftharpoons 1 \longrightarrow (3)$$

FROM EQ. (1), THE OUTPUT OF THE SYSTEM IS

$$Y(f) = 1 . H(f)$$

or

$$y(t) = h(t)$$

THEREFORE, THE FUNCTION $h(t)$ REPRESENTS THE OUTPUT OF THE SYSTEM WHEN THE INPUT IS AN IMPULSE FUNCTION; HENCE THE NAME 'IMPULSE RESPONSE'.

AMPLITUDE & PHASE RESPONSE:

THE TRANSFER FUNCTION MAY BE EXPRESSED IN THE FORM

$$H(f) = |H(f)| \exp[j\beta(f)] \longrightarrow (4)$$

WHERE $|H(f)|$ IS CALLED THE AMPLITUDE RESPONSE, AND $\beta(f)$ IS CALLED THE PHASE RESPONSE. THE PHASE RESPONSE IS GIVEN BY

$$\beta(f) = \text{Arg}[H(f)]. \longrightarrow (5)$$

WHEN THE IMPULSE RESPONSE $h(t)$ IS REAL, FROM THE PROPERTIES OF THE FOURIER TRANSFORM, IT FOLLOWS THAT

$$|H(f)| = |H(-f)| \quad \longrightarrow (6)$$

(SINCE $|C_n| = |C_{-n}|$ FOR FOURIER SERIES)

AND

$$\beta(f) = -\beta(-f) \longrightarrow (7)$$

i.e. THE AMPLITUDE RESPONSE IS AN EVEN FUNCTION OF FREQUENCY, ~~AND~~ WHEREAS THE PHASE RESPONSE IS AN ODD FUNCTION OF FREQUENCY.

THE AMPLITUDE RESPONSE IS SOMETIMES EXPRESSED IN THE DECIBEL (dB) UNITS BY WRITING

$$\alpha(f) = 20 \log_{10} |H(f)| = 10 \log_{10} |H(f)|^2 \longrightarrow (8)$$

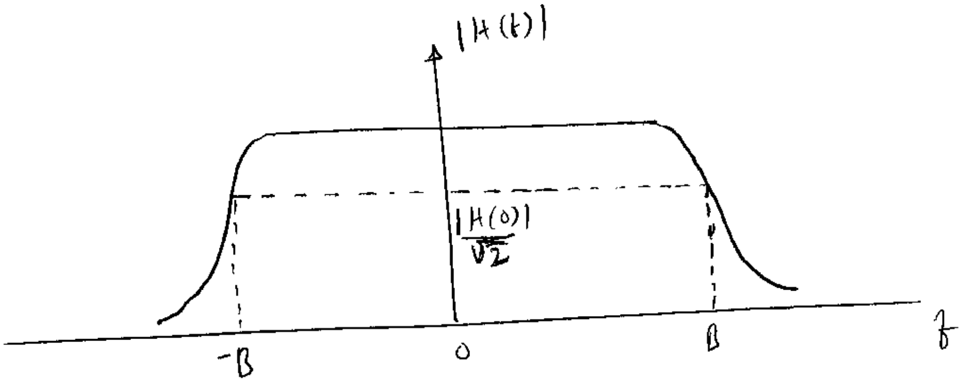
THE FUNCTION $\alpha(f)$ IS CALLED THE GAIN OF THE SYSTEM.

THE SQUARED AMPLITUDE RESPONSE $|H(f)|^2$ IS IDENTIFIED WITH POWER.

SYSTEM BANDWIDTH:

A COMMON DEFINITION OF SYSTEM BANDWIDTH IS THE 3-dB BANDWIDTH.

IN THE CASE OF A LOW-PASS SYSTEM, THE 3-dB BANDWIDTH IS DEFINED AS THE FREQUENCY AT WHICH THE AMPLITUDE RESPONSE DROPS TO A VALUE EQUAL TO $|H(0)|/\sqrt{2}$, AS SHOWN IN THE FIGURE.

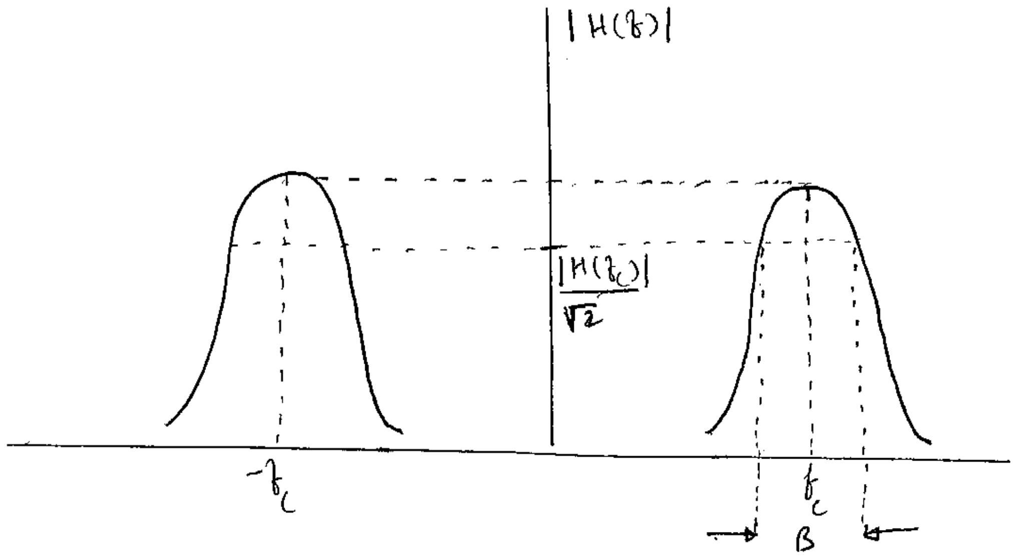


LOW-PASS SYSTEM

IN THE ABOVE FIGURE, AT $f = B$, THE AMPLITUDE RESPONSE DROPS TO $|H(0)|/\sqrt{2}$ AND THEREFORE B IS THE 3-dB BANDWIDTH. IT IS CALLED THE 3-dB BANDWIDTH BECAUSE THE RATIO OF SQUARED AMPLITUDE $|H(0)|^2$ AT $f=0$ AND AT $f = B$, $|H(0)|^2/2$ IS 3 dB IN LOGARITHMIC UNITS, i.e.

$$20 \log_{10} \frac{H(f)\big|_{f=B}}{H(f)\big|_{f=0}} = -3$$

IN THE CASE OF A BAND PASS SYSTEM, the 3-dB BANDWIDTH IS
DEFINED AS THE DIFFERENCE BETWEEN THE FREQUENCIES AT WHICH
THE AMPLITUDE RESPONSE DROPS TO $|H(f_c)|/\sqrt{2}$ WHERE f_c IS
THE MID-BAND FREQUENCY AS SHOWN IN THE FIGURE.

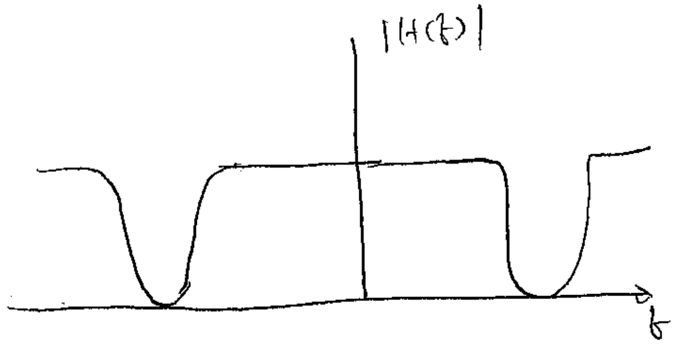


FILTER TYPES:

FILTERS CAN BE CLASSIFIED AS (i) LOW-PASS (ii) HIGH PASS
(iii) BAND PASS & (iv) BAND-STOP. LOW PASS & HIGH PASS FILTERS
TRANSMIT LOW AND HIGH FREQUENCIES, RESPECTIVELY. BAND PASS
AND BAND-STOP FILTERS TRANSMIT INTERMEDIATE AND ALL BUT
INTERMEDIATE FREQUENCIES, RESPECTIVELY. THE FREQUENCY
RESPONSE OF A FILTER IS CHARACTERIZED BY A **PASS BAND**
AND **STOP BAND**, WHICH ARE SEPARATED BY A **GUARD BAND**.

THE FREQUENCIES INSIDE THE PASSBAND ARE TRANSMITTED WITH LITTLE OR NO DISTORTION, WHEREAS THOSE IN THE STOP BAND ARE REJECTED.

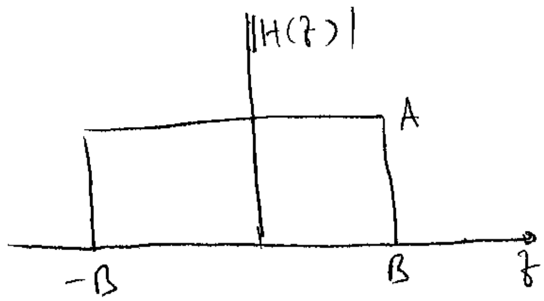
HIGH PASS FILTER

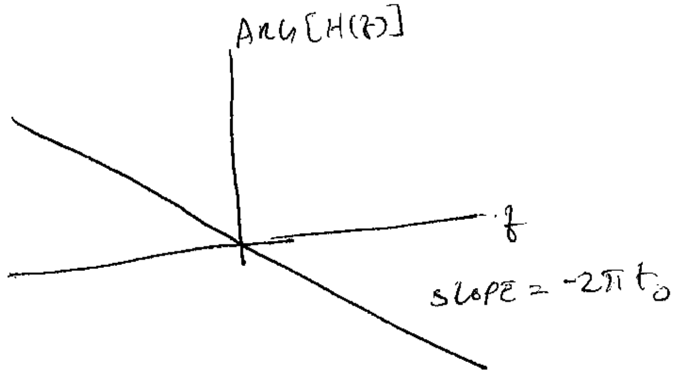


BAND-STOP FILTER

EXAMPLE 1: FIND THE IMPULSE RESPONSE OF AN IDEAL LOW PASS FILTER THAT HAS A TRANSFER FUNCTION

$$H(f) = A\,\mathrm{rect}(f/2B)\exp(-j2\pi f t_0) \quad \rightarrow (9)$$





LET $G(f) = A\,\mathrm{rect}(f/2B)$. WE KNOW THAT

$$A\,\mathrm{rect}(f/2B) \rightleftharpoons 2BA\,\mathrm{sinc}(2Bt) \quad \rightarrow (10)$$

EQ. (9) CAN BE WRITTEN AS

$$H(f) = G(f) \cdot \exp(-j2\pi f t_0)$$

USING THE TIME SHIFTING PROPERTY, WE ~~HAV~~ OBTAIN

$$h(t) = g(t - t_0)$$

USING EQ. (10), WE GET

$$h(t) = 2AB \operatorname{sinc}[2B(t - t_0)].$$

EXAMPLE 2: INPUT SIGNAL ~~g(t)~~ TO AN IDEAL LOW PASS FILTER IS GIVEN BY

$$g(t) = 5 \cos(2\pi \times 5t) + 2 \sin(2\pi \times 10t)$$

THE FILTER TRANSFER FUNCTION IS

$$H(f) = \operatorname{rect}(f/15)$$

FIND THE OUTPUT SIGNAL.

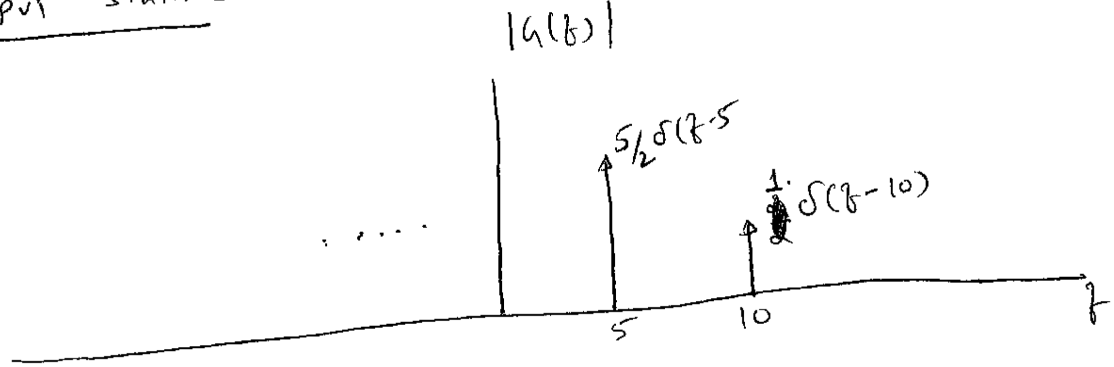
LET $g_1(t) = 5 \cos(2\pi \times 5t)$ and $g_2(t) = 2 \cdot \sin(2\pi \times 10t)$.

$$G_1(f) = \frac{5}{2} \cdot [\delta(f-5) + \delta(f+5)], \quad G_2(f) = \frac{2}{2j}[\delta(f-10) - \delta(f+10)]$$

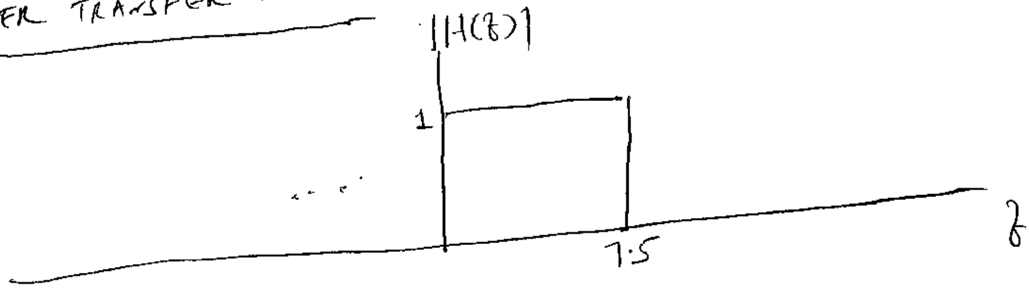
$$G(f) = G_1(f) + G_2(f).$$

THE FILTER HAS A BANDWIDTH OF 7.5 HZ AS SHOWN IN THE FIGURE. THEREFORE, IT TRASMITS $G_1(f)$ AND REJECTS $G_2(f)$.

INPUT SIGNAL



FILTER TRANSFER FUNCTION:



OUTPUT SIGNAL:

THEREFORE THE FILTER OUTPUT IS GIVEN BY

$$Y(f) = G(f)\,H(f)$$
$$= G_1(f)$$

IN TIME DOMAIN,

$$\cancel{y(t) = g_2(t) = 2.5\sin(2\pi\times 10t)}$$
$$y(t) = g_1(t) = 5\cos(2\pi\times 5\,t)$$