ANGLE MODULATION

Consider the signal

$$s(t) = A(t)\cos[\phi(t)]$$

Here the $A(t)$ is the amplitude and $\phi(t)$ is the angle of the carrier. Previously we have seen that the amplitude of the carrier is varied in accordance with the message in AM systems. Instead, if the angle of the carrier ~~varying~~ is varies in accordance with the message signal, it is called angle modulation.

The Concept of Frequency

Consider a sinusoid $c(t)$:

$$c(t) = A_c \cos(2\pi f_c t + \theta)$$

Note that the argument $\phi = 2\pi f_c t + \theta$ is a phase always expressed in radians.

In a certain time period Δt, the argument increases from $2\pi f_c t + \theta$ radians to $2\pi f_c(t+\Delta t)+\theta$ radians; i.e., the phase increases by $2\pi f_c \Delta t$

radians in Δt seconds. Thus, over the interval Δt, the phase changes at the rate of $2\pi f_c$ radians/sec.

$$\therefore \frac{\Delta\phi}{\Delta t} = 2\pi f_c$$

or

$$\frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = f_c \qquad (1)$$

Where $\Delta\phi$ is the phase change over the period Δt. Eq. (1) expresses the average rate-of-change of phase over a period Δt. We see that f_c is $\frac{1}{2\pi}$ × avg. rate of change of phase in radians.

But suppose the frequency varies continuously with time. Then, we can define instantaneous frequency $f_i(t)$ as the limit as $\Delta t \to 0$ in (1):

$$\boxed{f_i(t) = \frac{1}{2\pi}\frac{d\phi}{dt}}$$

$\therefore$ instantaneous freq. is $\frac{1}{2\pi}$ × the derivative of phase with respect to time.

ANGLE MODULATION

There are two common ways in which a message $m(t)$ may be embedded onto the angle of a carrier.

Phase Modulation (PM)

$$m(t) \longrightarrow \boxed{\text{PM}} \longrightarrow s_{PM}(t)$$

The PM generates an output sinusoid whose phase varies with the message signal. Therefore the phase signal $\phi(t)$ of the PM wave $s_{pm}(t)$ is :

$$\phi(t) = 2\pi f_c t + k_p m(t) \qquad \text{(radians)} \quad (2)$$

where k_p is the sensitivity in radians/volt. Since $s(t)$ for the angle-modulated case is

$$s(t) = A_c \cos[\phi(t)]$$

Then

$$s_{pm}(t) = A_c \cos\left(2\pi f_c t + k_p m(t)\right) \quad (3)$$

Frequency Modulation (FM)

The FM generates an output sinusoid $s_{FM}(t)$ whose instantaneous frequency $f_i(t)$ varies

$$m(t) \longrightarrow \boxed{\text{FM}} \longrightarrow s_{FM}(t)$$

with $m(t)$. Thus, the instantaneous freq.

$$f_i(t) = f_c + k_f\, m(t) \qquad (Hz) \qquad (4)$$

where k_f is the sensitivity in Hz/volt.

$$f_i(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

$$\therefore \quad \phi(t) = 2\pi \int_0^t f_i(t)\, dt + K \qquad (5)$$

In the following we take the constant $K = 0$.

By substituting (4) into (5) we have the $\phi(t)$ for an FM wave:

$$\phi(t) = 2\pi \int_0^t f_i(t)\, dt$$

$$= 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt \qquad (6)$$

Therefore the FM wave $s_{FM}(t)$ is given ~~by~~ as:

$$s_{FM}(t) = A_c \cos[\phi(t)]$$

$$= A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right) \quad (7)$$

Interelationships Between PM & FM

Suppose we differentiate $m(t)$ before applying to a freq. modulator. The resulting $\phi(t)$ from (6) is

$$\phi(t) = 2\pi f_c t + 2\pi k_f \int_0^t \frac{dm(t)}{dt}\,dt$$

$$= 2\pi f_c t + 2\pi k_f \, m(t) \quad \text{— (8)}$$

This phase term has exactly the same form as the PM version of $\phi(t)$ in (2), except for the presence of a constant 2π. By setting $2\pi k_f = k_p$, we see that differentiating the message signal first and then passing through a FM is equivalent to phase modulation (PM).

Thus, the following scheme is equivalent to PM :

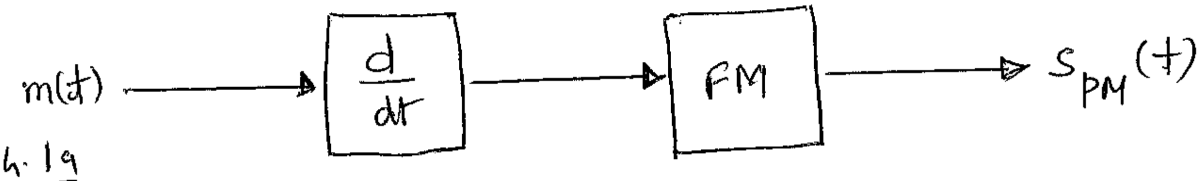


Fig. 1a

Further, suppose we integrate m(t) before applying it to a phase modulator. The result from (2) is

$$\phi(t) = 2\pi f_c t + k_p \int_0^t m(t)\,dt$$

This version of $\phi(t)$ now has the same form as $\phi(t)$ for the FM case. see (6), except for the presence of the 2π term which can be compensated by adjusting k_p. Thus, the following scheme is equivalent to FM :

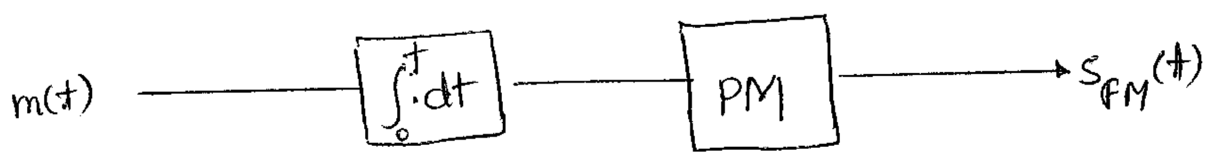


Fig. 1b

Thus, phase modulation and freq. modulation are interchangable. For this reason, they are called angle modulation schemes.

Examples

1. Sinusoidally modulated FM wave

This is shown in Fig. 2. Note that when the sinusoidally modulating wave (a) is large, the instantaneous freq. of the modulated wave (b) is large, and vice-versa.

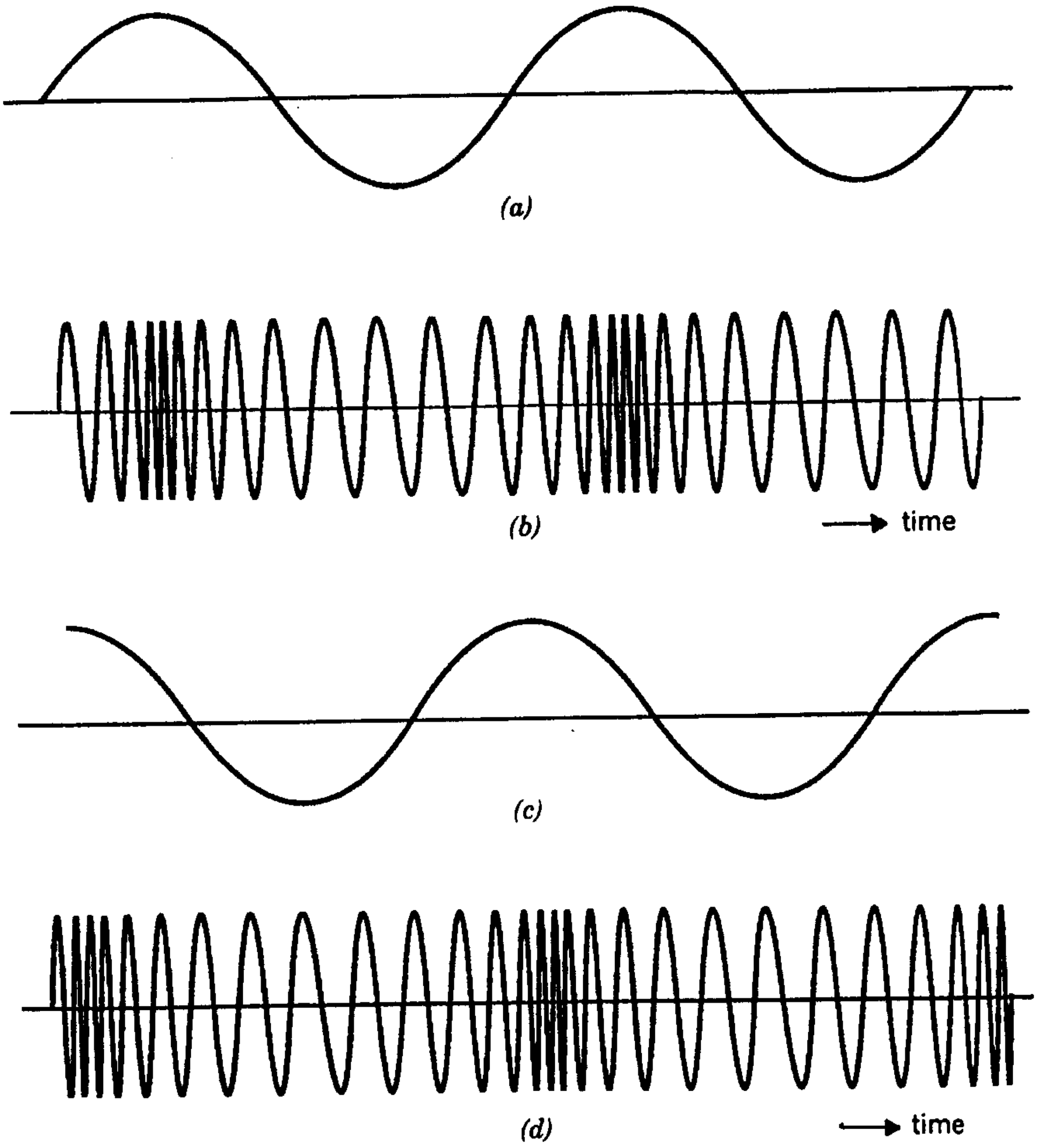
2. Sinusoidally modulated PM wave

Note that it is difficult to see the phase of the modulated wave by looking at the wave; to really see the phase we must have the unmodulated carrier drawn close beneath it for comparison. There are two ways to view the phase modulated wave ((d) in Fig. 2).

a) directly as a phase modulator: Here, the expression for the modulated wave (from eq. 4) is:

$$s(t) = A_c \cos(2\pi f_c t + k_p\, m(t))$$

since freq. is the derivative of phase, regions of $s(t)$ where $m(t)$ has positive slope will



~~Figure 7.36~~
(a) *Sinusoidal modulating wave* m(t). (b) *Frequency-modulated wave.* (c) *Derivative of* m(t) *with respect to time.* (d) *Phase-modulated wave.*

Fig. 2

demonstrate higher frequencies, and −ve slope will demonstrate lower frequencies.

(b) indirectly, as a freq. modulator (as in Fig. 1a)

Here, we differentiate the original message m(t) to give the signal shown in Fig. 2 part(c). This signal is used to frequency modulate the carrier.

Note that in AM systems, the frequency of the carrier remains constant, but the amplitude changes in accordance with the message signal. In contrast, in FM or PM systems amplitude remains constant, but the instantaneous frequency varies with the message signal.
