SAMPLING THEOREM:

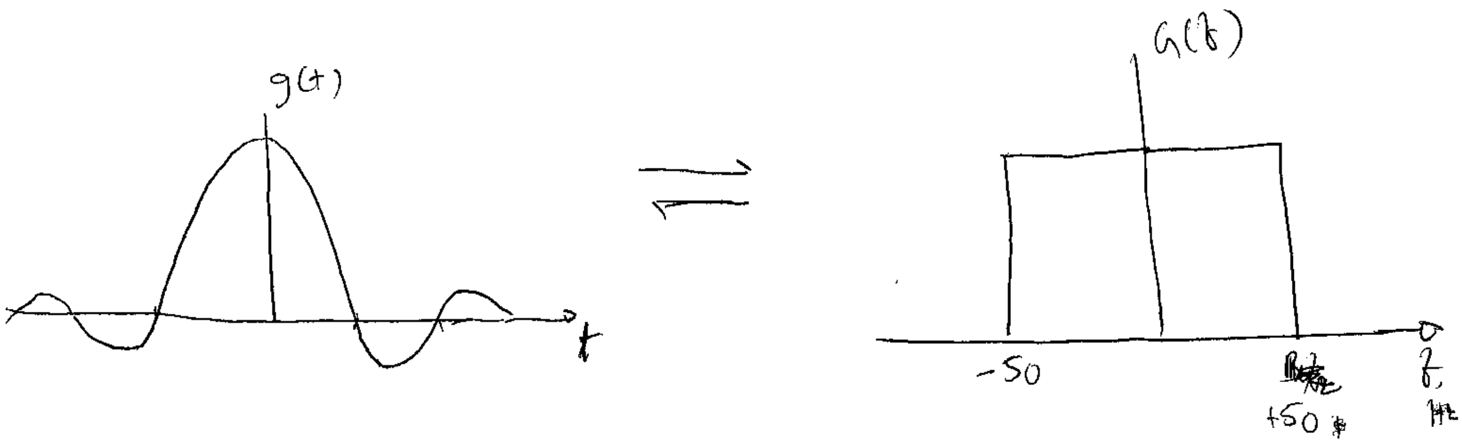
A BAND-LIMITED SIGNAL OF FINITE ENERGY, WHICH HAS NO FREQUENCY COMPONENTS HIGHER THAN ~~ARE~~ W HERTZ, ~~MAY~~ BE COMPLETELY RECOVERED FROM ~~THE~~ A KNOWLEDGE OF ITS SAMPLES TAKEN AT THE RATE OF 2W SAMPLES/s.

THE SAMPLING RATE OF 2W SAMPLES/s IMPLIES THAT THE INTERVAL BETWEEN SAMPLES IS $1/2W$ SECONDS. ~~THE IN~~ THIS INTERVAL IS CALLED NYQUIST INTERVAL. THE SAMPLING RATE OF 2W SAMPLES/s IS CALLED NYQUIST RATE.

EXAMPLE: CONSIDER A SIGNAL $g(t) = 100 \operatorname{sinc}(100t)$.

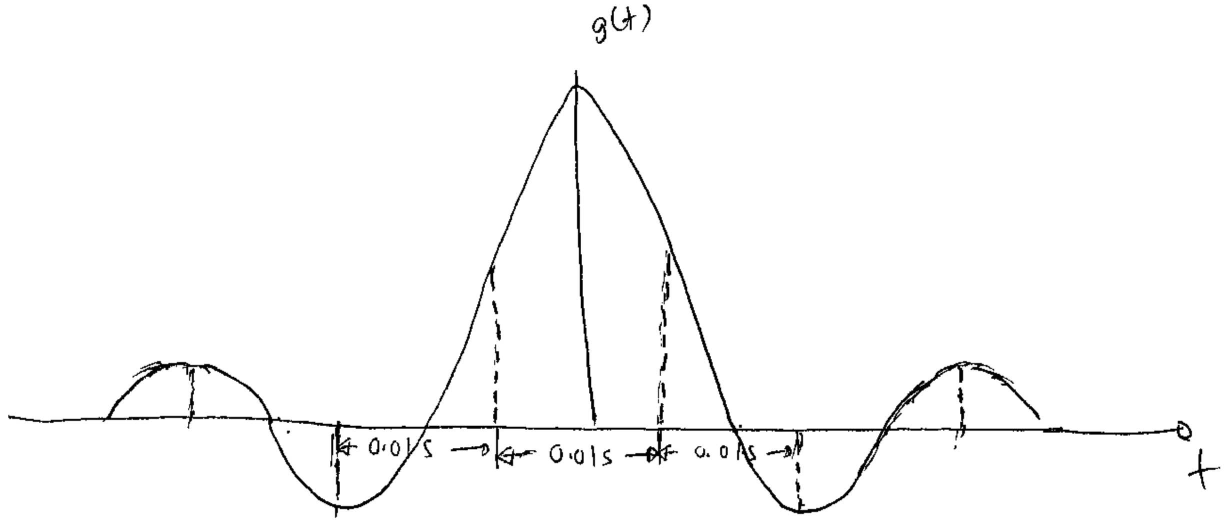
ITS FOURIER TRANSFORM IS

$$G(f) = \operatorname{rect}(f/100)$$



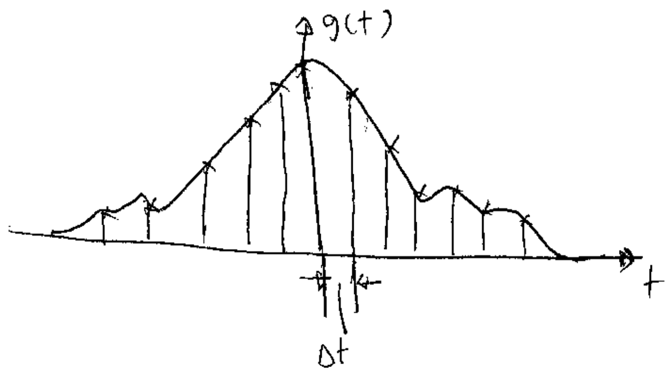
THE SIGNAL $g(t)$ HAS NO FREQUENCY COMPONENTS HIGHER THAN 50 Hz. THEREFORE, SAMPLING RATE OF 100 Hz IS SUFFICIENT TO COMPLETELY RECOVER THE SIGNAL.

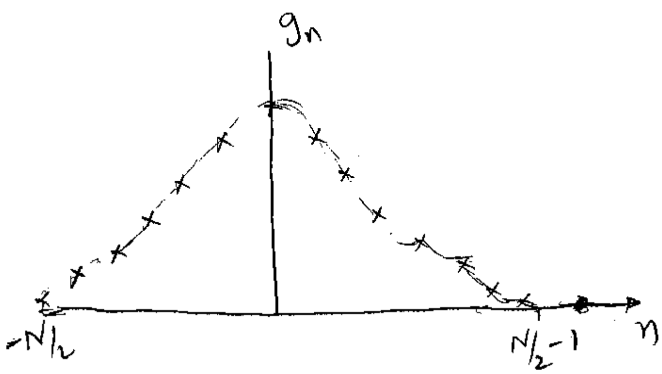
NYQUIST INTERVAL IN THIS CASE IS $\frac{1}{100} = 0.01\ s = \Delta t$



DISCRETE FOURIER TRANSFORM :

CONSIDER AN ANALOG SIGNAL $g(t)$ AS SHOWN BELOW



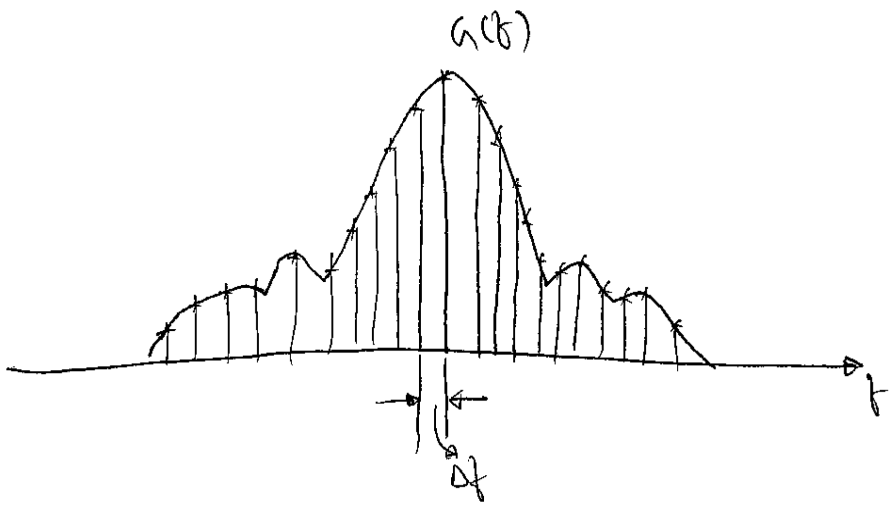


TO REPRESENT THIS SIGNAL ON A DIGITAL COMPUTER, LET US DISCRETISE THIS SIGNAL WITH AN INTERVAL OF Δt; i.e.

$$g(t) = g(n\Delta t), \quad n = -N/2, -N/2+1, \ldots, N/2-1 \qquad \rightarrow (39)$$
$$\triangleq g_n$$

WHERE N IS THE NUMBER OF SAMPLES.

LET THE SPECTRUM OF $g(t)$ be $G(f)$ AS SHOWN BELOW.



WE DISCRETISE THE SPECTRUM WITH AN INTERVAL OF Δf, i.e.

$$G(f) = G(k\Delta f), \quad k = -N/2, -N/2+1, \ldots, N/2-1 \qquad \rightarrow (40)$$
$$\triangleq G_k$$

THE HIGHEST FREQUENCY COMPONENT IS $N/2 \cdot \Delta f$. THEREFORE, THE NYQUIST SAMPLING RATE TO COMPLETELY RECOVER THE SIGNAL IS $N\Delta f$ AND THE NYQUIST INTERVAL IS $\frac{1}{N\Delta f}$.

WE CHOOSE THE INTERVAL Δt IN TIME DOMAIN TO BE EQUAL TO NYQUIST INTERVAL, i.e.

$$\Delta t = \frac{1}{N\Delta f} \quad \text{or} \quad \Delta t \cdot \Delta f = \frac{1}{N} \quad \rightarrow (41)$$

TO OBTAIN THE DISCRETE FOURIER TRANSFORM (DFT), FIRST CONSIDER THE FOURIER TRANSFORM

$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot \exp(-j2\pi f t)\, dt$$

REPLACING THE INTEGRAL BY SUM AND USING EQS. (39) & (40), WE HAVE

$$G(k\Delta f) = \sum_{n=-N/2}^{N/2-1} g(n\Delta t)\, e^{-j2\pi n\Delta t \cdot k\Delta f} \cdot \Delta t$$

USING EQ. (41), WE HAVE

$$G(k\Delta f) = \Delta t \sum_{n=-N/2}^{N/2-1} g(n\Delta t) \cdot e^{-j2\pi nk/N} \quad \rightarrow (42)$$

EQ. (42) DEFINES THE DISCRETE FOURIER TRANSFORM. (DFT).

SIMILARLY, THE INTEGRAL IN INVERSE FOURIER TRANSFORM MAY BE APPROXIMATED BY A SUM TO GIVE

$$g(n\Delta t) = \Delta f \sum_{k=-N/2}^{N/2-1} G(k\Delta f)\, e^{j2\pi nk/N} \quad \rightarrow (43)$$

EQ. (43) DEFINES THE INVERSE DISCRETE FOURIER TRANSFORM.