

MONDAY QUIZ 1

IF $\text{EXP}(-\pi x^2) \rightleftharpoons \text{EXP}(-\pi f^2)$ THEN

(a) FIND THE INVERSE FOURIER TRANSFORM OF-

$$\text{EXP}(-\pi(f-f_0)^2) \text{EXP}(j\alpha f)$$

(b) EVALUATE $\text{EXP}(-\pi x^2) * \text{EXP}(-\pi x^2)$ IN TIME DOMAIN.

* DENOTES CONVOLUTION.

(a) $\text{EXP}(-\pi x^2) \rightleftharpoons \text{EXP}(-\pi f^2)$

$$\text{EXP}(-\pi x^2) e^{j2\pi f_0 x} \rightleftharpoons \text{EXP}[-\pi(f-f_0)^2]$$

↑
FREQ. SHIFTING PROPERTY

$$g(t) \rightleftharpoons G(f)$$

TIME SHIFTING

$$g(t-t_0) \rightleftharpoons G(f) \text{EXP}(-j2\pi f t_0)$$

LET $\alpha = -2\pi t_0$ & $G(f) = \text{EXP}[-\pi(f-f_0)^2] \text{EXP}(j\alpha f)$

$$g(t) = \text{EXP}(-\pi x^2) e^{j2\pi f_0 x}$$

$$\begin{aligned} g(t-t_0) &= g\left(t + \frac{\alpha}{2\pi}\right) = \mathcal{F}^{-1}\left\{\text{EXP}[-\pi(f-f_0)^2] \text{EXP}(j\alpha f)\right\} \\ &= \text{EXP}\left[-\pi\left(t + \frac{\alpha}{2\pi}\right)^2\right] e^{j2\pi f_0\left(t + \frac{\alpha}{2\pi}\right)} \end{aligned}$$

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(b)

$$\text{EXP}(-\pi t^2) \iff \text{EXP}(-\pi f^2)$$

CONVOLUTION
PROPERTY

$$\begin{aligned} \text{EXP}(-\pi t^2) * \text{EXP}(-\pi t^2) &\iff \text{EXP}(-\pi f^2) \cdot \text{EXP}(-\pi f^2) \\ &= \text{EXP}(-\pi 2f^2) \\ &= \text{EXP}[-\pi(\sqrt{2}f)^2] \end{aligned}$$

$$\text{IF } g(t) \iff G(f)$$

$$g(at) \iff \frac{1}{|a|} G(f/a)$$

$$\text{LET } G(f) = \text{EXP}(-f^2 \pi), \quad g(t) = \text{EXP}(-\pi t^2)$$

$$\text{CHOOSE } a = \frac{1}{\sqrt{2}}$$

$$\text{EXP}\left[-\pi\left(\frac{t}{\sqrt{2}}\right)^2\right] \iff \sqrt{2} \text{EXP}[-\pi(\sqrt{2}t)^2]$$

or

$$\frac{1}{\sqrt{2}} \text{EXP}\left[-\frac{\pi t^2}{2}\right] \iff \text{EXP}[-\pi 2f^2]$$