SIGNAL TO NOISE RATIOS (SNR)

THE SNR AT A PARTICULAR POINT IN A SYSTEM IS DEFINED AS

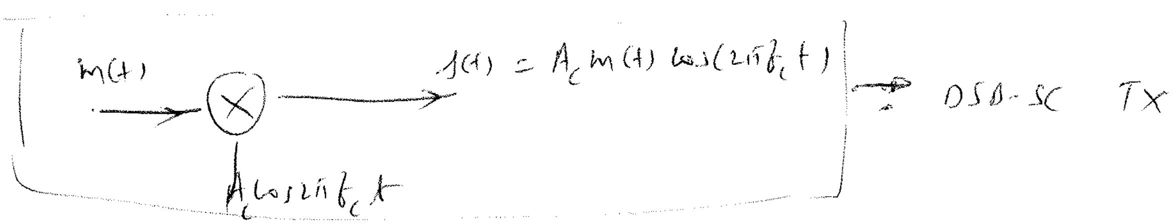
$$SNR = \frac{\text{AV. SIGNAL POWER}}{\text{AV. NOISE POWER}} = \frac{S}{N}$$

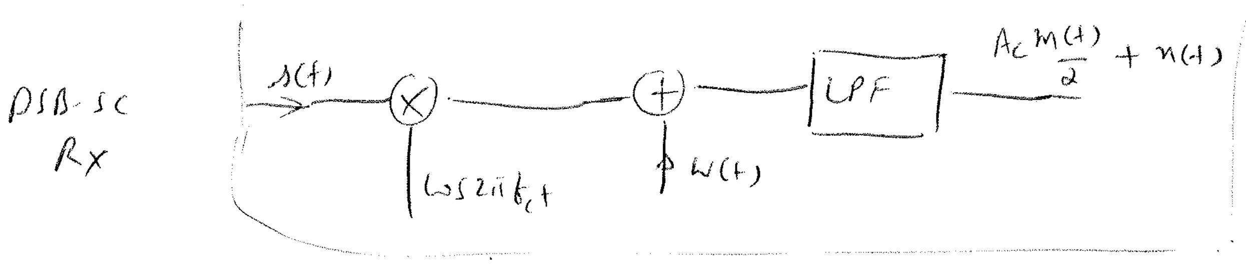
THE SNR IS USUALLY MEASURED IN dB, i.e.

$$SNR\,(dB) = 10 \log_{10} S/N$$

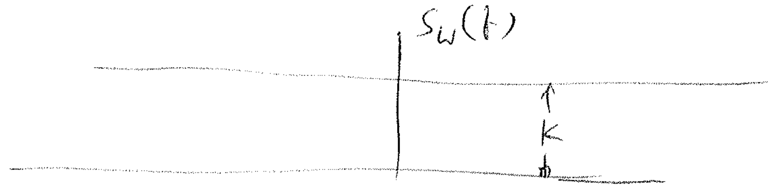
A SNR OF 10dB MEANS THE SIGNAL POWER IS **10** TIMES THE NOISE POWER.

EXAMPLE 1: SNR OF A DSB-SC SYSTEM

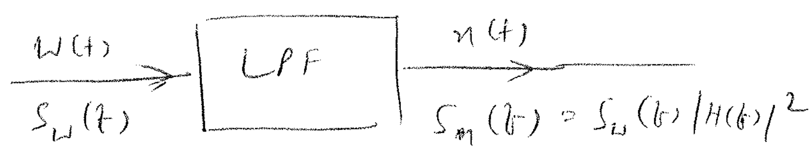




AT THE DSB-SC RX, THE SIGNAL IS MULTIPLIED BY THE LOCAL OSCILLATOR $\cos 2\pi f_c t$. THE RX CIRCUIT IS NOT PERFECT & ADDS NOISE. THIS IS REPRESENTED AS A WHITE NOISE WITH PSD $= K$.

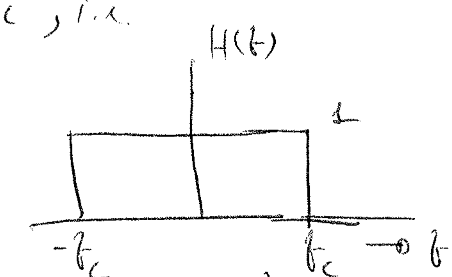


LET US FIRST CONSIDER THE NOISE FLOW (SET $s(t)=0$)



LET US ASSUME THAT LPF IS IDEAL, i.e.

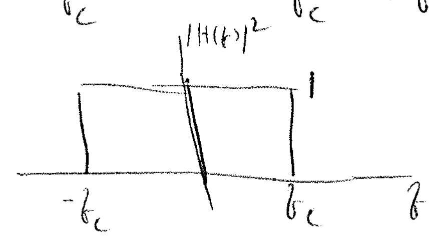
$$H(f) = \mathrm{rect}(f/2f_c)$$

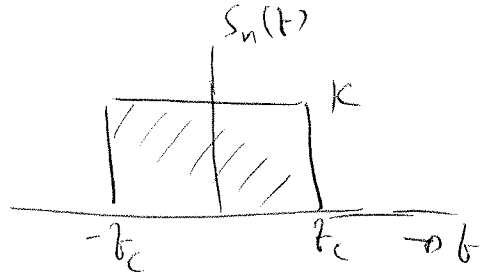


$$S_n(f) = S_w(f)\,|H(f)|^2$$

$$= S_w(f) \text{ if } |f| < f_c$$

$$= 0 \text{ OTHERWISE}$$





NOISE POWER, N = AREA UNDER $S_n(f)$

$$= K\, 2f_c$$

~~LET~~

NEXT CONSIDER THE SIGNAL FLOW (SET $n(t) = 0$)

$$\text{LPF INPUT} = s(t)\cos 2\pi f_c t$$

$$= A_c m(t) \cos^2(2\pi f_c t)$$

$$= \frac{A_c m(t)}{2}\{1 + \cos(4\pi f_c t)\}$$

AFTER PASSING THROUGH THE LPF, THE SIGNAL OUTPUT IS

$$\hat{m}(t) = \frac{A_c m(t)}{2}$$

CONSIDER SINGLE-TONE DSB-SC MODULATION

$$m(t) = A_m \cos(2\pi f_m t)$$

$$\hat{m}(t) = \frac{A_c A_m}{2}\cos(2\pi f_m t)$$

THE AVERAGE POWER (ASSUMING 1Ω RESISTOR) OF THE SIGNAL $\hat{m}(t)$ IS $\left(\frac{A_c A_m}{2}\right)^2 / 2 = A_c^2 A_m^2 / 8$

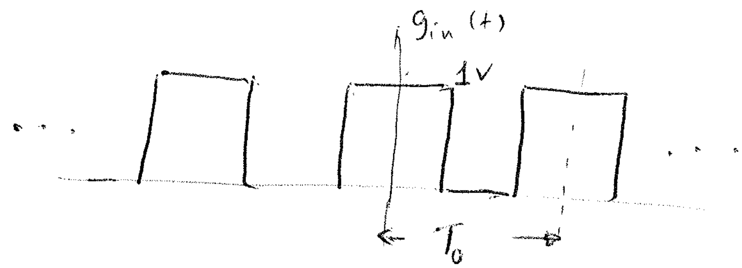
$\therefore$ AV. SIGNAL POWER $= S = A_c^2 A_m^2 / 8$

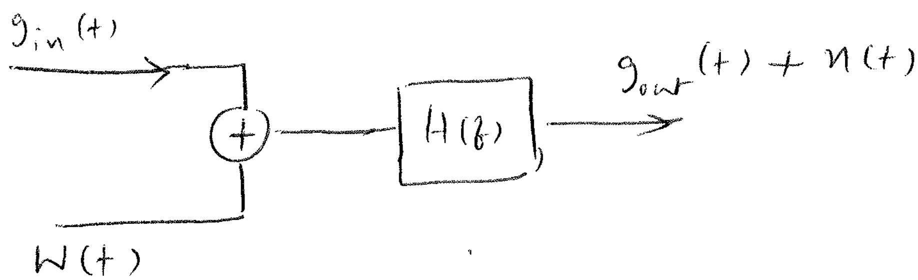
$$SNR = \frac{S}{N} = \frac{A_c^2 A_m^2/8}{2kf_c} = \frac{A_c^2 A_m^2}{16kf_c}$$

EXAMPLE

50% DUTY CYCLE

A SQUARE WAVE $g_{in}(t)$ & A WHITE NOISE PROCESS $W(t)$ ARE APPLIED TO AN IDEAL LOW PASS FILTER WITH DC GAIN EQUAL TO 1 & CUTOFF FREQUENCY $f_c = 1$ KHZ. FIND THE SNR AT THE FILTER OUTPUT. ASSUME



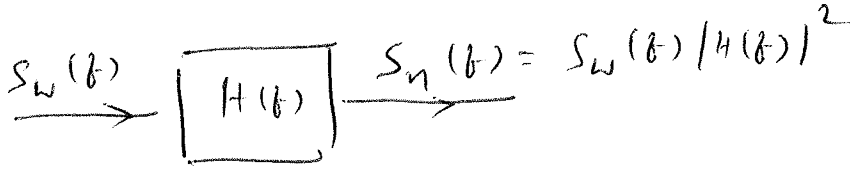


THE FUNDAMENTAL FREQUENCY OF THE SQUARE WAVE $= f_0 \left(= 1/T_0\right)$ $= 750$ Hz, & THE PSD OF $W(t) = 10^{-5}$ W/Hz.

SOLUTION:

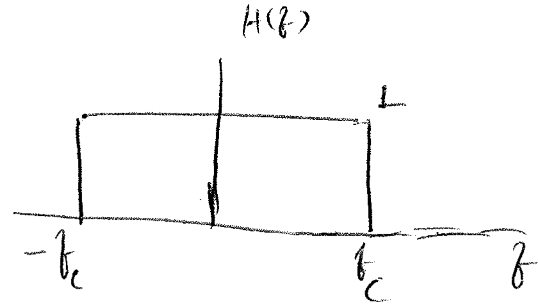
$$SNR = \frac{\text{SIGNAL POWER}}{\text{NOISE POWER}} = \frac{S}{N}$$

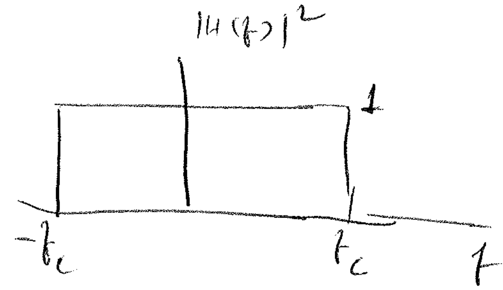
CALCULATION OF NOISE POWER:



$$S_w(f) = 10^{-5}\ \text{W/Hz}$$

$$H(f) = \mathrm{RECT}(f/2f_c)$$

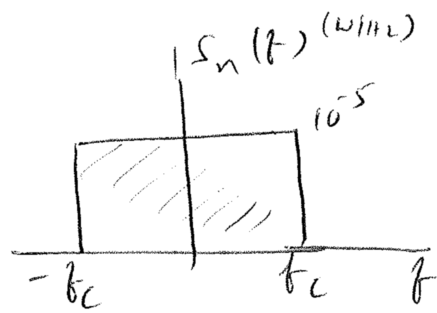




$$|H(f)|^2 = 1 \text{ if } |f| < f_c$$
$$= 0 \quad \text{OTHERWISE}$$

$$\therefore |H(f)|^2 = \mathrm{RECT}(f/2f_c)$$

$$\therefore S_n(f) = S_w(f)\,|H(f)|^2$$
$$= S_w(f)\,\mathrm{RECT}(f/2f_c)$$
$$= 10^{-5} \text{ if } |f| < f_c$$
$$= 0 \quad \text{OTHERWISE}$$



OUTPUT NOISE POWER $= \int_{-\infty}^{\infty} S_n(f)\,df$ = AREA OF THE SHADED REGION

$$= 10^{-5} \times 2f_c$$
$$= 2\times 10^{-2}\ \text{W}$$
$$= N$$

CALCULATION OF SIGNAL POWER:

$f_0 = 750$ Hz.

THE SQUARE WAVE CAN BE ~~BE~~ EXPANDED AS A FOURIER SERIES

$$s_{in}(t) = \sum_{n=-\infty}^{\infty} c_n^{in} e^{+j2\pi n f_0 t}$$

$$c_n^{out} = c_n^{in} \, H(f)$$

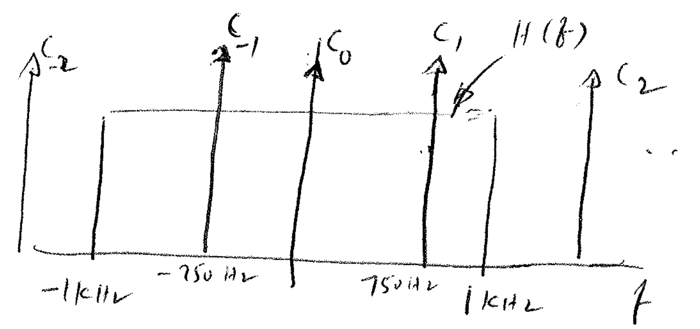
SINCE $H(f) = 0$ FOR $|f| > f_c (= 1\text{ kHz})$, 2^{ND} $(= 1.5\text{ kHz})$ & HIGHER HARMONICS OF THE SQUARE WAVE ~~WAVE~~ DO NOT APPEAR AT THE FILTER OUTPUT.

$$c_0^{out} = c_0^{in} \cdot 1 \quad (= H(f))$$

$$c_1^{out} = c_1^{in} \cdot 1$$

$$c_2^{out} = c_2^{in} \cdot 0$$

$$c_3^{out} = 0 \quad \text{\& so on}$$



THE SIGNAL PART OF THE FILTER OUTPUT CAN BE WRITTEN AS

$$s_{out}(t) = \sum_{n=-\infty}^{\infty} c_n^{out} e^{j2\pi n f_0 t}$$

$$= c_0^{out} + c_1^{out} e^{j2\pi f_0 t} + c_{-1}^{out} e^{-j2\pi f_0 t}$$

$$= c_0^{in} + c_1^{in} e^{j2\pi f_0 t} + c_{-1}^{in} e^{-j2\pi f_0 t} \quad \rightarrow (*)$$

FOR A 50% DUTY CYCLE SQUARE WAVE,

$$C_n^{in} = \frac{1}{2} \operatorname{sinc}(n/2)$$

$$C_0^{in} = \frac{1}{2} \operatorname{sinc}(0) = 1/2$$

$$C_1^{in} = \frac{1}{2} \operatorname{sinc}(1/2) = 0.3183$$

$$= C_{-1}^{in}$$

SUBSTITUTING THIS IN EQ. (*), WE OBTAIN

$$g_{out}(t) = \frac{1}{2} + 2\cos(2\pi f_0 t) \times 0.3183$$

AV SIGNAL POWER CORRESPONDING TO THE DC PART

$$= \frac{1}{4}\ \text{W}$$

AV. SIGNAL POWER CORRESPONDING TO THE SINUSOID

$$= (2 \times 0.3183)^2/2 = 0.2026\ \text{W}$$

TOTAL AVERAGE SIGNAL POWER = 0.4526 W = S

$$SNR = \frac{S}{N} = \frac{0.4526}{2 \times 10^{-2}} = 22.63$$

$$SNR(dB) = 10 \log_{10} 22.63 = 13.54\ \text{dB}.$$