Note Title 2/20/2008

SOLUTIONS

$$x(t) = \sum_{n=-\infty}^{\infty} \mathrm{SINC}\left[5(t - nT_s - 2)\right]$$

$$\mathrm{RECT}(t) \rightleftharpoons \mathrm{SINC}(f)$$

$$\mathrm{SINC}(t) \rightleftharpoons \mathrm{RECT}(-f) = \mathrm{RECT}(f) \rightarrow \text{DUALITY}$$

$$\mathrm{SINC}(5t) \rightleftharpoons \frac{1}{5}\mathrm{RECT}(f/5) \longrightarrow \text{SCALING}$$

$$\mathrm{SINC}(5(t - nT_s - 2)) \rightleftharpoons \frac{1}{5}\mathrm{RECT}(f/5)\,\mathrm{EXP}\left[j2\pi f(-nT_s - 2)\right]$$

$\rightarrow$ TIME SHIFTING

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$$\therefore x(t) = \sum_{n=-\infty}^{\infty} \mathrm{SINC}\left[5\left(t - nT_s - 2\right)\right] \rightleftharpoons \sum_{n=-\infty}^{\infty} \frac{1}{5}\,\mathrm{RECT}(f/5)\, e^{j2\pi f(-nT_s - 2)}$$

$$= \frac{1}{5}\,\mathrm{RECT}(f/5)\, e^{-j2\pi f 2} \sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_s}$$

$$= X(f)$$

OUTPUT $Y(f) = X(f)\,H(f)$

$$y(t) = \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left[3(t - nT_s + 6)\right] \rightleftharpoons \sum_{n=-\infty}^{\infty} \frac{1}{3}\,\mathrm{RECT}(f/3)\, e^{j2\pi f(-nT_s + 6)}$$

$$= \frac{1}{3}\,\mathrm{RECT}(f/3)\, e^{+j2\pi f 6} \sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_s}$$

$$= Y(f)$$

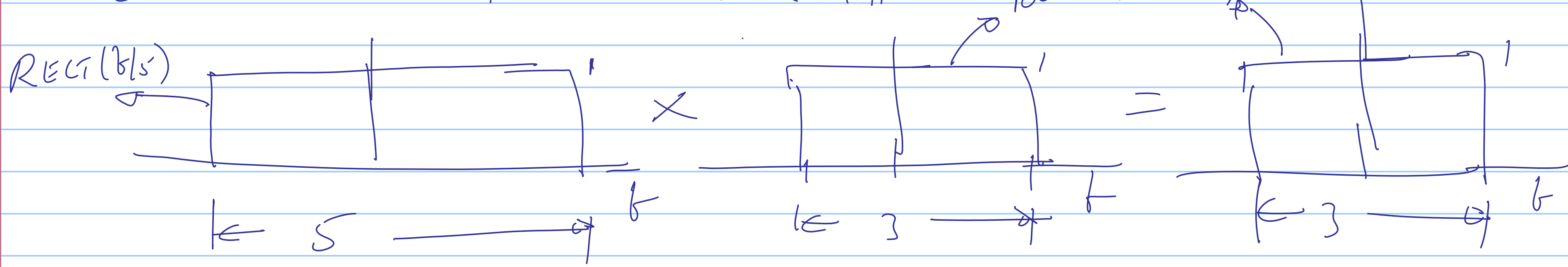
$$H(f) = \frac{Y(f)}{X(f)} = \frac{1/3\,\mathrm{RECT}(f/3) \cancel{\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_s}} \cdot e^{+j2\pi f 6}}{1/5\,\mathrm{RECT}(f/5) \cancel{\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_s}}\, e^{-j2\pi f 2}}$$

$$H(f) = \frac{1/3 \, \mathrm{RECT}(f/3)\, e^{+j2\pi f 8}}{1/5 \, \mathrm{RECT}(f/5)}$$

OR

$$1/3 \, \mathrm{RECT}(f/3)\, e^{+j2\pi f 8} = 1/5 \, \mathrm{RECT}(f/5) \cdot H(f) \longrightarrow (*)$$

EQ.(5) CAN BE SOLVED BY NOTING THAT



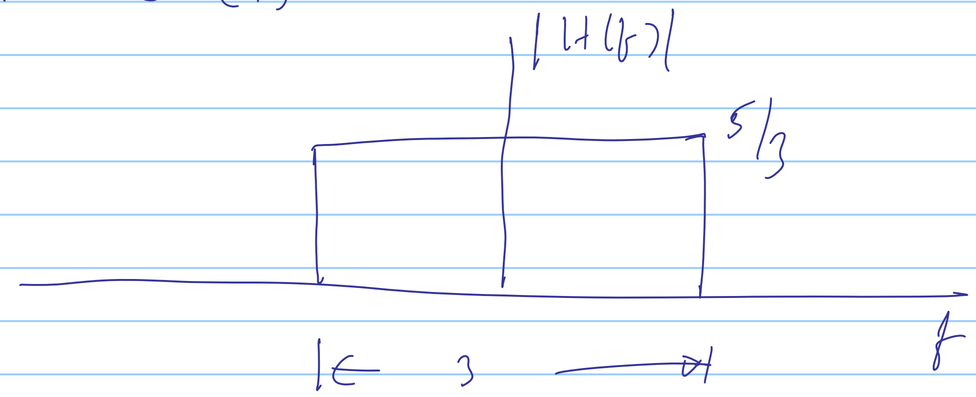
$$\mathrm{RECT}(f/5)\, \mathrm{RECT}(f/3) = \mathrm{RECT}(f/3)$$

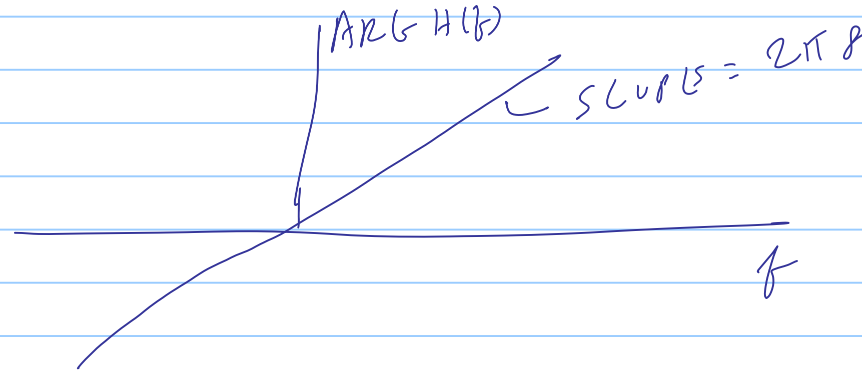
FROM (*)

$$\mathrm{RECT}(f/5)\; H(f) = \mathrm{RECT}(f/3) \cdot 5/3 \; e^{j2\pi f 8}$$

∴ IF WE CHOOSE $H(f) = \mathrm{RECT}(f/3)\, 5/3\, e^{j 2\pi f 8}$

IT SATISFIES EQ. (*).





THIS TRANSFER FUNCTION IS NOT UNIQUE. FOR EXAMPLE, CONSIDER A TRANSFER FUNCTION

$$H'(f) = \frac{5}{3}\,\mathrm{RECT}(f/3)\, e^{j2\pi f 8} + K(f)$$

WHERE

$$K(f) = 0 \quad \text{FOR } |f| \leq 5$$
$$= 1 \quad \text{OTHERWISE}$$

IT IS EASY TO SHOW THAT

$$Y(f) = X(f)\, H'(f)$$

2. FIRST CONSIDER $g_1(t) = A\,\mathrm{SINC}(at)\,\mathrm{SIN}(2\pi f_0 t)\cos(2\pi f_0 t)$

SINCE $2\sin A \cos A = \sin(2A)$

$$g_1(t) = \frac{A}{2}\,\mathrm{SINC}(at)\,\mathrm{SIN}(4\pi f_0 t)$$

$$\mathrm{RECT}(t) \rightleftharpoons \mathrm{SINC}(f)$$

DUALITY $\quad \mathrm{SINC}(t) \rightleftharpoons \mathrm{RECT}(-f) = \mathrm{RECT}(f)$

TIME SCALING $\quad \mathrm{SINC}(at) \rightleftharpoons \frac{1}{|a|}\,\mathrm{RECT}(f/a)$

$$\mathrm{SINC}(at)\,e^{j2\pi f_0 t} \rightleftharpoons \frac{1}{a}\,\mathrm{RECT}\left(\frac{f-f_0}{a}\right), \quad a > 0$$

$$\mathrm{sinc}(at)\, e^{j2\pi 2f_0 t} \rightleftharpoons \frac{1}{a}\, \mathrm{rect}\left(\frac{f-2f_0}{a}\right)$$

$$\mathrm{sinc}(at)\, e^{-j4\pi f_0 t} \rightleftharpoons \frac{1}{a}\, \mathrm{rect}\left(\frac{f+2f_0}{a}\right)$$

$$\mathrm{sinc}(at)\, \frac{\left[e^{j4\pi f_0 t} - e^{-j4\pi f_0 t}\right]}{2j} \rightleftharpoons \frac{1}{a}\, \frac{\left\{\mathrm{rect}\left(\frac{f-2f_0}{a}\right) - \mathrm{rect}\left(\frac{f+2f_0}{a}\right)\right\}}{2j}$$

$$\therefore \underset{2}{\underline{A}}\, \underbrace{\mathrm{sinc}(at)\, \sin(4\pi f_0 t)}_{g_1(t)} \rightleftharpoons \frac{A}{\cancel{4}\, aj} \left\{\mathrm{rect}\left(\frac{f-2f_0}{a}\right) - \mathrm{rect}\left(\frac{f+2f_0}{a}\right)\right\}$$

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$$g(t) = g_1(t - t_0)$$

$$\therefore G(f) = G_1(f)\, EXP(-j\,2\pi f t_0)$$

$$= \frac{A}{4a}\left\{ RECT\left(\frac{f-2f_0}{a}\right) - RECT\left(\frac{f+2f_0}{a}\right)\right\} e^{-j(2\pi f t_0 + \pi/2)}$$