

EE 3TI4 Lab II : Computing the DTFT using the MATLAB `fft` Function

In this exercise, you will compute the Discrete-Time Fourier Transform (DTFT) using the MATLAB Fast Fourier Transform `fft` built-in function. The Fast Fourier Transform implements the DTFT in a computationally efficient manner (see Haykin's book for a description of the FFT). You are required to submit a report for this lab.

The Discrete-Time Fourier Transform (DTFT) of a sequence $g(n)$, $0 \leq n \leq M - 1$ is defined as

$$G(e^{j2\pi f}) = \sum_{n=0}^{M-1} g(n) \exp(-j2\pi f n) \quad (1)$$

Note that $G(e^{j2\pi f})$ can only be computed for a finite number of frequencies. The more frequency points you use, the better is the DTFT. However, since the DTFT is periodic with period 2π , it only needs to be evaluated for frequencies in the range $0 \leq f \leq 1$. For computational efficiency, the frequency points are chosen to be evenly spaced. An N -point DTFT is then given by

$$G(e^{j2\pi f_k}) = \sum_{n=0}^{M-1} g(n) \exp(-j2\pi k n / N), \quad k = 0, \dots, N - 1 \quad (2)$$

If g is the vector containing $g(n)$, $1 \leq n \leq M$ and $N \geq M$ then $\mathbf{G} = \text{fft}(g, N)$ computes N evenly spaced samples of the DTFT of g in G . Note that MATLAB does not support the zero index i.e. the first element of a vector has index 1.

1 Experiments

1. Calculate analytically the DTFT of the rectangular pulse defined by

$$g(n) = \begin{cases} 1, & 0 \leq n \leq 10 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

and plot the magnitude and phase of your expression over $N = 128$ frequency points. Neglect the case of $k = 0$. Explain the plots.

2. Use the `fft` function to calculate the DTFT of g over $N = 128$ frequency points and plot the magnitude and phase spectra. Compare with your previous plots.
3. Use the time-shifting property of the DTFT to calculate analytically the DTFT of the signal $g(n + 5)$, i.e. the centred pulse. Your expression should be purely real. Verify this by multiplying the FFT of $g(n)$ by the appropriate factor. Plot and interpret the new magnitude and phase spectra.
4. Use `fft` to compute the DTFT of the signal

$$g(n) = \begin{cases} (5 - |n|), & -5 \leq n \leq 5 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

Hint: The DTFT should be purely real. Explain how you calculate the DTFT of the signal and plot and interpret the magnitude and phase spectra.

2 Lab Write-Up

Your report should contain equations, MATLAB codes, plots and explanations, all presented in a coherent fashion. You should show that you understand the material by linking the theory to your observations.