ECE 712 Take–home final exam 2009

Due: Friday Dec. 11, 2009 at 4:00pm.

James P. Reilly

Please hand in completed exam to Cheryl Gies in ITB A112 by Friday Dec. 11, 2009 by 4:00pm. This exam is based on the honour system. As such, please sign the following statement:

This exam is entirely my own work. I may consult any written material or text, but I have NOT received nor offered any help from any other person.

Name: Date:

Signature:

Given that individuals may be sitting closely together in the same lab, doing this exam at the same time, it may be difficult to avoid discussion of this exam. I therefore urge you avoid this situation and work e.g., in the library. It is your responsibility to work in an environment where you can honour the above oath. Students can feel free to contact me by email for clarification of any questions.

For each question, explain your method fully, but as precisely and concisely as possible. Include any matlab code if appropriate.

1. Given $\boldsymbol{A} \in \mathbb{R}^{m \times n}$, and a set $\boldsymbol{x}_i \in \mathbb{R}^m, \boldsymbol{y}_i \in \mathbb{R}^n$, $i = 1, \ldots, k,\ k \leq \min(m, n)$

a. find a set of coefficients a_i so that

$$\left|\left| \boldsymbol{A} - \sum_{i=1}^{k} a_i \boldsymbol{x}_i \boldsymbol{y}_i^T \right|\right|_F^2 \tag{1}$$

is minimized.

b. What are the set $\boldsymbol{x}_i$, $\boldsymbol{y}_i$ that minimize the minimum in (1) for a given set of coefficients a_i?

2. Derive an analytical expression for the eigendecomposition of a square symmetric rank one matrix.

3. Download the 7×5 matrix $\boldsymbol{A}$ found on the course website, associated with this exam. Using the SVD, identify an orthonormal basis for each of the four subspaces associated with $\boldsymbol{A}$. Verify using matlab that these subspaces have their required properties. Submit your code with your response.

4. Let the matrix $\boldsymbol{X}$ be partitioned into the following blocks:

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} \tag{2}$$

Find a block lower triangular matrix $\boldsymbol{L}$ and a block upper triangular matrix $\boldsymbol{U}$ so that

$$\boldsymbol{X} = \boldsymbol{L} \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_A \end{bmatrix} \boldsymbol{U}, \tag{3}$$

where the diagonal blocks of $\boldsymbol{L}$ and $\boldsymbol{U}$ are the identity, and $\boldsymbol{S}_A$ is the *Schur complement* of $\boldsymbol{A}$, given as $\boldsymbol{S}_A = \boldsymbol{D} - \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B}$. Then, using this result, find the upper left block of $\boldsymbol{X}^{-1}$.