ECE 712 Assignment on Gaussian Elimination

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1. Suggest a sequence of modified Gauss transforms $\boldsymbol{N}_i, i = 1, \ldots, n$, so that

$$\boldsymbol{D} = \boldsymbol{N}\boldsymbol{U} \tag{1}$$

where $\boldsymbol{D} \in \Re^{n \times n}$ is diagonal, $\boldsymbol{U} \in \Re^{n \times n}$ is full-rank upper triangular, and $\boldsymbol{N} = \prod_{i=1}^{n} \boldsymbol{N}_i$. Use this procedure to calculate the inverse of a full–rank square matrix $\boldsymbol{A}$. Show that $\boldsymbol{N}_i\boldsymbol{U}$ differs from $\boldsymbol{U}$ only in the ith column. Use this fact to propose an efficient means of calculating $\boldsymbol{N}$ from the $\boldsymbol{N}_i$.

2. Prove that Gaussian elimination with full pivoting on a rank deficient matrix will create a block of zeros in the lower right-hand corner of $\boldsymbol{U}$

3. Let $\boldsymbol{A} \in \Re^{n \times n} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}$, where $\boldsymbol{A}_{11} \in \Re^{k \times k}$ is non–singular. Then $\boldsymbol{S} = \boldsymbol{A}_{22} - \boldsymbol{A}_{21}\boldsymbol{A}_{11}^{-1}\boldsymbol{A}_{12}$ is called the Schur complement of $\boldsymbol{A}$. Show that after k steps of the Gaussian elimination algorithm without pivoting, $\boldsymbol{A}_{22}$ has been overwritten by $\boldsymbol{S}$.

4. Prove that the diagonal elements of a positive definite square matrix must all be positive.

5. **4.** Let the matrix $\boldsymbol{X}$ be partitioned in the following blocks:

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} \tag{2}$$

Find a block lower triangular matrix $\boldsymbol{L}$ and a block upper triangular matrix $\boldsymbol{U}$ so that

$$\boldsymbol{X} = \boldsymbol{L} \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_A \end{bmatrix} \boldsymbol{U}, \tag{3}$$

where the diagonal blocks of $\boldsymbol{L}$ and $\boldsymbol{U}$ are the identity, and $\boldsymbol{S}_A$ is the *Schur complement* of $\boldsymbol{A}$, given as $\boldsymbol{S}_A = \boldsymbol{D} - \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B}$. Then, using this result, find the upper left block of $\boldsymbol{X}^{-1}$.