CoE4TN3
Image Processing

Chapter 11

Image Representation & Description

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Image Representation & Description

- After an image is segmented into regions, the regions are represented and described in a form suitable for computer processing (descriptors).
- Representing a region:
  1. In terms of its external characteristics (boundary)
  2. In term of its internal characteristics

  Exp: A region might be represented by the length of its boundary.
  - External representations are used when the focus is on shape of the region.
  - Internal representations are used when the focus is on color and texture.
  - Representations should be insensitive to rotation and translation.

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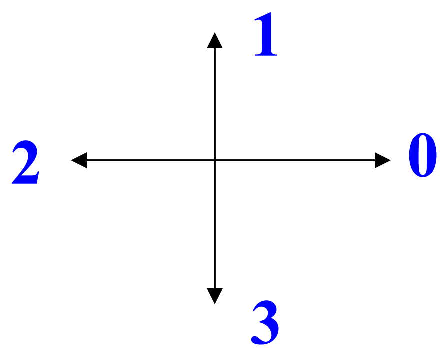
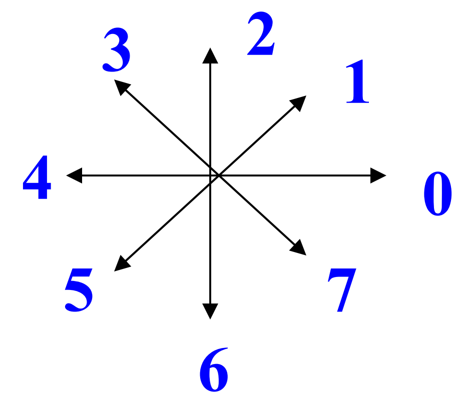
Chain Code

- Chain Code: Used to represent a boundary by a connected sequence of straight line segments.
  - 4 or 8 connectivity is used
  - The direction of each segment is coded by a numbering scheme.



Method:

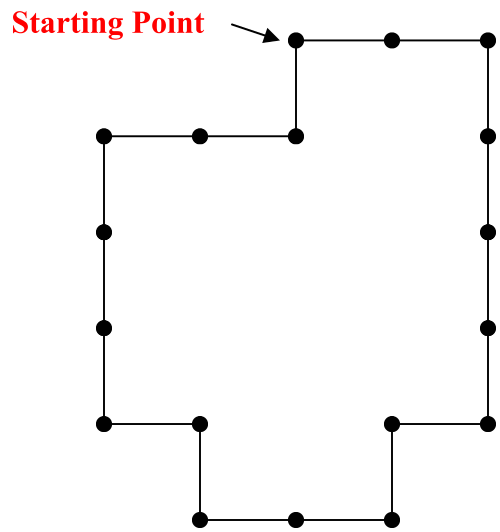
- Follow the boundary in a specific (clockwise) direction.
- Assign a direction to the segment connecting every pair of pixels.

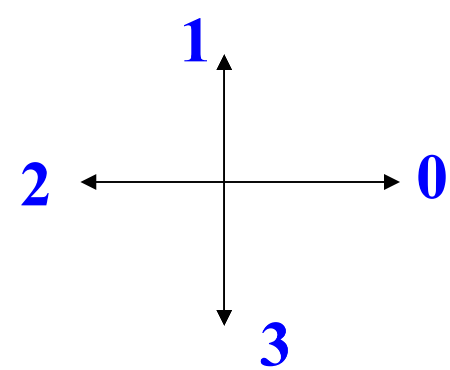


Chain Code

Exp: 00333323221212111001

- Problems:
  - The chain code depends on the starting point.
  - It changes with rotation.



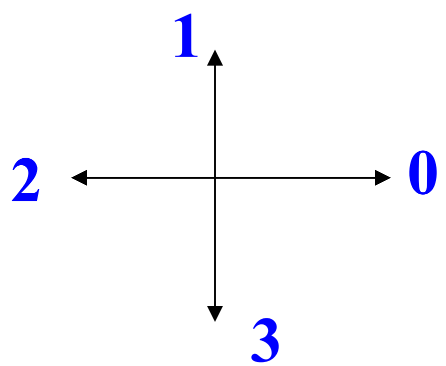


Chain Code

Solutions:

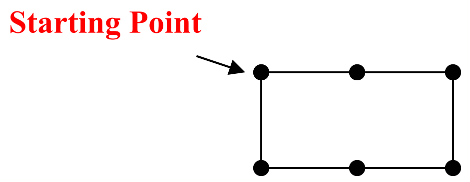
- Treat the chain code as a circular sequence of numbers. Circulate until the number is of minimum magnitude.
- Use the difference of chain code instead of the code itself: count counterclockwise the number of directions that separates two adjacent elements (First difference)

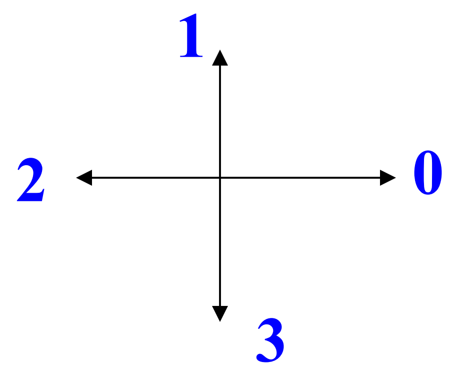
- Exp: 10103322
- First difference code: 33133030



Shape Number

- Shape number: first difference of smallest magnitude (in the chain code)
  - Exp: chain code 003221
  - First difference code: 303303
  - Shape number: 033033



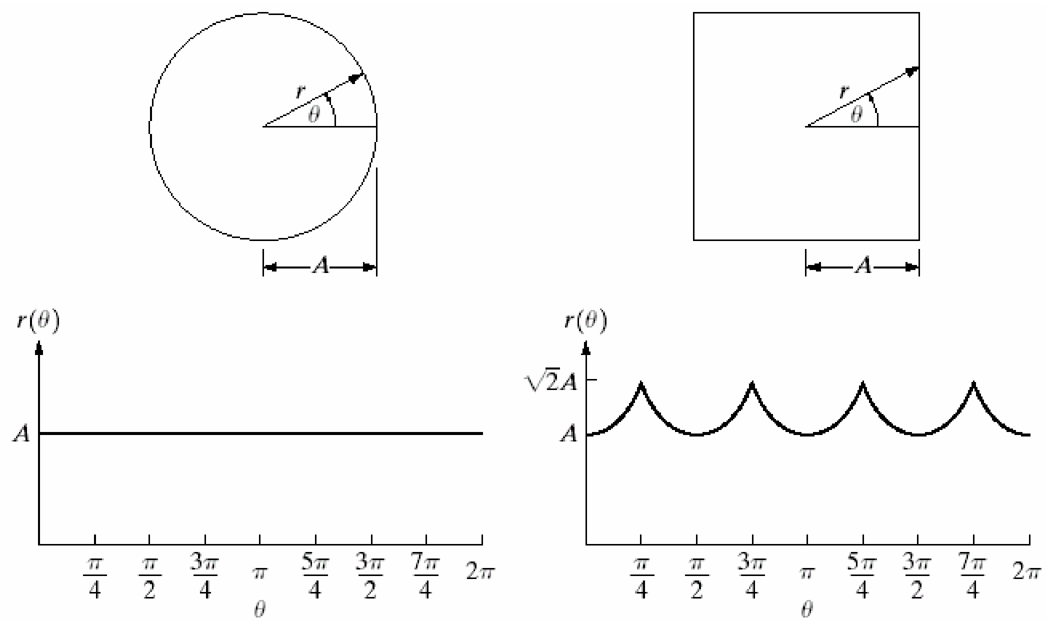


Signature

- Signature: a 1-D functional representation of a boundary
- Different ways of generating signature
- Plot distance form centroid to boundary as a function of angle

a b

FIGURE 11.5 Distance-versus-angle signatures. In (a) $r(\theta)$ is constant. In (b), the signature consists of repetitions of the pattern $r(\theta) = A \sec\theta$ for $0 \leq \theta \leq \pi/4$ and $r(\theta) = A \csc\theta$ for $\pi/4 < \theta \leq \pi/2$.



Signature

- Invariant to translation, depend on rotation and scaling
- To make it invariant to rotation we should select the same starting point regardless of the orientation
  - Select starting point farthest from centroid (if unique)
- To make it invariant to scaling we can normalize to a particular range
- Other signatures: traverse the boundary, at each point plot the angle between a line tangent to the boundary and a reference line
- Slope-density-function: histogram of tangent-angle values
- Straight segments will form the peaks of histogram

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Skeletons

- An important approach to representing structural shape of a plane region is to reduce it to a graph
- This may be accomplished by obtaining the skeleton of the region via a thinning algorithm.
- Applications in automated inspection ….
- Definition of skeleton is based on medial axis transformation (MAT)

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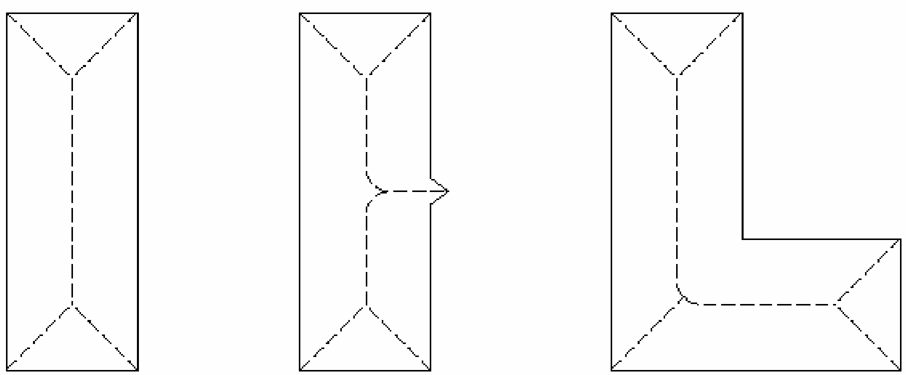
Skeletons

- MAT of a region R with border B: for each point p in R, find the closest neighbor in B. If p has more than one such neighbor, it belongs to medial axis (skeleton)
- MAT is based on “prairie fire concept”.
- Direct implementation of MAT is computationally expensive
- Alternative algorithms have been proposed that “thin” the boundary of a region until the skeleton is left



a b c

FIGURE 11.7 Medial axes (dashed) of three simple regions.

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Skeletons

- An algorithm for thinning binary regions (assume region points are 1 and background points are 0)
- The algorithm has two steps which are applied to all the pixels on the contour of the region
- A contour point is any pixel with value 1 and having at least one 8-neighbor valued 0.
- In each step the boundary point that satisfy a set of conditions are flagged and then deleted

Skeletons

- Step 1 flags a contour point p1 if the following conditions are satisfied:

  $a)\quad 2 \le N(p1) \le 6$

  $b)\quad T(p1) = 1$

  $c)\quad p2.p4.p6 = 0$

  $d)\quad p4.p6.p8 = 0$

p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

FIGURE 11.8 Neighborhood arrangement used by the thinning algorithm.

- N(p1): number of nonzero neighbors of p1
- T(p1) : number of 0 to 1 transitions in ordered sequence p2,p3,….p8,p9,p2.

FIGURE 11.9 Illustration of conditions (a) and (b) in Eq. (11.1-1). In this case $N(p_1) = 4$ and $T(p_1) = 3$.

0	0	1
1	p_1	0
1	0	1

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Skeletons

- After step 1 is applied to all border points those that are flagged are deleted (changed to 0)
- In step 2 conditions (a) and (b) remain the same but (c) and (d) are changed to:

$$c') \qquad p2.p4.p8 = 0$$
$$d') \qquad p2.p6.p8 = 0$$

- After step 2 is applied to all border points remaining after step 1, those that are flagged are deleted (changed to 0)
- This procedure is applied iteratively until no further points are deleted.

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Skeletons

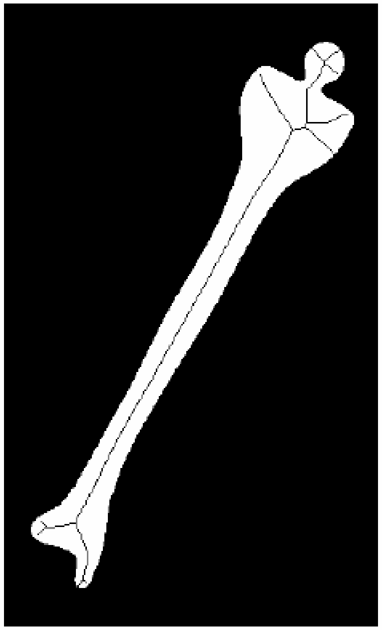
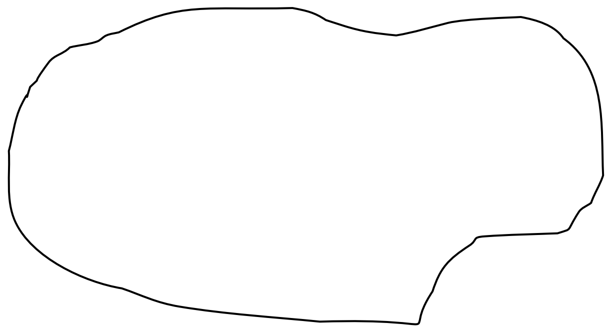


FIGURE 11.10 Human leg bone and skeleton of the region shown superimposed.

Simple boundary descriptors

- Length: number of pixels along the contour of a region
- Diameter: $Dima(B)=\max[D(p_i,p_j)]$
  - p_i,p_j are points on the boundary.



- Curvature: rate of change of slope.
- For digital images: the difference between the slopes of adjacent boundary segments.

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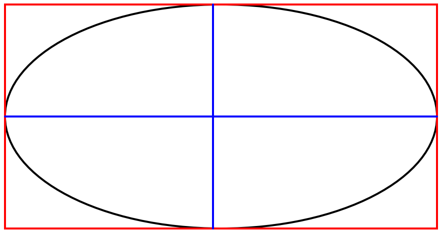
Simple boundary descriptors

- Major axis: straight line segment joining the two points farthest from each other on the boundary
- Minor axis: Perpendicular to the major axis and of such length that a box could be formed to enclose the boundary.
- Eccentricity=Major axis/Minor axis
- Basic rectangle (bounding box): the rectangle formed by minor and major axes enclosing the boundary.



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Fourier Descriptor

N point boundary

$(x_0, y_0), (x_1, y_1), \ldots.(x_{N-1}, y_{N-1})$

$s(k) = x_k + jy_k$

N point DFT of s(k) :

$$a(u) = \frac{1}{N}\sum_{k=0}^{N-1} s(k)\exp(-j2\pi uk/N)$$

$a(u)$ are called Fourier descriptors.

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Fourier Descriptor

If P of the Fourier descrptors are used

$$\hat{s}(k) = \sum_{u=0}^{P-1} a(u) \exp(j2\pi uk / N)$$

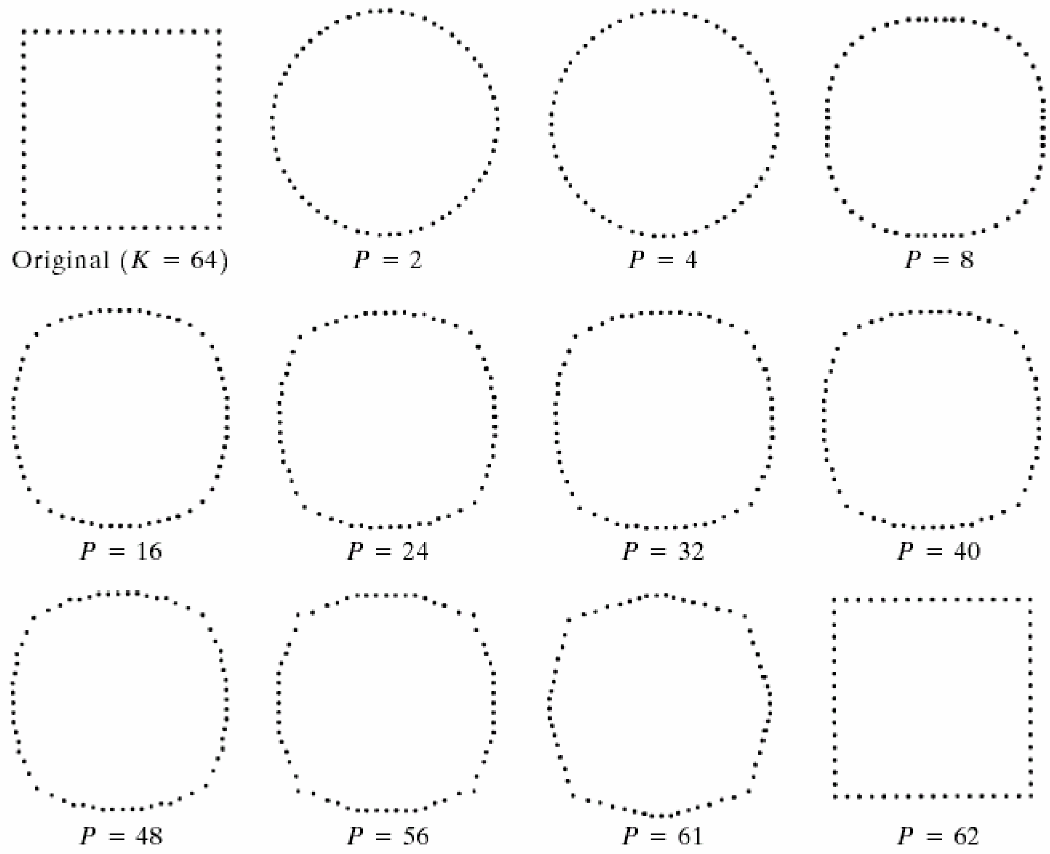
$P < N \Rightarrow$ High frequency details of

the boundary (e.g., corners) are removed.

- Fourier descriptors are not directly insensitive to translation, rotation and scaling.
- Magnitude of the Fourier descriptors is insensitive to rotation.

Fourier Descriptor

FIGURE 11.14 Examples of reconstruction from Fourier descriptors. *P* is the number of Fourier coefficients used in the reconstruction of the boundary.



Regional Descriptors

- Area: number of pixels contained within a region
- Compactness: $(\text{perimeter})^2/\text{area}$
- Min and Max of the gray levels in the region
- Mean and median of gray levels

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Topological Descriptors

- Topology: study of properties of a figure that are unaffected by any deformation, as long as there is no tearing or joining of the figure (rubber sheet distortions)
- Since stretching affects distance, topological properties do not depend on the notion of distance
- Number of holes in a region (H)
- Number of connected components (C)

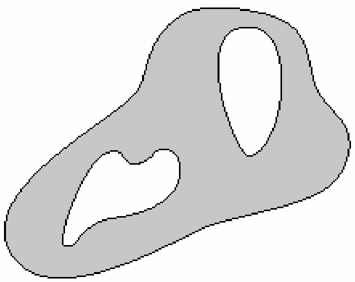

FIGURE 11.17 A region with two holes.

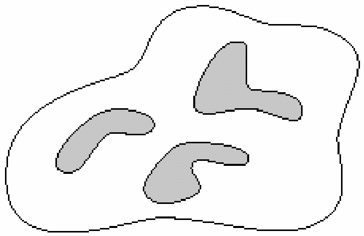

FIGURE 11.18 A region with three connected components.

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Topological Descriptors

- Euler number: E=C-H
- Sometimes a region is represented by straight-line segments (polygonal network)
- V: number of vertices, Q: number of edges, F: number of faces => V-Q+F=C-H=E
- For the figure below right: 7-11+2=1-3=-2

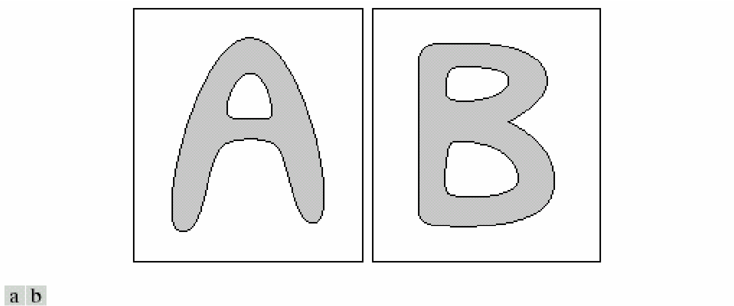


FIGURE 11.19 Regions with Euler number equal to 0 and −1, respectively.

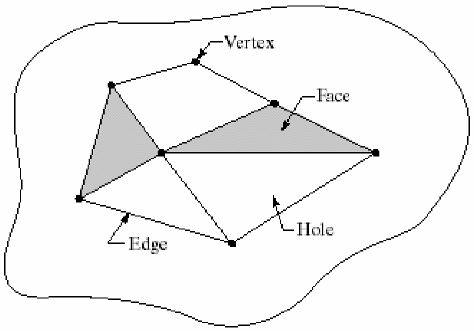
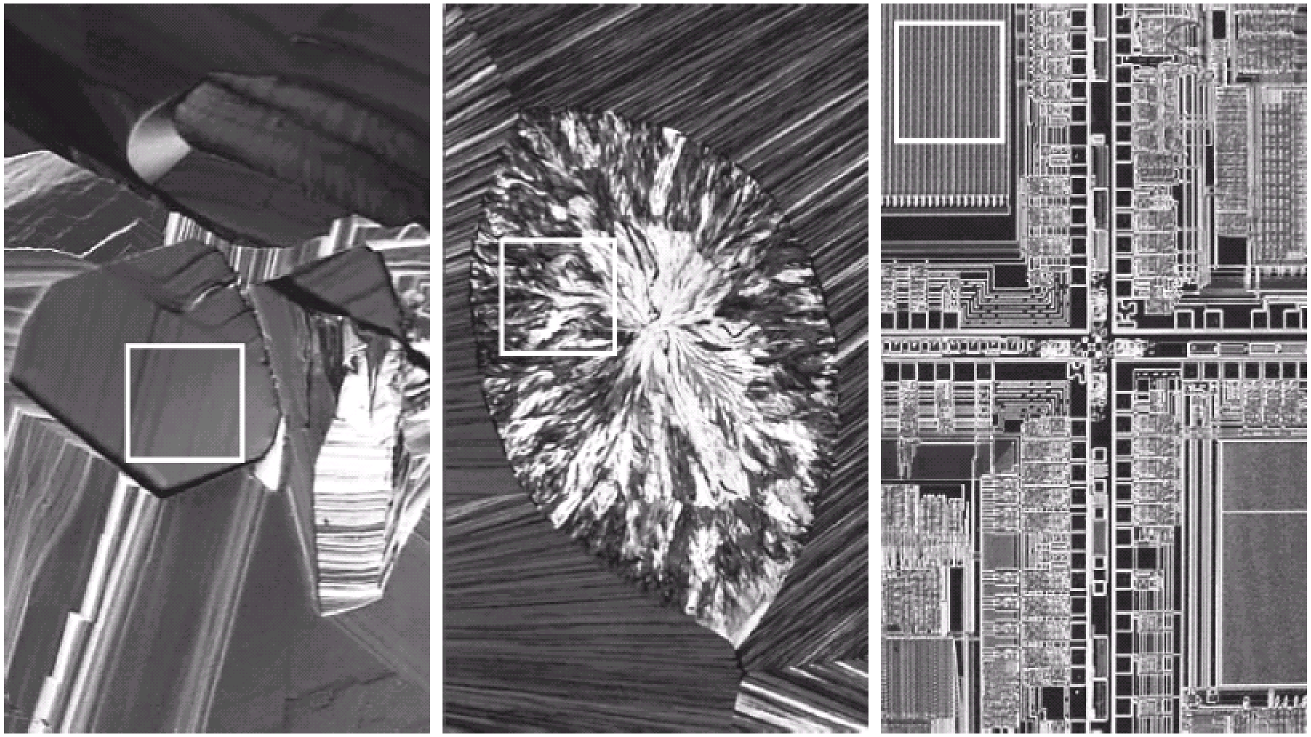


FIGURE 11.20 A region containing a polygonal network.

Texture



a b c

FIGURE 11.22 The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

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Texture

- An important approach to region description is to quantify its texture content
- This descriptor provides measures of smoothness, coarseness and regularity
- 3 approaches in describing the texture of a region: statistical, structural, and spectral.

Texture (statistical)

- One of the simplest approaches for describing texture is to use statistical moments of the histogram of an image or a region

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$

$$m = \sum_{i=0}^{L-1} z_i p(z_i)$$

- $\sigma^2(z) = \mu_2(z)$ is a measure of contrast

$$R = 1 - \frac{1}{1+\sigma^2(z)}$$

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Texture (statistical)

- Third moment is a measure of skewness of histogram

$$\mu_3(z) = \sum_{i=0}^{L-1} (z_i - m)^3 p(z_i)$$

- Measure of uniformity: U is maximum for an image in which all gray levels are equal

$$U = \sum_{i=0}^{L-1} p^2(z_i)$$

- Average entropy: a measure of variability and is zero for constant image

$$e = -\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)$$

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Texture (statistical)

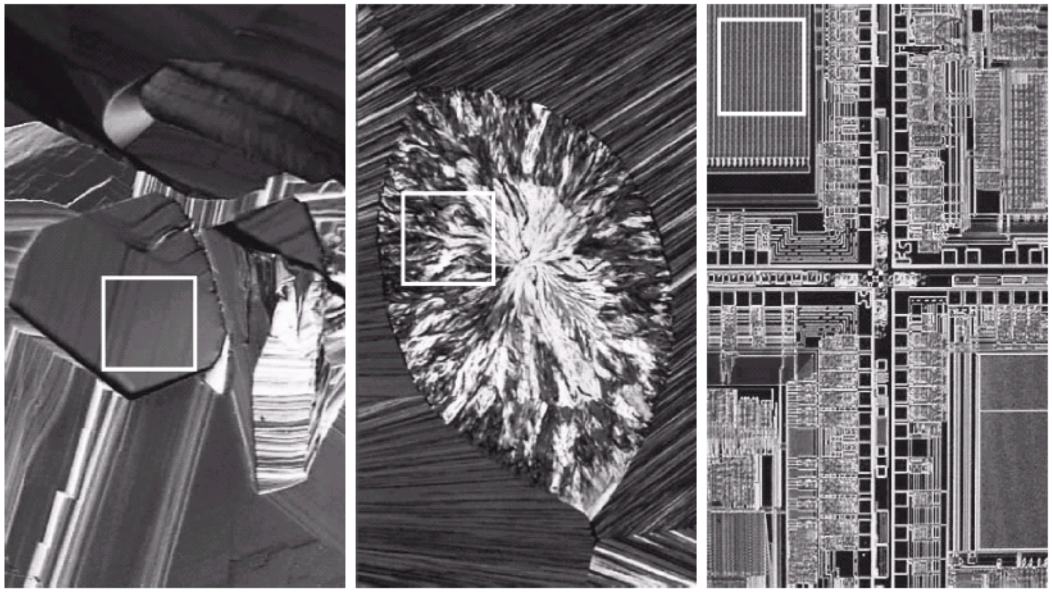

TABLE 11.2 Texture measures for the subimages shown in Fig. 11.22.

Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

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Texture (statistical)

- Measures of texture computed using histogram carry no information regarding the relative position of pixels with respect to each other
- Let P be a position operator and A a kxk matrix whose element a_{ij} is the number of times that points with gray level z_i occur (in position specified by P) relative to points with gray level z_j.
- Exp: an image with three gray levels, 0, 1, 2

0 0 0 1 2

1 1 0 1 1

2 2 1 0 0

1 1 0 2 0

0 0 1 0 1

P: one pixel to the right and one pixel below

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

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Texture (statistical)

- Let n be the total number of point pairs in the image that satisfy P (n=16 in the previous example)
- C=A/n : gray-level co-occurrence matrix
- C depends on P
- C is analyzed to categorize texture over which C was computed

$$\max_{i,j}(c_{ij})$$

$$\sum_i \sum_j (i-j)^k c_{ij}$$

$$\sum_i \sum_j c_{ij} / (i-j)^k$$

$$\sum_i \sum_j c_{ij}^2$$

$$-\sum_i \sum_j c_{ij} \log(c_{ij})$$

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Texture (structural)

- A simple texture primitive (texture elements) can be used to form more complex texture patterns by means of some rules
- Exp: S -> aA
  - S, A: variables (symbols or primitives)
  - a: some operation for example "put a circle to the right"
  - Rule S -> aA says that starting point can be replaced by a circle to the right and a variable
- Exp:
  1. S->aA
  2. A-> bA,
  3. A-> cA,
  4. A->a,
  5. A-> aA,
  - a: circle right
  - b: circle down
  - c: circle left

155233254 is a 3x3 square of circles

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Texture (spectral)

- Three features of Fourier spectrum are useful in texture description:
  1. Peaks in spectrum give the principle direction of texture pattern
  2. Location of the peaks in the frequency plane gives spatial period of the pattern
  3. Eliminating periodic components via filtering leaves non-periodic image elements that can be described by statistical techniques
- Spectrum is sometimes considered in polar coordinates:

$$S(r,\theta)$$

$$S(r) = \sum_{\theta} S(r,\theta)$$

$$S(\theta) = \sum_{r} S(r,\theta)$$

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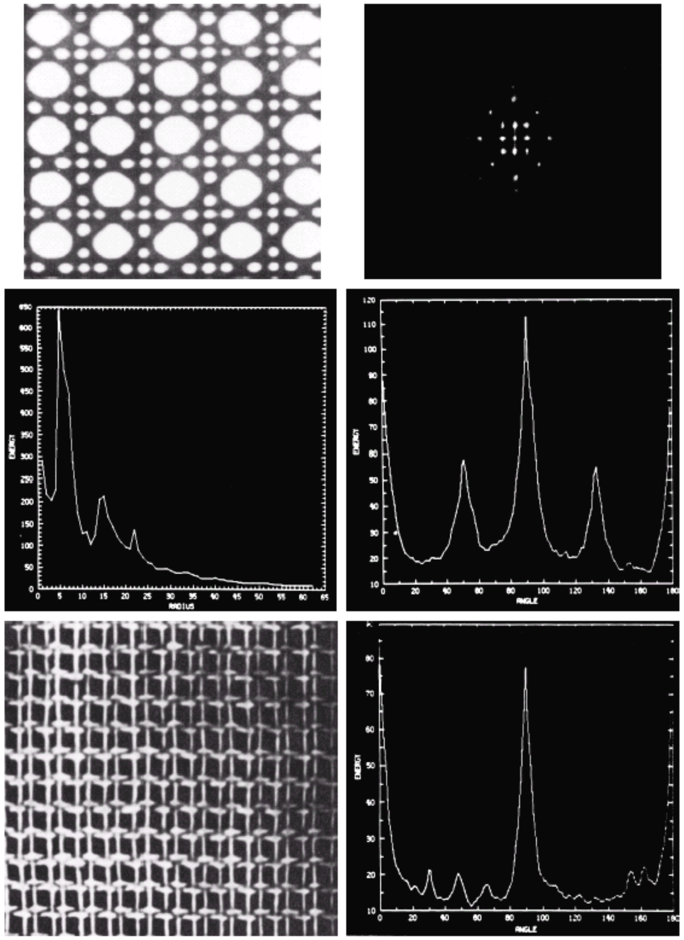
Texture (spectral)



a b
c d
e f

FIGURE 11.24 (a) Image showing periodic texture. (b) Spectrum. (c) Plot of $S(r)$. (d) Plot of $S(\theta)$. (e) Another image with a different type of periodic texture. (f) Plot of $S(\theta)$. (Courtesy of Dr. Dragana Brzakovic, University of Tennessee.)

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Moments

$$m_{pq} = \sum_{(x,y)\in R} \sum x^p y^q f(x,y)$$ Moment of order p + q

$$\mu_{pq} = \sum_{(x,y)\in R} \sum (x-\bar{x})^p (y-\bar{y})^q f(x,y)$$ Central Moments

$$\bar{x} = \frac{m_{10}}{m_{00}} \qquad \bar{y} = \frac{m_{01}}{m_{00}}$$

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}}$$ Normalized Central Moments

$$\gamma = \frac{\mathrm{p}+\mathrm{q}}{2} + 1$$

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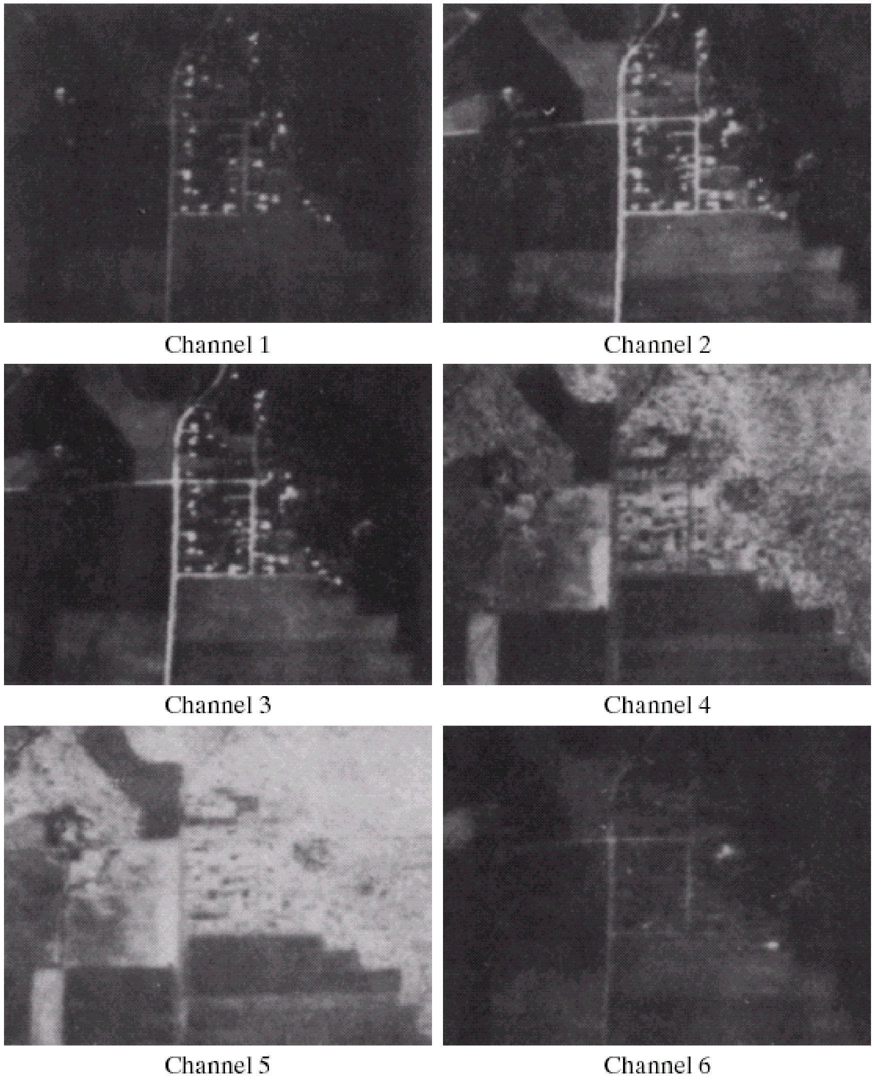
Principle component analysis

FIGURE 11.26 Six spectral images from an airborne scanner. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)



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Principle component analysis

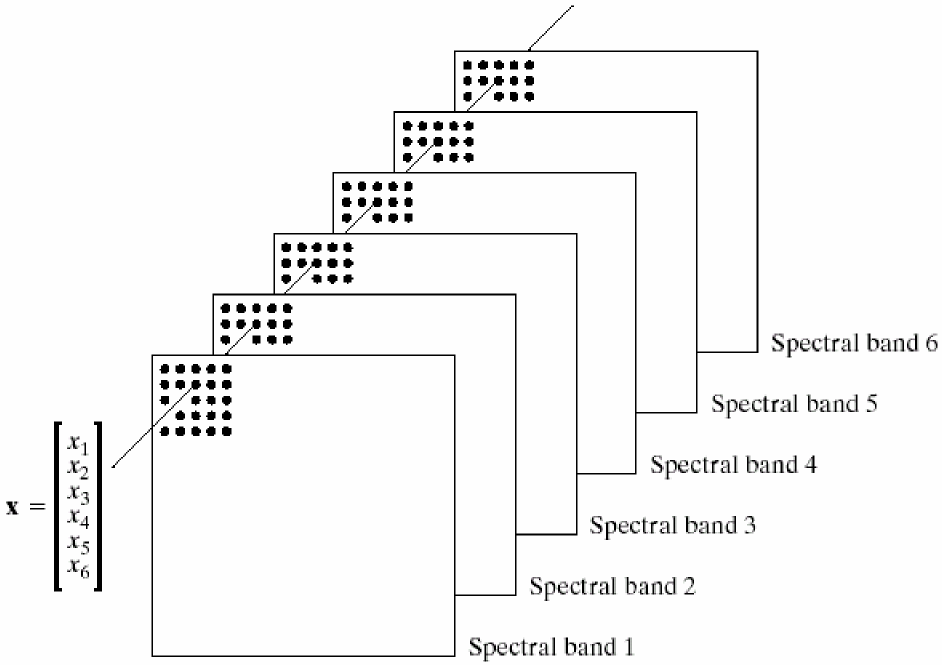

FIGURE 11.27 Formation of a vector from corresponding pixels in six images.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
3210	931.4	118.5	83.88	64.00	13.40

TABLE 11.5 Eigenvalues of the covariance matrix obtained from the images in Fig. 11.26.

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Principle component analysis

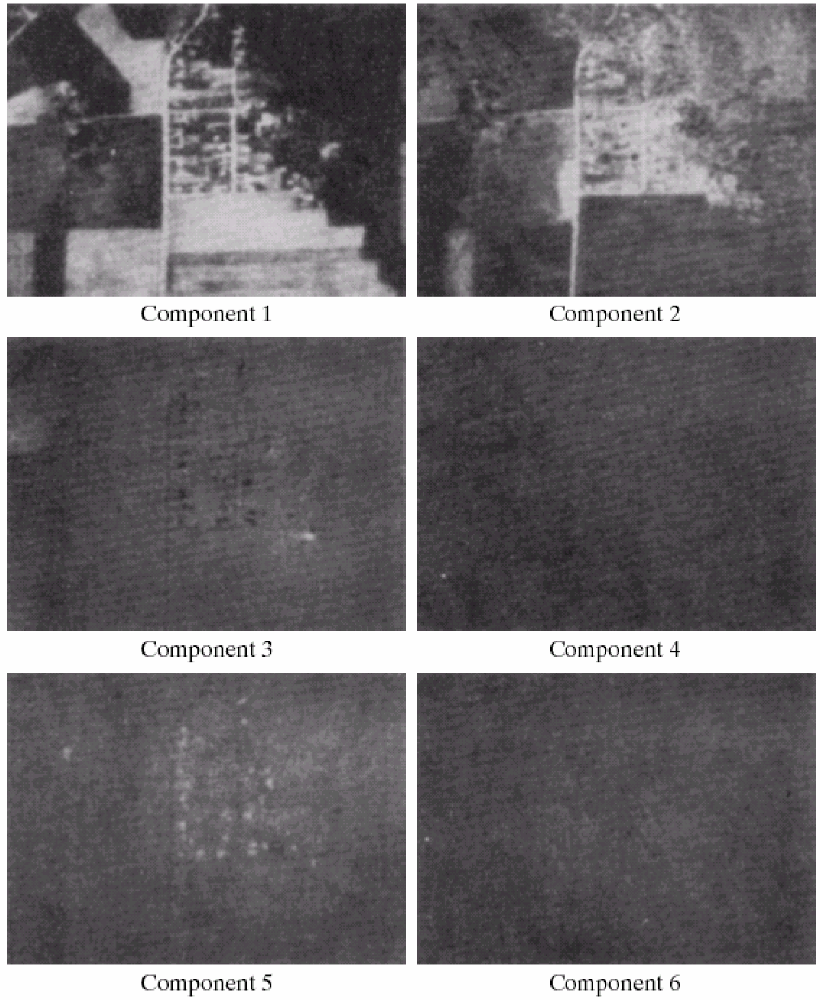

FIGURE 11.28 Six principal-component images computed from the data in Fig. 11.26. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)

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