CoE4TN3
Image Processing

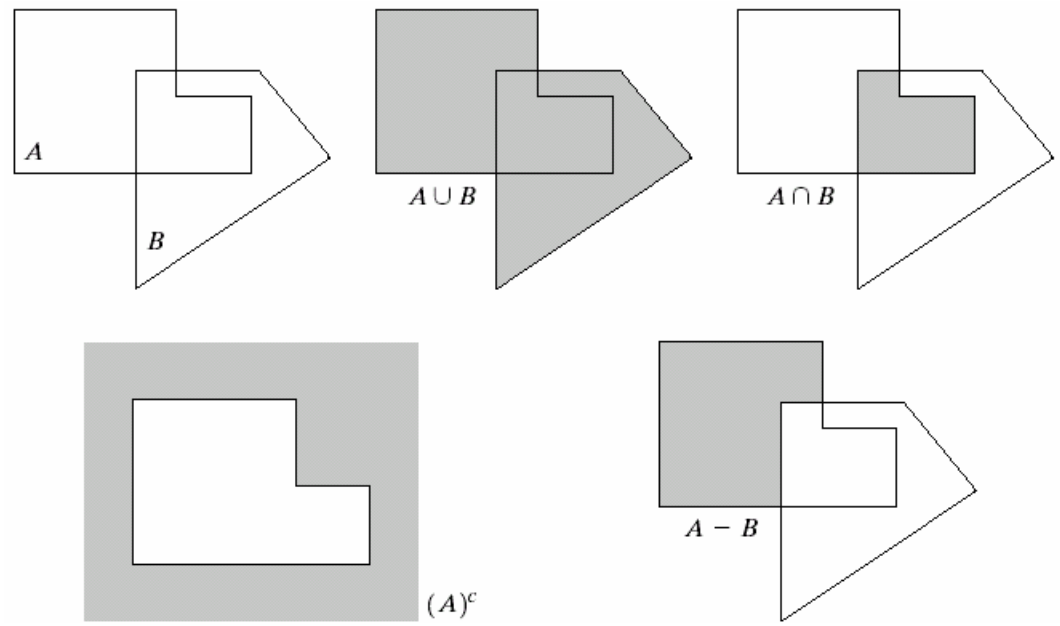
Chapter 9

Morphological Image Processing

Image Morphology

- Morphology: a branch of biology that deals with the form and structure of animals and plants
- A tool for extracting image components that are useful in representation and description
- Language of morphology: set theory
- Objects in an image are represented by a sets
- For binary images each element of the set is a 2-D vector with the (x,y) coordinates of a black (or white depending on the object) pixel

Preliminaries



a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

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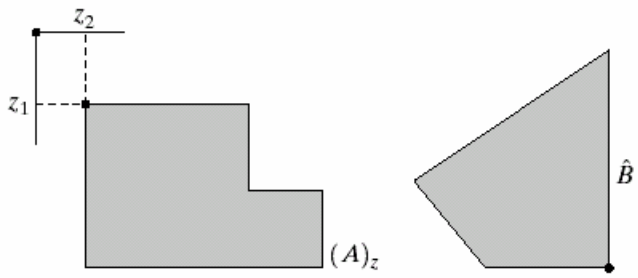
Preliminaries

- Reflection of a set:

$$\hat{B} = \{w \mid w = -b, \; for \quad b \in B\}$$

- Translation:

$$(A)_z = \{c \mid c = a + z, \quad for \;\; a \in A\}$$



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

Dilation & Erosion

- Dilation:

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

- Obtain the reflection of B and shift it by z.
- Dilation of A by B is the set of all displacements z such that $B^{\wedge}$ and A overlap at least one element
- B is called the structuring element
- One of the applications of dilation is for bridging gaps

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Dilation

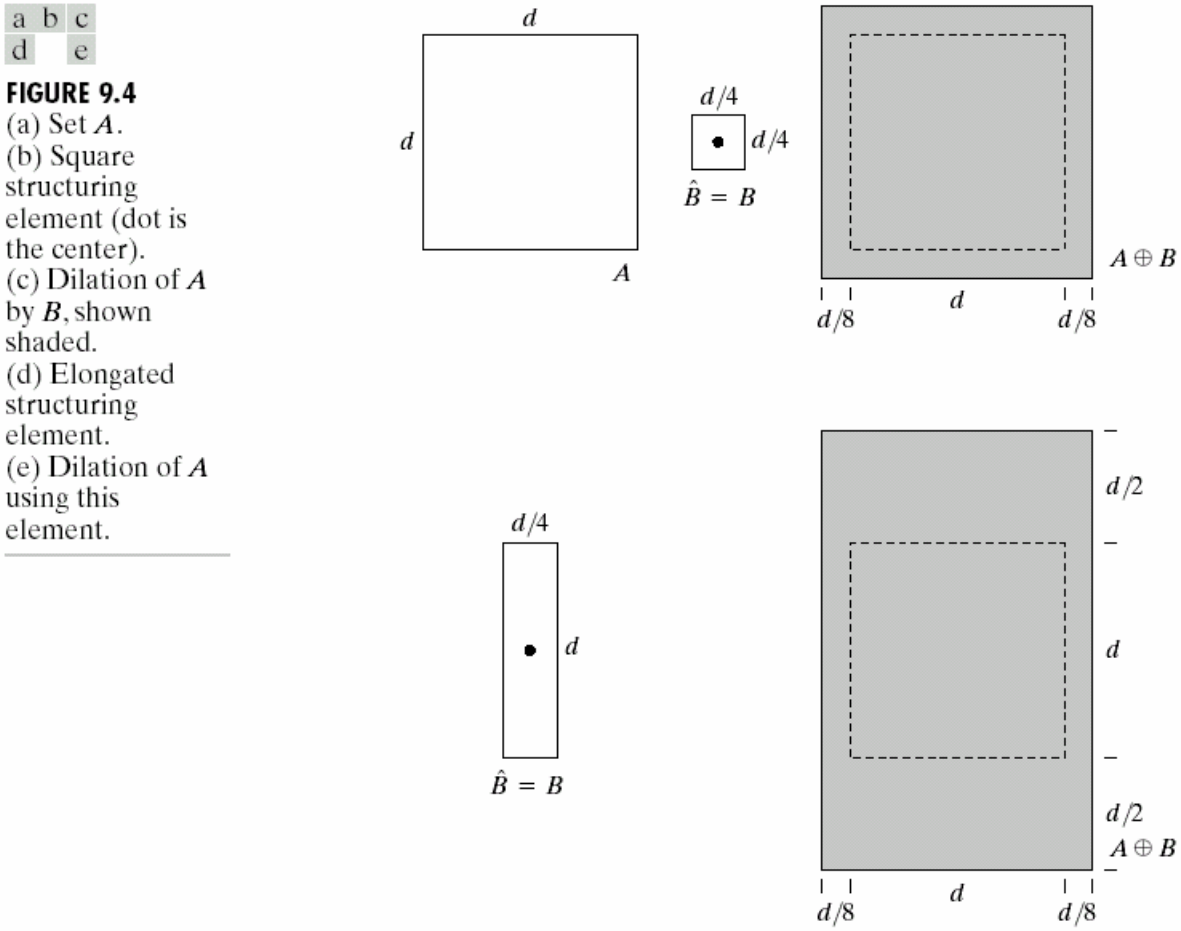
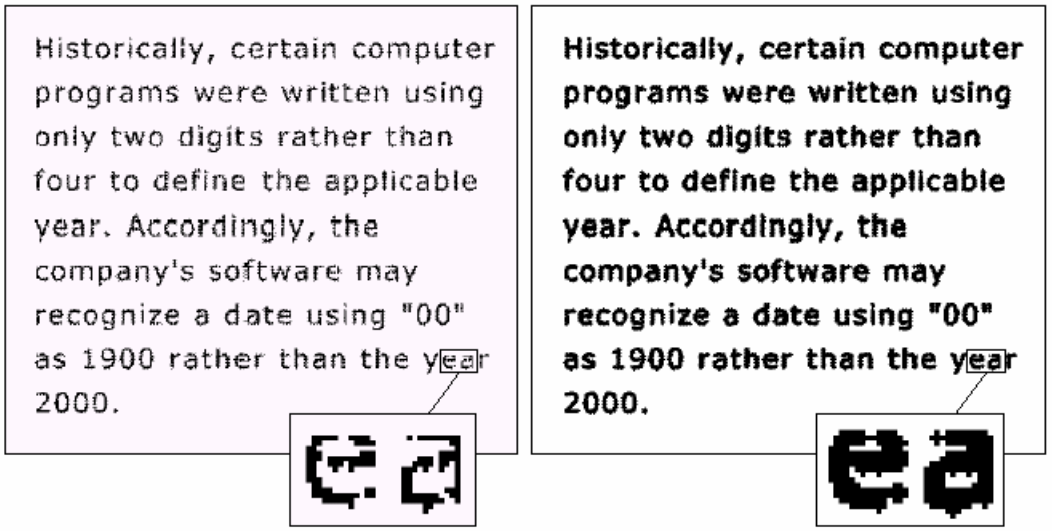


FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.

Dilation



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

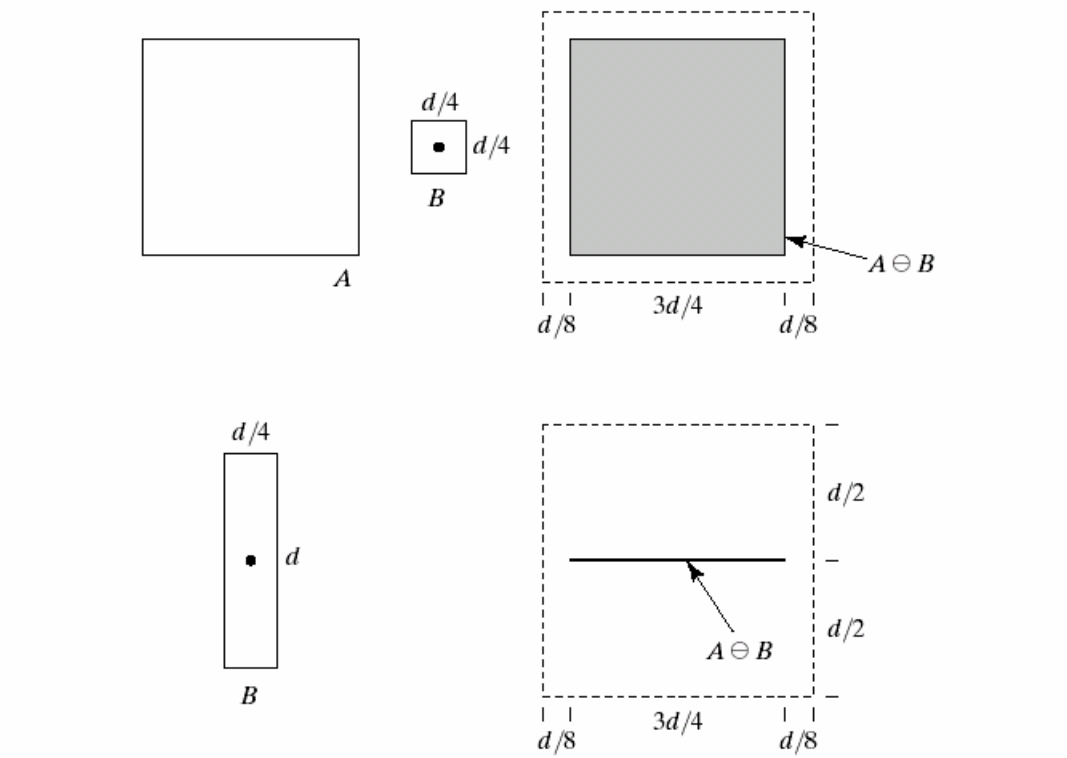
Dilation & Erosion

- Erosion:

$$A(-)B = \{z \mid (B)_z \subseteq A\}$$

- Erosion of A by B is the set of all displacements z such that B translated by z is contained in A
- B is called the structuring element
- One of the applications of erosion is elimination of irrelevant details

Erosion



a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

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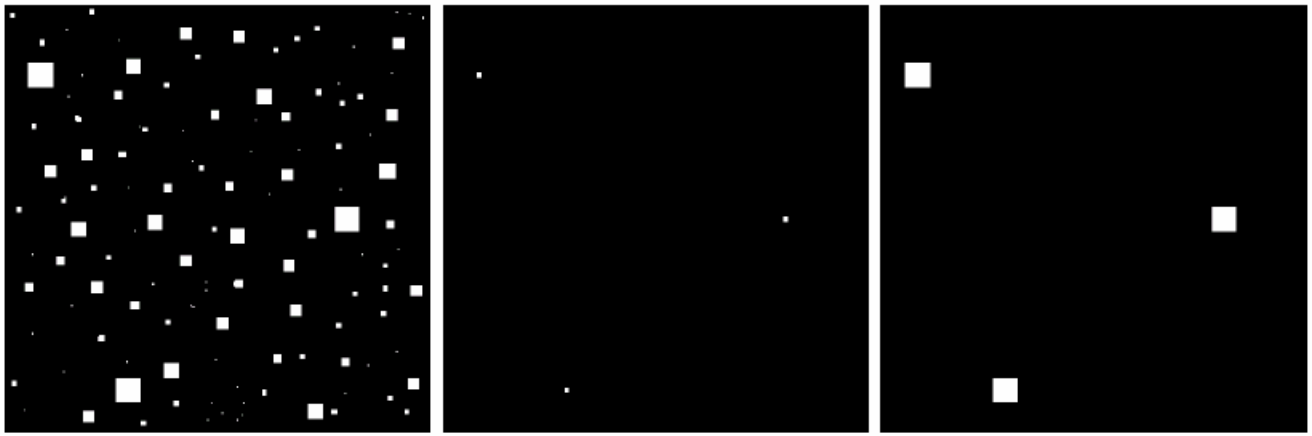
Dilation & Erosion



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

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Opening and closing

- Opening: smoothes the contour of an object, breaks narrow strips and eliminates thin protrusions (bulges)
- Opening of set A by structuring element B is defined as:

$$A \circ B = (A(-)B) \oplus B$$

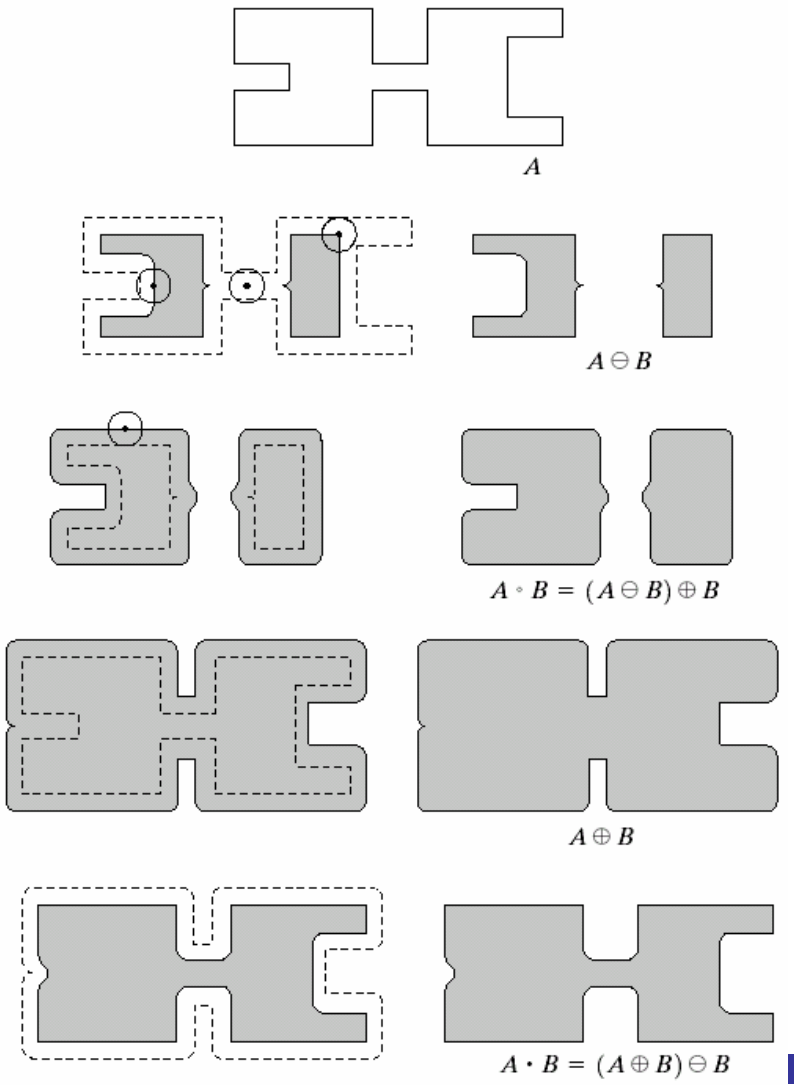
- Closing of A by structuring element B is defined as:

$$A \bullet B = (A \oplus B)(-)B$$

Opening and closing

a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



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Hit-or-Miss transform

- Hit-or-miss transform is a basic tool for shape detection
- Set A consists of three shapes: X, Y, Z
- The objective is to find the location of X
- Let X be enclosed by a small window W
- Local background of X with respect to W is W-X
- Set of locations for which X exactly fits inside A is the intersection of the erosion of A by X and the erosion of A^c by (W-X)

$$A(*)B = (A(-)X) \cap [A^c(-)(W - X)]$$

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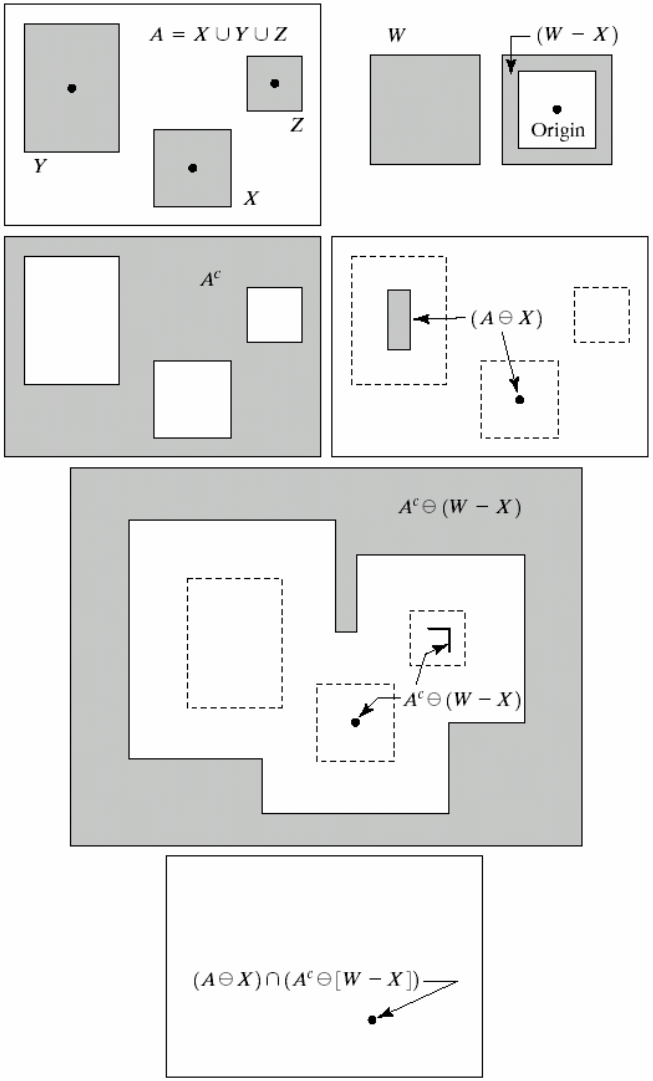
Hit-or-Miss transform



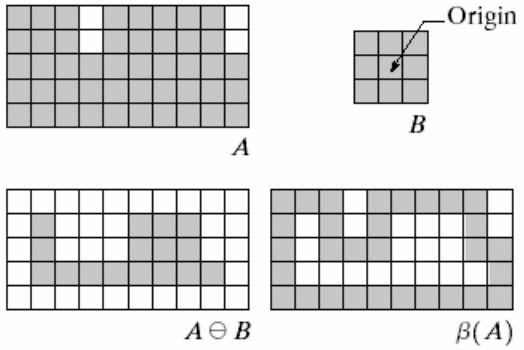
Boundary Extraction

- Boundary of a set A, denoted by β(A) can be obtained by first eroding A by B and then performing the set difference between A and its erosion:

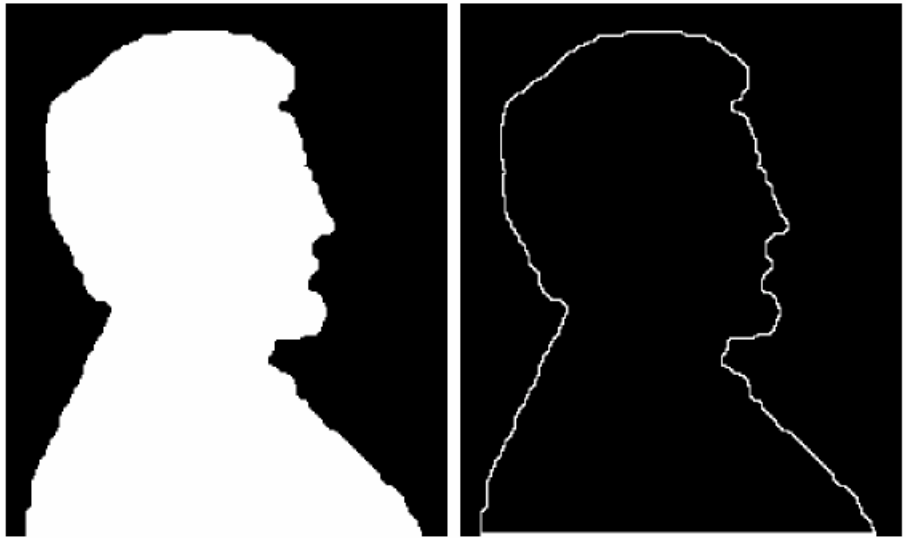
$$\beta(A) = A - (A(-)B)$$

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



Boundary



a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

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Region filling

- Let's A denote the boundary points of a region and p is a point inside the boundary
- Objective is to fill the entire region with 1s.

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

X_0=p

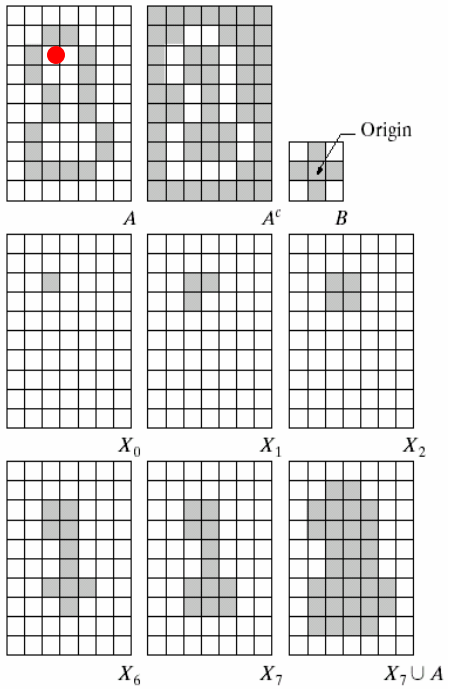
B is the structuring element

Algorithm terminates if X_k=X_{k-1}

Union of X_k and A is the region filled

a b c
d e f
g h i

FIGURE 9.15
Region filling.
(a) Set *A*.
(b) Complement of *A*.
(c) Structuring element *B*.
(d) Initial point inside the boundary.
(e)–(h) Various steps of Eq. (9.5-2).
(i) Final result [union of (a) and (h)].



Extraction of connected components

- Extraction of connected components in a binary image is central to many automated image analysis applications
- Let Y represent a connected component contained in a set A and assume that a point p of Y is known. Then the following iterative expression yields all the elements of Y:

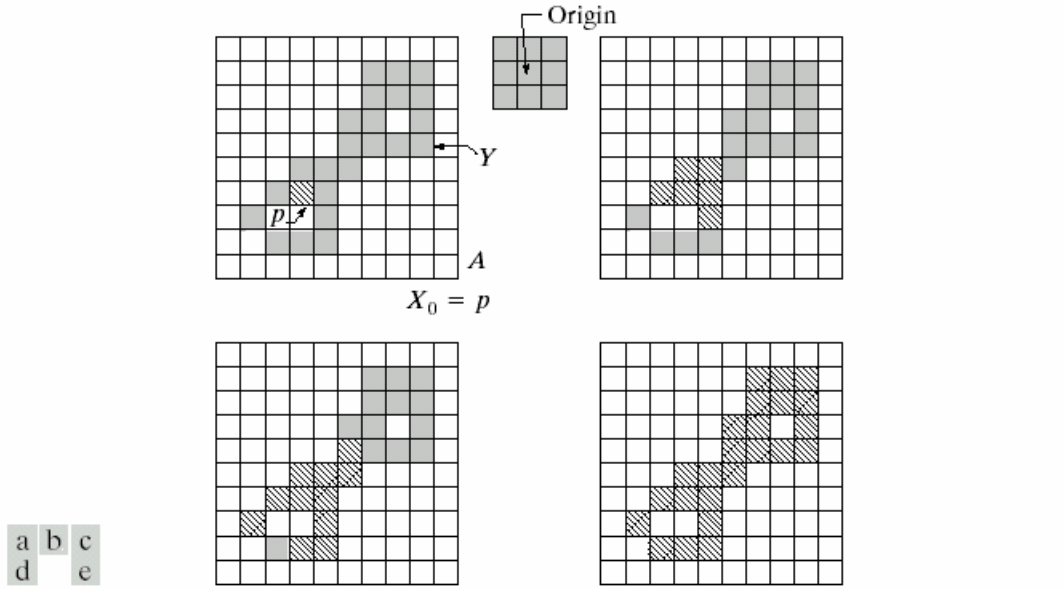
$$X_k = (X_{k-1} \oplus B) \cap A$$

X_0=p

B is the structuring element

Algorithm terminates if X_k=X_{k-1}

Extraction of connected components



a b c
d e

FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

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