McMaster University
Department of Electrical and Computer Engineering

ELEC 728: Multimedia Communications

Term I, 2008-2009

Final Exam

Time: 3 hours

Name:
Student Number:

Problem 1: (10 points) Given an i.i.d. source of six symbols {a, b, c, d, e, f} and the corresponding probabilities { 0.0625, 0.125, 0.0625, 0.125, 0.5, 0.125},

- Calculate the entropy of this source.
- Is it possible to construct a uniquely decodable binary code for this source with code lengths {1, 2, 3, 4, 4, 5}?
- Construct a Huffman code for this source.
- Use the binary tree and design a prefix code which is decodeable in forward and reverse directions both. This code does NOT have to be optimal. Hint: one way of achieving this is to make every codeword symmetric.

Problem 2: (10 points) Let $\{Z_k\}$ be a binary iid source with alphabet $A_Z = \{0,1\}$ and $p_Z(1) = q$. Suppose $\{X_k\}$ is a stationary binary source with alphabet $A_X = \{0,1\}$, with $p_{X_k}(1) = 1/2$, $p_{X_k,X_{k-1}}(0,0) = p_{X_k,X_{k-1}}(0,1) = p_{X_k,X_{k-1}}(1,0) = p_{X_k,X_{k-1}}(1,1)$. Also assume that $X_k, = X_{k-1} \oplus X_{k-2} \oplus Z_k$ where $\oplus$ denotes modulo 2 addition, and Z_k is independent of X_{k-1}, X_{k-2}.

- Show that the $\{X_k\}$ is Nth-order Markov for some appropriately chosen N.
- Find an expression for H_k (k-th order entropy) and H_∞ (Hint: use the chain rule: $H(X,Y) = H(X) + H(Y \mid X)$)

Problem 3: (10 points) Suppose that a 4×4 image has already been transformed and the 16 wavelet coefficients are as shown below. Do a EZW coding (only the first three passes).

28	-6	18	-7
5	15	3	1
2	4	-9	3
2	-2	4	-2

Problem 4: (20 points) In this problem we consider designing VQ's for a different measure of distortion, i.e. not MSE. Suppose that each source sample X_i is binary valued, specifically $X_i = 0$ or 1. Suppose further that instead of $(x - y)^2$ as the measure of dissimilarity of x and y, we use Hamming distance $d_H(x, y) = 0 \quad if \quad x = y$ and $d_H(x, y) = 1 \quad if \quad x \neq y$. Then if a VQ produces $\mathbf{Y}$ as the reproduction of $\mathbf{X}$, the average distortion (D) is the number of places in which $\mathbf{X}$ and $\mathbf{Y}$ disagree. As usual, a VQ is described by a partition $S = \{S_1, ..., S_M\}$ and codebook $C = \mathbf{w_1}, ..., \mathbf{w_M}$. However, here we assume that the codebook is binary valued, i.e. each component of each codevector is either 0 or 1, and S is a partition of $(0, 1)^k$, the set of all binary sequences of length k.

1. Derive an optimality condition for the best partition $S = S_1, ..., S_M$ for a given codebook $C = \mathbf{w_1}, ..., \mathbf{w_M}$. (Assume that the probability mass function $p_{\mathbf{X}}(\mathbf{x})$ of $\mathbf{X}$ is given.)

2. Derive an optimality condition for the best codebook $C = \mathbf{w_1}, ..., \mathbf{w_M}$ for a given partition $S = S_1, ..., S_M$. (Assume that the probability mass function $p_{\mathbf{X}}(\mathbf{x})$ of $\mathbf{X}$ is given.)

3. Describe an LGB-like algorithm that designs an M-codeword k-dimensional VQ based on a training sequence $t_1, ..., t_N$ by iterating two optimality criteria. The goal should be to minimize the distortion D defined above.

4. Apply your algorithm to design a 4-dimensional, 2-codeword VQ for the following training sequence 1111, 1110, 1110, 0001, 1001, 0001, 1000, 0010, 0001, 1101 Start with initial codebook $C1 = \{\mathbf{w_1}, \mathbf{w_2}\} = \{1100, 0011\}$.

Problem 5: (10 points) Consider a binary source with estimated $p(0) = 0.25$. A particular output of the source is "10111011001011101011".

1. Encode the source output using the LZW coder. What is the average code length? Please do not decode the LZW output.

2. Encode and decode the first 6 bits of the source output using an 8-bit arithmetic coder as discussed in class.

Problem 6: (10 points) Suppose that a random variable X has the two-sided exponential pdf $f_X(x) = \frac{\delta}{4} e^{-2\lambda|x|}$. A 3-level quantizer q for X has the form: $q(x) = a$ for $x > b$, $q(x) = 0$ for $-b \leq x \leq b$, and $q(x) = -a$ for $x < -b$.

1. Find an expression for a as a function of b so that the centroid condition is met.
2. For what values of b will the quantizer using a as above satisfy both the LLoyd conditions for optimality? What is the resulting MSE?

Problem 7: (15 points) Design a 2-dimensional fixed rate transform code with rate R=2 bit/sample for a first order Gaussian autoregressive source with mean 0, variance 1 and correlation coefficient $\rho = 0.9$. The code should have as small MSE as possible. You need to specify the transform and the scalar quantizers, as well as give an overall block diagram of the encoder and decoder. Assume the scalar quantizers have integer rates and are uniform. Compute the MSE of this code. Compare the MSE to that of an optimal fixed rate uniform scalar quantizer for this source with the same rate.

Hint: use the KLT transform and the scalar quantization tables of the textbook.

Problem 8: (15 points) Consider the following lossy compression scheme for a binary source. We divide the binary sequence into blocks of size M. For each block we count the number of 0s. If this number is greater than or equal to M/2 we send a 0; otherwise we send a 1.

1. If the source is modeled as iid with P(0)=0.6, compute the rate-distortion function for M=2 and M=4
2. Assume the output of this encoder is encoded at a rate equal to the entropy of the output. Find the rate-distortion function.
3. Now assume we do the same coding but the source is modeled as a two-state Markov model with $P(0) = 0.6, P(1) = 0.4, P(0 \mid 0) = 0.8, P(1 \mid 1) = 0.8$. Find the rate-distortion function for M=2 and M=4. Ignore the dependency between the blocks.
4. Assume the output of this second encoder is encoded at a rate equal to the entropy of the output. Find the rate-distortion function.
5. Plot the RD curves for these encoders on the same graph.