Logic Design

Chapter 1: Binary Numbers

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Machine Representation of Numbers/Data

- In logic circuits, numbers (and data in general) are coded as electronic signals
  - Each signal provides one bit of information
  - A bit can take on only two possible values, 0 and 1
- Human cultures are more familiar with decimal numbers consisting of digits of 10 possible values, 0, 1, …, 9

$$8547 = 8 \times 10^3 + 5 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

Decimal vs. Binary Numbers

- Decimal number system

$D = d_{n-1}d_{n-2}\cdots d_1 d_0$
$V(D) = d_{n-1} \times 10^{n-1} + d_{n-2} \times 10^{n-2} + \cdots + d_1 \times 10^1 + d_0 \times 10^0$

As a digit has 10 possible values (human hands!), decimal numbers are said to be of base-10 or radix-10

- Electronic circuits can conveniently code two possible values, hence binary or base-2 numbers are used in hardware

$B = b_{n-1}b_{n-2}\cdots b_1 b_0$
$V(B) = b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \cdots + b_1 \times 2^1 + b_0 \times 2^0$

Binary Numbers

$B = 1101$
$V(B) = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$(1101)_2 = (13)_{10}$

- In a binary number the right most bit is called least-significant bit (LSB)
- The left most bit is called the most significant bit (MSB)
- A group of four bits: nibble
- A group of eight bits: byte

Conversion between Decimal and Binary

- Converting a decimal number into a binary number: successively divide the decimal number by 2.

$V = b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \cdots + b_1 \times 2^1 + b_0 \times 2^0$
$\frac{V}{2} = b_{n-1} \times 2^{n-2} + b_{n-2} \times 2^{n-3} + \cdots + b_1 + \frac{b_0}{2}$

- Do an example on the board