1 Problem Description

In tone mapping of grey scale images, an input gray level $x$ is mapped to an output grey level $y$. This mapping is carried out by a transfer function $y = T(x)$. Suppose that the number of grey levels is 256. The transfer function performs an integer-to-integer mapping

$$T : \{0, 1, \cdots, 255\} \rightarrow \{0, 1, \cdots, 255\}$$

(1)

The transfer function $T$ is monotonically non-decreasing, because $T$ should never reverse the order of intensities. In other words, we must have

$$T(j) \geq T(i) \text{ if } j > i$$

(2)

Therefore, any transfer function satisfying the required monotonicity has the form

$$T(i) = \sum_{0 \leq j \leq i} s_j, \quad 0 \leq i < 256$$

$$0 \leq s_j < 256$$

$$\sum_{0 \leq j < 256} s_j < 256$$

(3)

In (3), $s_j$ is the increment in output intensity versus a unit step up in input level $j$. Therefore, $s_j$ can be interpreted as contrast at level $j$, which is the rate of change in output intensity without considering the pixel context. Note that a transfer function is completely determined by the integer-valued vector $s = (s_0, s_1, \cdots, s_{N_x-1})$. Having associated the transfer function $T$ with contrasts $s_j$'s at different levels, we induce from (3) a definition of expected contrast of $T$ for an image $I$:

$$C(s) = \sum_{0 \leq j < N_x} p_j s_j$$

(4)

where $p_j$ is the probability that a pixel in $I$ has input gray level $j$.

2 Tasks and Requirements

a. Formulate the problem of contrast enhancement as a linear programming problem of maximizing the expected contrast \( C(s) \) while satisfying the monotonicity condition (2) and the range of the mapping function \( T \) in (1).

b. Solve the above formulated linear programming problem using MatLab or any other solver, and run your program on test images provided to you. Are the results good? Explain your observations.

c. Improve your results by using additional constraints (hints: forcing the average intensity of the output image to be close to the average intensity of the corresponding input image; limiting the difference between the output grey level \( T(i) \) and the input grey level \( i \)).

d. Compute the transfer function that minimizes the expected contrast using linear programming; observe and explain the resulting image.

e. Write a project report to detail your algorithm development and discuss the roles of different constraints and their impacts on visual effects of the processed image. Your conclusions and claims have to be backed up by experimental results, such as output images and plots of the input histograms and optimized transfer functions.

Notes:

- Submit your project report together with your codes.
- This project is to be carried out by individual.

Due: midnight, Thursday, April 3, 2014.