When displaying low-resolution images on high-resolution monitors, for example, watching standard definition TV programs on high definition TV sets, the viewers or players have to interpolate the missing pixels as illustrated by the following figure. In the figure, the yellow dots are the pixels of the low-resolution image and the white dots are the missing pixels to be interpolated.

a) Develop a bi-cubic interpolation algorithm that is described by the following formula:

\[ f(x, y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{i,j} x^i y^j \]

where \( a_{i,j} \) are the coefficients of the bi-cubic interpolation function. The two-dimensional function \( f(x, y) \) interpolates the low-resolution pixels, and it is used to estimate the missing pixels.

b) Present your algorithm in Matlab or C or pseudo code. Analyze the complexity of your algorithm, and show the number of operations (additions, multiplication(s), and division(s) if any) required to interpolate a pixel. Is the number of operations required to interpolate the center missing pixels (the hashed white dots) the same as that to interpolate the other missing pixels (the empty white dots)?

c) Write a program (either in C or in Matlab) to read an image, perform image interpolation using your algorithm, and then display the interpolated image.

d) In your experiments, down sample a grey-scale image (the original) by a factor of 2 (dropping every other row and every other column of pixels) to generate its low-
resolution version (the input image for your algorithm). Compare the output image of your program with the original image and compute the mean-squared error.

e) Evaluate the accuracy of your algorithm on images of different types, and explain your findings.

Requirements:

1. This project is to be carried out by individual and independently.

2. Submit a written report to detail the development of your algorithm(s), the source codes, your experimental results, and comment on your findings.

3. Demonstrate your program to your TA face to face.