Answer for assignment 2 (3SK3)

(a)

$$\overline{E}(k,c) = \frac{1}{2\Delta}\int_{x_0-\Delta}^{x_0+\Delta}[\cos x - (kx+c)]^2\,dx$$

$$\frac{\delta\overline{E}}{\delta k} = -\frac{1}{\Delta}\int_{x_0-\Delta}^{x_0+\Delta}(\cos x - kx - c)x\,dx = 0$$

$$\frac{\delta\overline{E}}{\delta c} = -\frac{1}{\Delta}\int_{x_0-\Delta}^{x_0+\Delta}(\cos x - kx - c)\,dx = 0$$

$$\int_{x_0-\Delta}^{x_0+\Delta}(\cos x)x\,dx - k\int_{x_0-\Delta}^{x_0+\Delta}x^2\,dx - c\int_{x_0-\Delta}^{x_0+\Delta}x\,dx = 0$$

$$\int_{x_0-\Delta}^{x_0+\Delta}(\cos x)\,dx - k\int_{x_0-\Delta}^{x_0+\Delta}x\,dx - c\int_{x_0-\Delta}^{x_0+\Delta}dx = 0$$

Form the above equations, we can see that it is a system of two linear equations in variables k and c

(b)

K = 3/2*(cos(x0+h)+sin(x0+h)*h-cos(x0-h)+sin(x0-h)*h)/h^3

c= -1/2*(-sin(x0+h)*h^2+3*x0*cos(x0+h)+3*x0*sin(x0+h)*h-3*x0*cos(x0-h)+3*x0*sin(x0-h)*h+sin(x0-h)*h^2)/h^3

(c)

$$\overline{f(x)} = \cos x_0 - \sin x_0 (x - x_0)$$

(d)

k_* is not equal to the slope of the linear Taylor expansion.

(e)

S1 =

-1/4/h^4*(6*cos(x0+h)*sin(x0+h)*h-6*sin(x0+h)*cos(x0-h)*h-6*cos(x0-h)*cos(x0+h)+4*sin(x0-h)^2*h^2+4*sin(x0+h)^2*h^2+3*cos(x0+h)^2+3*cos(x0-h)^2+4*sin(x0+h)*sin(x0-h)*h^2+cos(x0-h)*sin(x0-h)*h^3-cos(x0+h)*sin(x0+h)*h^3+6*cos(x0+h)*sin(x0-h)*h-2*h^4-6*cos(x0-h)*sin(x0-h)*h)

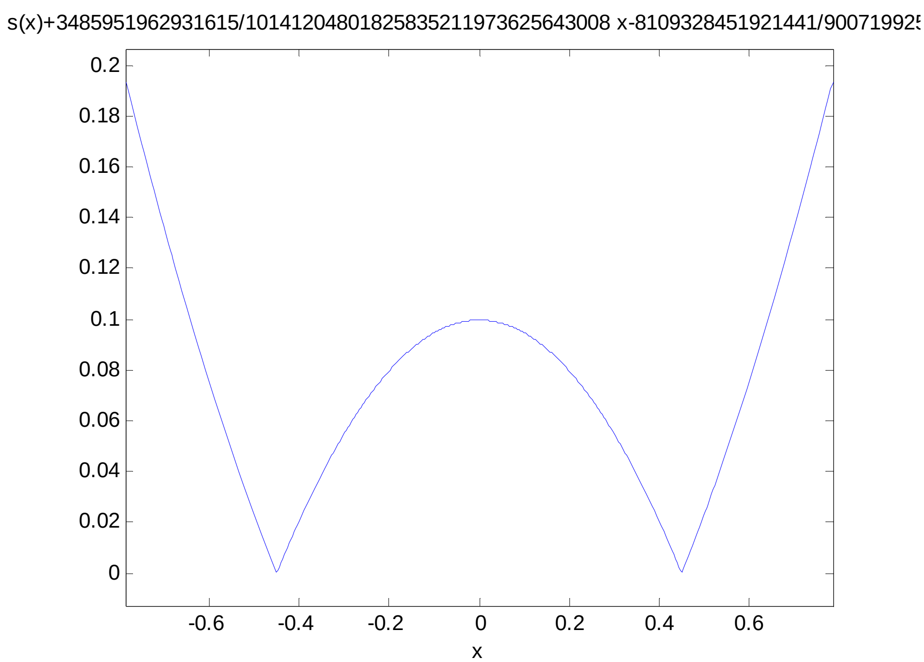
(f)

They are different, and $g_*(x) = k_* x + c_*$ is a better linear approximation. The reason is $g_*(x) = k_* x + c_*$ is the optimal one among all possible linear approximation.

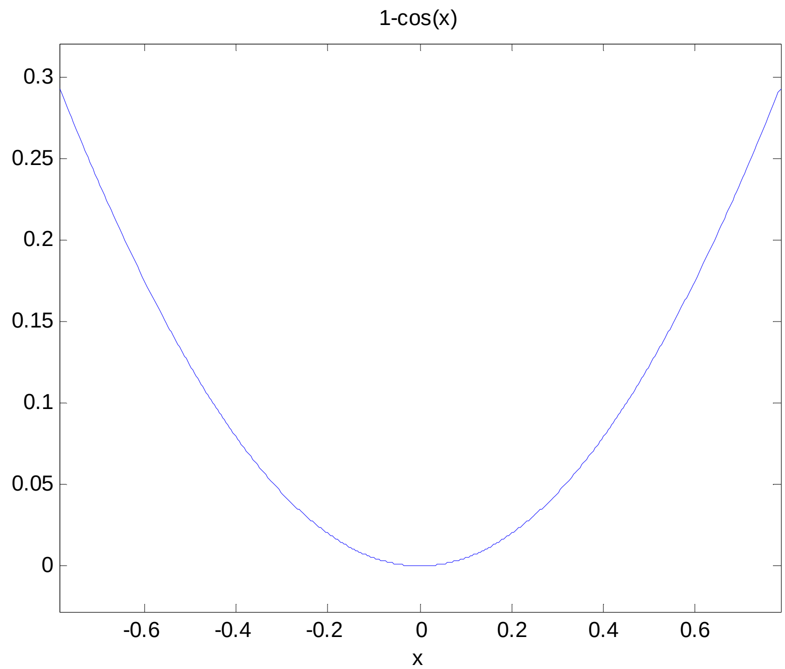
(g)

The $e(x)$ computed based on $g_*(x)$ has two roots which are not at x_0, while its counterpart has only one root which is at x_0. The $e(x)$ computed based on $g_*(x)$ has a local maximum around x_0, while the counterpart has the global minimum around x_0.

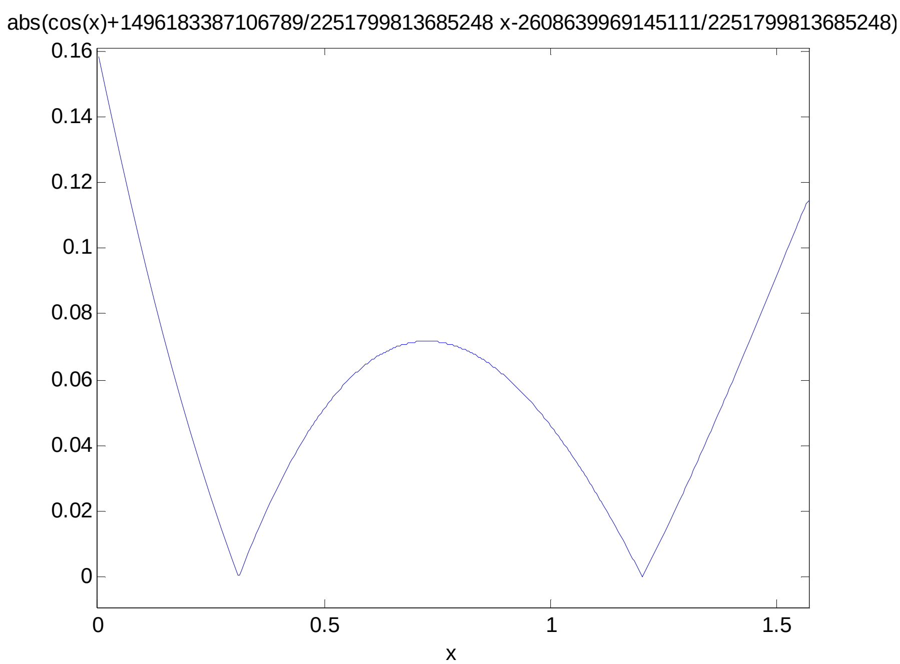
The figure $e(x)$ computed based on $g_*(x)$ when $\Delta = \pi / 4, x_0 = 0$



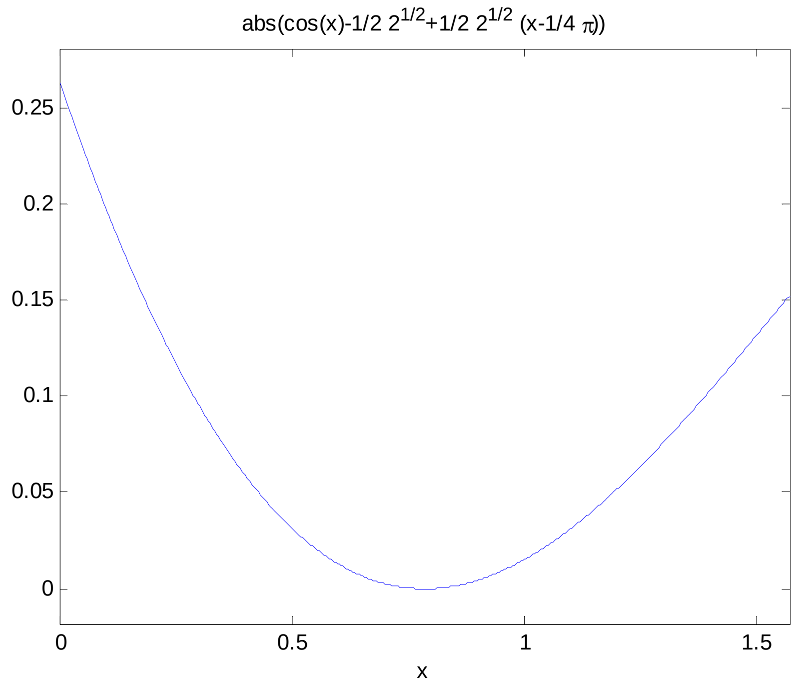
The figure $e(x)$ computed based on first order Taylor approximation when $\Delta = \pi / 4, x_0 = 0$



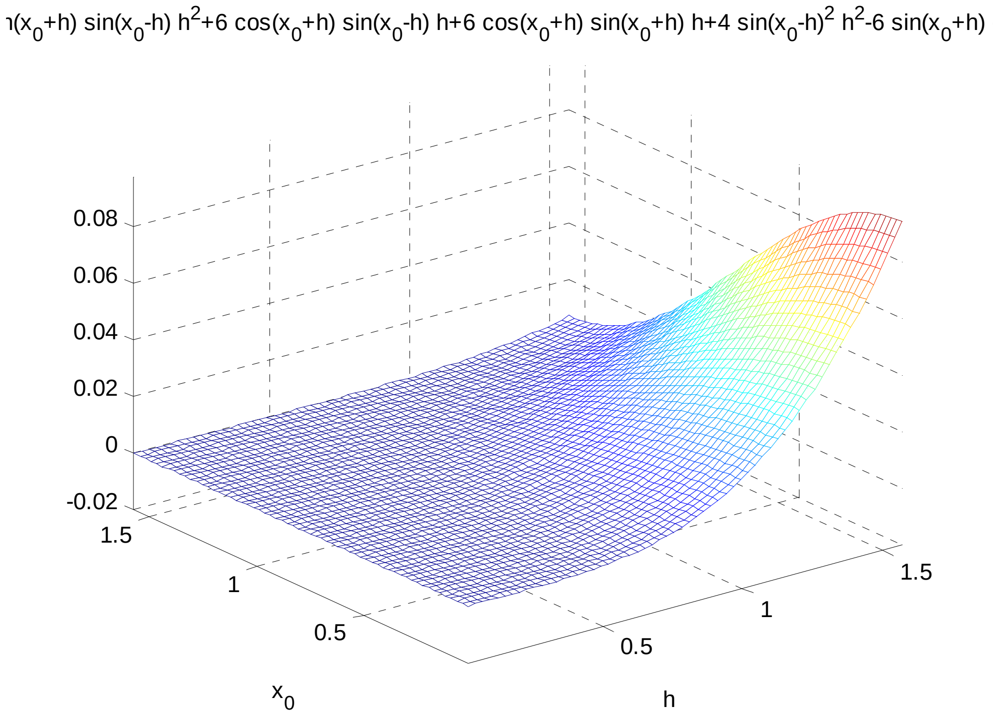
The figure $e(x)$ computed based on $g_*(x)$ when $\Delta = \pi / 4, x_0 = \pi / 4$



The figure $e(x)$ computed based on first order Taylor approximation when $\Delta = \pi / 4, x_0 = \pi / 4$



(h)



Matlab Code:

```
clear;
syms x0 h  k  x b real

f1 = (cos(x)-k*x-b)*x;
term1 = int(f1, x, x0-h, x0+h);

f2 = (cos(x)-k*x-b);
term2 = int(f2, x, x0-h, x0+h);

A = solve(term1, term2, k, b);   % question (b) k and b are
got

S1_term = (cos(x) - A.k*x - A.b)^2;
S1 = int(S1_term, x, x0-h, x0+h)/(2*h);  % question (e)

figure, ezmesh(S1);  % question (h)


S2_term = (cos(x) + sin(x0)*x - cos(x0) - sin(x0)*x0)^2;
S2 = int(S2_term, x, x0-h, x0+h)/(2*h);   %MSE formula of
the first-order Taylor approximation

%figure, ezmesh(S2, [pi/200, pi/2, pi/200, pi/2]);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

sub = S2 - S1;                      %compare the two MSEs on
pi/4 and pi/3
sub_1 = subs(sub, x0, pi/4);
sub_2 = subs(sub, x0, pi/3);
xx = pi/200:pi/200:pi/4;
ss = size(xx, 2);
ff1 = zeros(1, ss);
ff2 = zeros(1, ss);
for i=1:ss
    ff1(i) = subs(sub_1, h, xx(i));
    ff2(i) = subs(sub_2, h, xx(i));
end

figure, plot(xx, ff1, '-.r*', xx, ff2, '--mo');
xlabel('0 \leq h \leq \pi/4');
ylabel('f(h)');
title('f(h)');
legend('x0=\pi/4', 'x0=\pi/3');               % question (f)
```

```
% question (g)
clear;
syms x0 h  k  x b real

f1 = (cos(x)-k*x-b)*x;
term1 = int(f1, x, x0-h, x0+h);

f2 = (cos(x)-k*x-b);
term2 = int(f2, x, x0-h, x0+h);

A = solve(term1, term2, k, b);

x0_store = [0, pi/4];

h_value = pi/4;

for i = 1:2
    k = subs(A.k, {h, x0}, {h_value, x0_store(i)});
    b = subs(A.b, {h, x0}, {h_value, x0_store(i)});

    error1 = abs(cos(x) - k*x - b);
    figure, ezplot(error1, [x0_store(i)-h_value,
x0_store(i)+h_value]);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
apx = cos(x0) - sin(x0)*(x-x0);
error_formula = abs(cos(x) - apx);

for i = 1:2
    error2 = subs(error_formula, x0, x0_store(i));
    figure, ezplot(error2, [x0_store(i)-h_value,
x0_store(i)+h_value]);
end
```