COMP ENG 3SK3 Winter 2011

Assignment #2 (Solutions)

```
Problem 4.1
------------
True value: cos(pi/3) = 0.5
Maclaurin series approximation:
Stopping criterion, es = 0.5%
Terms  Result        |ea| (%)
1      1.000000      100.000000
2      0.451689      121.391441
3      0.501796      9.985639
4      0.499965      0.366353
```

```
disp('Problem 4.1')
disp('------------')

tvalue = cos(pi/3); % Compute true value
disp(['True value: cos(pi/3) = ', num2str(tvalue)])

n = 2;              % Correct to at least 2 significant figures
es = 0.5*10^(2-n);  % Stopping criterion
ea = 100;           % Initialize approximate error estimate
i = 0;
cos_x = 0;
disp('Maclaurin series approximation:')
disp(['Stopping criterion, es = ', num2str(es), '%'])
fprintf('Terms\tResult\t\t|ea| (%%)\n');
while (ea >= es)    % Process continued till |ea|<es
    % Compute Maclaurin series approximation
    cos_x = cos_x + (-1)^i*(pi/3)^(2*i)/factorial(2*i);
    % Compute absolute value of the approximate error estimate
    if(i)
        ea = abs((cos_x - cos_x_prev)*100/cos_x);
    end
    fprintf('%d\t\t%f\t%f\n', i+1, cos_x, ea);
    cos_x_prev = cos_x;
    i = i + 1;
end
```

```
Problem 4.2
------------
True value: sin(pi/3) = 0.86603
Maclaurin series approximation:
Stopping criterion, es = 0.5%
Terms  Result        |ea| (%)
1      1.047198      100.000000
2      0.855801      22.364641
3      0.866295      1.211423
4      0.866021      0.031640
```

```
disp('Problem 4.2')
disp('------------')

tvalue = sin(pi/3); % Compute true value
disp(['True value: sin(pi/3) = ', num2str(tvalue)])

n = 2;              % Correct to at least 2 significant figures
es = 0.5*10^(2-n);  % Stopping criterion
ea = 100;           % Initialize approximate error estimate
i = 0;
sin_x = 0;
```

```
disp('Maclaurin series approximation:')
disp(['Stopping criterion, es = ', num2str(es), '%'])
fprintf('Terms\tResult\t\t|ea| (%%)\n');
while (ea >= es)    % Process continued till |ea|<es
    % Compute Maclaurin series approximation
    sin_x = sin_x + (-1)^i*(pi/3)^(2*i+1)/factorial(2*i+1);
    % Compute absolute value of the approximate error estimate
    if(i)
        ea = abs((sin_x - sin_x_prev)*100/sin_x);
    end
    fprintf('%d\t\t%f\t%f\n', i+1, sin_x, ea);
    sin_x_prev = sin_x;
    i = i + 1;
end
```

Problem 4.3(a)

Taylor series expansion [Eq. (4.7)]:

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \frac{f^{(3)}(x_i)}{3!}h^3 + \cdots + \frac{f^{(n)}(x_i)}{n!}h^n, \qquad h = x_{i+1} - x_i$$

With $x_i = 0$ and $h = x_{i+1} = x$, $\qquad f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$

$$f(x) = f'(x) = f''(x) = f^{(3)}(x) = \cdots = f^{(n)}(x) = e^x,$$
$$f(0) = f'(0) = f''(0) = f^{(3)}(0) = \cdots = f^{(n)}(0) = e^0 = 1$$
$$\therefore e^x = f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

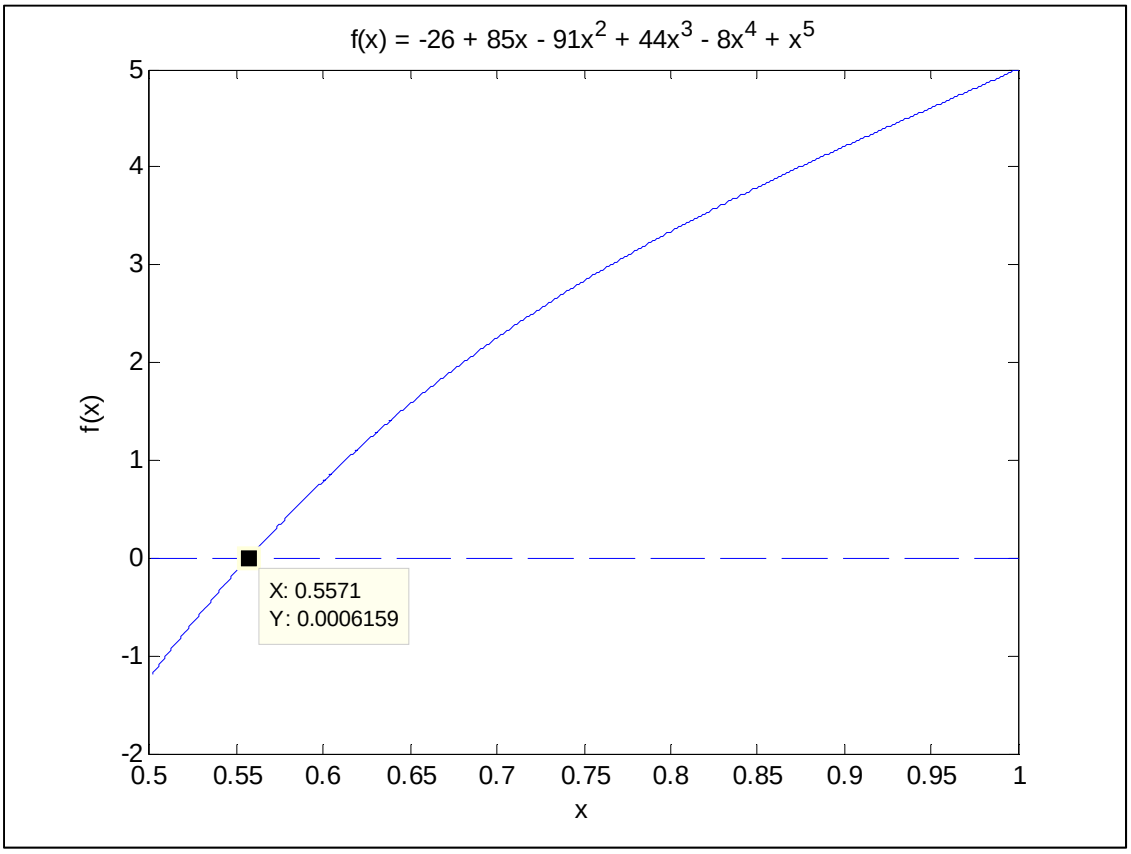
```
Problem 4.3(b)
------------
True value: e^(-1) = 0.36788
Taylor series approximation:
Order  Result       |et| (%)
0      0.818731     122.554093
1      0.163746     55.489181
2      0.425740     15.728128
3      0.355875     3.263154
```

```
disp('Problem 4.3(b)')
disp('------------')

tvalue = exp(-1);       % Compute true value
disp(['True value: e^(-1) = ', num2str(tvalue)])

syms x;
f = exp(-x);            %f(x)
xi1 = 1;   xi = 0.2;
e_1 = 0;
disp('Taylor series approximation:')
fprintf('Order\tResult\t\t|et| (%%)\n');
% Compute Taylor series approximation
for i = 0:3             % Order
    if(i)
        f = diff(f);    % Differentiate f(x)
    end
    e_1 = e_1 + subs(f,xi)*(xi1 - xi)^i/factorial(i);   % Compute e^(-1)
    et = abs((e_1 - tvalue)*100/tvalue);                % Compute |et|
    fprintf('%d\t\t%f\t%f\n', i, e_1, et);
end
```

Problem 5.3(a)




```
x = linspace(0.5,1,1000);
f = -26 + 85*x - 91*x.^2 + 44*x.^3 - 8*x.^4 + x.^5;
figure, plot(x,f)
hold on
plot(x,zeros(1,1000),'--')
xlabel('x')
ylabel('f(x)')
title('f(x) = -26 + 85x - 91x^2 + 44x^3 - 8x^4 + x^5')
hold off
```

```
Problem 5.3(b)
------------
Bisection method
Iteration     xl            xu            xr            |ea| (%)
1             0.500000      1.000000      0.750000      100.000000
2             0.500000      0.750000      0.625000      20.000000
3             0.500000      0.625000      0.562500      11.111111
4             0.500000      0.562500      0.531250      5.882353
```

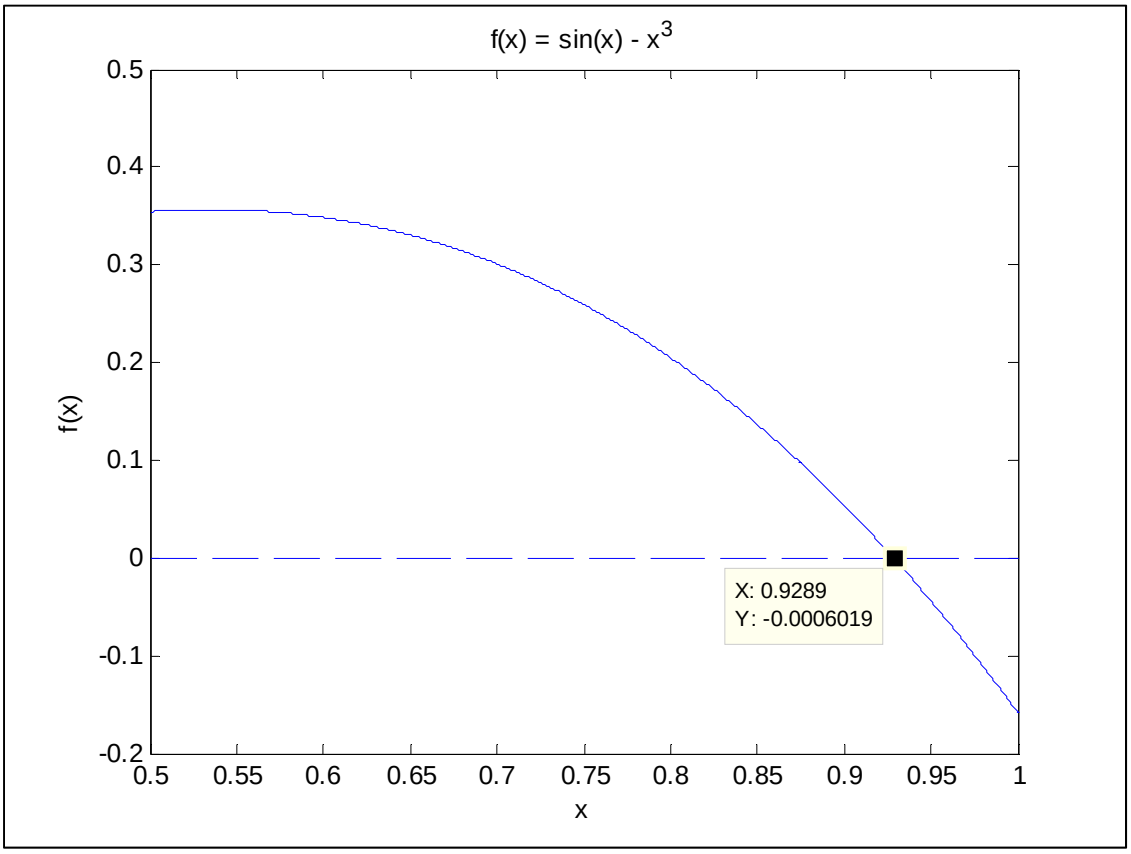
```
disp('Problem 5.3(b)')
disp('------------')
xl = 0.5;   xu = 1.0;
es = 10;    ea = 100;
i = 0;
disp('Bisection method')
fprintf('Iteration\txl\t\t\txu\t\t\txr\t\t\t|ea| (%%)\n');
while (ea >= es)
    xr = (xl+xu)/2;
```

```
    if(i)
        ea = abs((xr - xr_prev)*100/xr);    % Compute |ea|
    end
    xr_prev = xr;
    i = i+1;
    fprintf('%d\t\t\t%f\t%f\t%f\t%f\n', i, xl, xu, xr, ea);
    fxr = -26 + 85*xr - 91*xr^2 + 44*xr^3 - 8*xr^4 + xr^5;
    fxl = -26 + 85*xl - 91*xl^2 + 44*xl^3 - 8*xl^4 + xl^5;
    fxu = -26 + 85*xu - 91*xu^2 + 44*xu^3 - 8*xu^4 + xu^5;
    if(fxr*fxl < 0)     % Check if f(x) changes sign over lower half interval
        xu = xr;        % Update xu
    elseif(fxr*fxu < 0) % Check if f(x) changes sign over upper half interval
        xl = xr;        % Update xl
    end
end
```

```
Problem 5.3(c)
------------
False position
Iteration     xl            xu            xr            |ea| (%)
1             0.500000      1.000000      0.597990      100.000000
2             0.500000      0.597990      0.560643      6.661457
3             0.500000      0.560643      0.557338      0.592956
4             0.500000      0.557338      0.557052      0.051288
```

```
disp('Problem 5.3(c)')
disp('------------')
xl = 0.5;   xu = 1.0;
es = 0.2;   ea = 100;
i = 0;
disp('False position method')
fprintf('Iteration\txl\t\t\txu\t\t\txr\t\t\t|ea| (%%)\n');
while (ea >= es)
    fxl = -26 + 85*xl - 91*xl^2 + 44*xl^3 - 8*xl^4 + xl^5;
    fxu = -26 + 85*xu - 91*xu^2 + 44*xu^3 - 8*xu^4 + xu^5;
    xr = xu - fxu*(xl-xu)/(fxl-fxu);
    if(i)
        ea = abs((xr - xr_prev)*100/xr);    % Compute |ea|
    end
    xr_prev = xr;
    i = i+1;
    fprintf('%d\t\t\t%f\t%f\t%f\t%f\n', i, xl, xu, xr, ea);
    fxr = -26 + 85*xr - 91*xr^2 + 44*xr^3 - 8*xr^4 + xr^5;
    if(fxr*fxl < 0)     % Check if f(x) changes sign over lower half interval
        xu = xr;        % Update xu
    elseif(fxr*fxu < 0) % Check if f(x) changes sign over upper half interval
        xl = xr;        % Update xl
    end
end
```

Problem 5.5




```
Problem 5.5
------------
Bisection method
Iteration     xl           xu           xr           |ea| (%)
1             0.500000     1.000000     0.750000     100.000000
2             0.750000     1.000000     0.875000     14.285714
3             0.875000     1.000000     0.937500     6.666667
4             0.875000     0.937500     0.906250     3.448276
5             0.906250     0.937500     0.921875     1.694915
Error check: x = 0.92188, sin(x) - x^3 = 0.013277
```

```
disp('Problem 5.5')
disp('------------')

% Graphical technique
x = linspace(0.5,1,1000);
f = sin(x) - x.^3;
figure, plot(x,f)
hold on
plot(x,zeros(1,1000),'--')
xlabel('x')
ylabel('f(x)')
title('f(x) = sin(x) - x^3')
hold off

% Bisection
xl = 0.5;    xu = 1;
es = 2;      ea = 100;
i = 0;
disp('Bisection method')
```

```
fprintf('Iteration\txl\t\t\txu\t\t\txr\t\t\t|ea| (%%)\n');
while (ea >= es)
    xr = (xl+xu)/2;
    if(i)
        ea = abs((xr - xr_prev)*100/xr);    % Compute |ea|
    end
    xr_prev = xr;
    i = i+1;
    fprintf('%d\t\t\t%f\t%f\t%f\t%f\n', i, xl, xu, xr, ea);
    fxr = sin(xr) - xr^3;
    fxl = sin(xl) - xl^3;
    fxu = sin(xu) - xu^3;
    if(fxr*fxl < 0)     % Check if f(x) changes sign over lower half interval
        xu = xr;        % Update xu
    elseif(fxr*fxu < 0) % Check if f(x) changes sign over upper half interval
        xl = xr;        % Update xl
    end
end

% Error check
disp(['Error check: x = ', num2str(xr),', sin(x) - x^3 = ', num2str(sin(xr)-xr^3)])
```

```
Problem 5.17
------------
False position method
Iteration     hl (m)        hu (m)        hr (m)        |ea| (%)
1             0.000000      3.000000      1.591549      100.000000
2             1.591549      3.000000      1.986575      19.884755
3             1.986575      3.000000      2.023904      1.844409

Depth of water in tank required = 2.0239m
```

```
disp('Problem 5.17')
disp('------------')
R = 3;      V = 30;
hl = 0;     hu = R;
ea = 100;
disp('False position method')
fprintf('Iteration\thl (m)\t\thu (m)\t\thr (m)\t\t|ea| (%%)\n');
for i = 0:2
    fhl = V - pi*hl^2*(3*R-hl)/3;
    fhu = V - pi*hu^2*(3*R-hu)/3;
    hr = hu - fhu*(hl-hu)/(fhl-fhu);
    if(i)
        ea = abs((hr - hr_prev)*100/hr);    % Compute |ea|
    end
    hr_prev = hr;
    fprintf('%d\t\t\t%f\t%f\t%f\t%f\n', i+1, hl, hu, hr, ea);
    fhr = V - pi*hr^2*(3*R-hr)/3;
    if(fhr*fhl < 0)     % Check if f(h) changes sign over lower half interval
        hu = hr;        % Update hu
    elseif(fhr*fhu < 0) % Check if f(h) changes sign over upper half interval
        hl = hr;        % Update hl
    end
end
disp(' ')
disp(['Depth of water in tank required = ', num2str(hr), 'm'])
```