**CoE 3SK3 Assignment 3**

A linear approximation $g(x) = kx + c$ of $f(x) = \cos^n x$ in interval $[x_0 - \Delta, x_0 + \Delta]$ has the mean square error:

$$\begin{aligned} \bar{E}(k,c) &= \frac{1}{2\Delta}\int_{x_0-\Delta}^{x_0+\Delta} [f(x) - g(x)]^2 dx \\ &= \frac{1}{2\Delta}\int_{x_0-\Delta}^{x_0+\Delta} [\cos^n x - (kx+c)]^2 dx. \end{aligned} \tag{1}$$

We want to find optimal linear approximation $g_*(x) = k_* x + c_*$ that minimizes $\bar{E}(k,c)$, by solving the following optimization problem:

$$\min_{k,c} \bar{E}(k,c) = \min_{k,c} \frac{1}{2\Delta}\int_{x_0-\Delta}^{x_0+\Delta} [\cos^n x - (kx+c)]^2 dx. \tag{2}$$

(a) To minimize $\bar{E}(k,c)$, let $\frac{\partial \bar{E}}{\partial k} = 0$ and $\frac{\partial \bar{E}}{\partial c} = 0$. Show that these two conditions on partial derivatives correspond to a system of two linear equations in variables $k$ and $c$.

(b) For $n = 3, 4$, solve the system of two linear equations in (a) *symbolically* in MATLAB to determine the parameters $k_*$ and $c_*$ for the optimal linear approximation $g_*(x) = k_* x + c_*$. In your solution, $k_*$ and $c_*$ are expressed in terms of $x_0$ and $\Delta$.

(c) Derive the formula of the first-order (i.e., linear) Taylor expansion of $f(x) = \cos^n x$ at $x_0$.

(d) Comparing $g_*(x) = k_* x + c_*$ with the formula of part (c) for $n = 3$, is $k_*$ equal to the slope of the linear Taylor expansion?

(e) Let $\Delta = \pi/4$ and respectively for $x_0 = 0$ and $x_0 = \pi/4$, plot and visualize the error function $e(x) = |\cos^3 x - g_*(x)|$ in the interval $x \in [x_0 - \Delta, x_0 + \Delta]$.

(f) Plot the two-dimensional error function

$$\bar{E}(k_*, c_*) = \frac{1}{2\Delta}\int_{x_0-\Delta}^{x_0+\Delta} [\cos^3 x - (k_* x + c_*)]^2 dx. \tag{3}$$

in $x_0$ and $\Delta$, and comment on your observations.

Due: 6:00pm, Monday, Feb. 28, 2011.