Image Enhancement

Enhancement: to process an image for more suitable output for a specific application.

Image Enhancement

- Image enhancement techniques:
  - Spatial domain methods
  - Frequency domain methods

- Spatial (time) domain techniques are techniques that operate directly on pixels.

- Frequency domain techniques are based on modifying the Fourier transform of an image.

Joseph Fourier (1768-1830)

Fourier was obsessed with the physics of heat and developed the Fourier transform theory to model heat-flow problems.

Fourier Transform: a review

- Basic ideas:
  - A periodic function can be represented by the sum of sines/cosines functions of different frequencies, multiplied by a different coefficient.
  - Non-periodic functions can also be represented as the integral of sines/cosines multiplied by weighing function.

Fourier transform basis functions

Approximating a square wave as the sum of sine waves.
Any function can be written as the sum of an even and an odd function

\[ E(x) = \frac{f(x) + f(-x)}{2} \]
\[ O(x) = \frac{f(x) - f(-x)}{2} \]
\[ f(x) = E(x) + O(x) \]

**Fourier Cosine Series**

Because \( \cos(mt) \) is an even function, we can write an even function, \( f(t) \), as:

\[ f(t) = \frac{1}{\pi} \sum_{n=0}^{\infty} F_n \cos(mt) \]

where \( F_n \) is computed as

\[ F_n = \int_{-\pi}^{\pi} f(t) \cos(mt) \, dt \]

Here we suppose \( f(t) \) is over the interval \((-\pi, \pi)\).

**Fourier Sine Series**

Because \( \sin(mt) \) is an odd function, we can write any odd function, \( f(t) \), as:

\[ f(t) = \frac{1}{\pi} \sum_{n=1}^{\infty} F'_n \sin(mt) \]

where the series \( F'_n \) is computed as

\[ F'_n = \int_{-\pi}^{\pi} f(t) \sin(mt) \, dt \]

**Fourier Series**

So if \( f(t) \) is a general function, neither even nor odd, it can be written:

\[ f(t) = \frac{1}{\pi} \sum_{n=0}^{\infty} F_n \cos(mt) + \frac{1}{\pi} \sum_{n=0}^{\infty} F'_n \sin(mt) \]

where the Fourier series is

\[ F_n = \int f(t) \cos(mt) \, dt \quad F'_n = \int f(t) \sin(mt) \, dt \]

**The Fourier Transform**

Let \( F(m) \) incorporates both cosine and sine series coefficients, with the sine series distinguished by making it the imaginary component:

\[ F(m) = F_e - jF'_e = \int f(t) \cos(mt) \, dt - j \int f(t) \sin(mt) \, dt \]

Let’s now allow \( f(t) \) range from \(-\infty\) to \(\infty\), we rewrite:

\[ \Re [f(t)] = F(u) = \int f(t) \exp(-j2\pi u \cdot t) \, dt \]

\( F(u) \) is called the Fourier Transform of \( f(t) \). We say that \( f(t) \) lives in the “time domain,” and \( F(u) \) lives in the “frequency domain.” \( u \) is called the frequency variable.

**The Inverse Fourier Transform**

We go from \( f(t) \) to \( F(u) \) by

\[ \Re [f(t)] = F(u) = \int_{-\infty}^{\infty} f(t) \exp(-j2\pi u \cdot t) \, dt \]

Given \( F(u) \), \( f(t) \) can be obtained by the inverse Fourier transform

\[ \Re^{-1} [F(u)] = f(t) = \int_{-\infty}^{\infty} F(u) \exp(j2\pi u \cdot t) \, du \]
2-D Fourier Transform

Fourier transform for \( f(x,y) \) with two variables

\[
\mathcal{F}\{f(x,y)\} = F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \exp(-j2\pi(ux+vy)) \, dx \, dy
\]

and the inverse Fourier transform

\[
\mathcal{F}^{-1}\{F(u,v)\} = f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) \exp(j2\pi(ux+vy)) \, du \, dv
\]

Discrete Fourier Transform (DFT)

• A continuous function \( f(x) \) is discretized as:

\[
\{ f(x_0), f(x_0 + \Delta x), f(x_0 + 2\Delta x), \ldots, f(x_0 + (M-1)\Delta x) \}
\]

Discrete Fourier Transform (DFT)

Let \( x \) denote the discrete values \( x=0,1,2,\ldots,M-1 \), i.e.

\[
f(x) = f(x_0 + x\Delta x)
\]

then

\[
\{ f(x_0), f(x_0 + \Delta x), f(x_0 + 2\Delta x), \ldots, f(x_0 + (M-1)\Delta x) \}
\]

\[
\{ f(0), f(1), f(2), \ldots, f(M-1) \}
\]

Discrete Fourier Transform (DFT)

• The discrete Fourier transform pair that applies to sampled functions is given by:

\[
F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) \exp(-j2\pi ux / M)
\]

and

\[
f(x) = \sum_{u=0}^{M-1} F(u) \exp(j2\pi ux / M)
\]

2-D Discrete Fourier Transform

• In 2-D case, the DFT pair is:

\[
F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \exp(-j2\pi(ux / M + vy / N))
\]

and:

\[
f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) \exp(j2\pi(ux / M + vy / N))
\]

Polar Coordinate Representation of FT

• The Fourier transform of a real function is generally complex and we use polar coordinates:

\[
F(u,v) = R(u,v) + j \cdot I(u,v)
\]

Polar coordinate

\[
F(u,v) = |F(u,v)| \exp(j\phi(u,v))
\]

Magnitude:

\[
|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}
\]

Phase:

\[
\phi(u,v) = \tan^{-1}\left( \frac{I(u,v)}{R(u,v)} \right)
\]
Fourier Transform: shift

- It is common to multiply input image by \((-1)^{x+y}\) prior to computing the FT. This shifts the center of the FT to \((M/2, N/2)\).

\[
\mathcal{F}\{f(x,y)\} = F(u,v)
\]

\[
\mathcal{F}\{f(x,y)(-1)^{x+y}\} = F(u - M/2, v - N/2)
\]

Symmetry of FT

- For real image \(f(x,y)\), FT is conjugate symmetric:

\[
F(u,v) = F^*(-u,-v)
\]

- The magnitude of FT is symmetric:

\[
|F(u,v)| = |F(-u,-v)|
\]

Simple Cases

Magnitude (how much) & Phase (where)
The central part of FT, i.e. the low frequency components are responsible for the general gray-level appearance of an image.

The high frequency components of FT are responsible for the detail information of an image.

The high frequency components of FT are responsible for the detail information of an image.
Frequency Domain Filtering

- Edges and sharp transitions (e.g., noise) in an image contribute significantly to high-frequency content of FT.
- Low frequency contents in the FT are responsible to the general appearance of the image over smooth areas.
- Blurring (smoothing) is achieved by attenuating range of high frequency components of FT.

Ideal low-pass filter (ILPF)

\[ H(u, v) = \begin{cases} 1 & D(u, v) \leq D_0 \\ 0 & D(u, v) > D_0 \end{cases} \]

\( D(u, v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2} \)

(M/2,N/2): center in frequency domain

\( D_0 \) is called the cutoff frequency.

Convolution Theorem

\[ G(u, v) = F(u, v) \bullet H(u, v) \]

\[ g(x, y) = h(x, y) \ast f(x, y) \]

- \( f(x, y) \) is the input image
- \( g(x, y) \) is the filtered
- \( h(x, y) \): impulse response

- Filtering in Frequency Domain with \( H(u, v) \) is equivalent to filtering in Spatial Domain with \( f(x, y) \).

Shape of ILPF

Examples of Filters

 Ideal in frequency domain means non-ideal in spatial domain, vice versa.

FT

ringing and blurring

FIGURE 4.12 (a) Original image, (b-e) Results of ideal lowpass filtering with cutoff frequencies set at mid-values of 3, 13, 31, 81, and 223, as shown in Fig. 4.12(d). The power retained by these filters was 8.3, 3.3, 2, and 0.35% of the total, respectively.
Butterworth Lowpass Filters (BLPF)
• Smooth transfer function, no sharp discontinuity, no clear cutoff frequency.

\[ H(u, v) = \frac{1}{1 + \left[ \frac{D(u, v)}{D_0} \right]^{2n}} \]

Gaussian Lowpass Filters (GLPF)
• Smooth transfer function, smooth impulse response, no ringing

\[ H(u, v) = e^{-\frac{D^2(u, v)}{2D_0^2}} \]

No serious ringing artifacts
Examples of Lowpass Filtering

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Arminingly, the company’s software may recognize a date using “99” as 1990 rather than the year 2000.

Examples of Lowpass Filtering

Original image and its FT  Filtered image and its FT

Sharpening High-pass Filters

- $H_{hp}(u,v) = 1 - H_{lp}(u,v)$
- Ideal: $H(u,v) = \begin{cases} 1 & D(u,v) > D_0 \\ 0 & D(u,v) \leq D_0 \end{cases}$
- Butterworth: $|H(u,v)|^2 = \frac{1}{1 + \left(\frac{D(u,v)}{D_0}\right)^{2n}}$
- Gaussian: $H(u,v) = 1 - e^{-\gamma^2D(u,v)^2/2\sigma^2}$

Ideal High-pass Filtering

ringing artifacts
Butterworth High-pass Filtering

In Fig. 43(a) using a BHPF of order 2 with $R_0 = 15$, 30, and 60, respectively. These results are identical to those obtained with an ILPF.

Gaussian High-pass Filtering

In Fig. 43(b) using a GHPF of order 2 with $R_0 = 15$, 30, and 60, respectively. Compare with Figs. 42b and 42c.

Laplacian in Frequency Domain

\[
\hat{f}(u, v) = (u^2 + v^2)F(u, v)
\]

\[
F(u, v) = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}
\]

Spatial domain

Laplacian operator

Frequency domain

Subtract Laplacian from the Original Image to Enhance It

In the spatial domain,

\[
g(x, y) = f(x, y) - \nabla^2 f(x, y)
\]

In the frequency domain,

\[
G(u, v) = F(u, v) - (u^2 + v^2)F(u, v)
\]

new operator $H_2(u, v) = 1 + (u^2 + v^2) - H_1(u, v)$

Laplacian
An image formation model

- We can view an image \( f(x,y) \) as a product of two components:
  \[
  f(x, y) = i(x, y) \cdot r(x, y)
  \]
- \( i(x,y) \): illumination. It is determined by the illumination source.
- \( r(x,y) \): reflectance (or transmissivity). It is determined by the characteristics of imaged objects.

Unsharp Masking, High-boost Filtering

- Unsharp masking: \( f_{hp}(x,y) = f(x,y) - f_{lp}(x,y) \)
- \( H_{hp}(u,v) = 1 - H_{lp}(u,v) \)
- High-boost filtering: \( f_{hb}(x,y) = Af(x,y) - f_{lp}(x,y) \)
- \( f_{hb}(x,y) = (A-1)f(x,y) + f_{hp}(x,y) \)
- \( H_{hb}(u,v) = (A-1) + H_{hp}(u,v) \)

Homomorphic Filtering

- In some images, the quality of the image has reduced because of non-uniform illumination.
- Homomorphic filtering can be used to perform illumination correction.
  \[
  f(x, y) = i(x, y) \cdot r(x, y)
  \]
- The above equation cannot be used directly in order to operate separately on the frequency components of illumination and reflectance.

Homomorphic Filtering

\[
\begin{align*}
\text{In:} & & z(x,y) &= \ln f(x,y) = \ln i(x,y) + \ln r(x,y) \\
\text{DFT:} & & Z(u,v) &= F_i(u,v) + F_r(u,v) \\
\text{H}(u,v) : & & S(u,v) &= H(u,v)Z(u,v) \\
\text{(DFT)}^t : & & s(x,y) &= \hat{i}(x,y) + r'(x,y) \\
\text{exp:} & & g(x,y) &= e^{s(x,y)} = \hat{i}(x,y)\hat{r}(x,y)
\end{align*}
\]
Homomorphic Filtering

- By separating the illumination and reflectance components, homomorphic filter can then operate on them separately.
- Illumination component of an image generally has slow variations, while the reflectance component vary abruptly.
- By removing the low frequencies (highpass filtering) the effects of illumination can be removed.

Homomorphic Filtering: Example 1

Homomorphic Filtering: Example 2

Original image

Filtered image

End of Lecture