CoE 3SK3 Computer Aided Engineering Tutorial: Unconstrained Optimization

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Outline

What is unconstrained and constrained optimization problems?

Unconstrained optimization

- $\min_x F(x)$ or $\max_x F(x)$

Constrained Optimization

- $\max_x F(x)$ or $\max_x F(x)$
- subject to $g(x) = 0$
- and/or $h(x) < 0$ or $h(x) > 0$

Ways to get there

One-dimensional Unconstrained Optimization

- Analytical method
- Golden-section search method
- Newton's method

Multi-dimensional Unconstrained Optimization

- Analytical method
- Gradient method - steepest ascent (descent) method
- Newton's method

Analytical method

Question 13.2 (5th Edition)
Given
$f(x) = -1.5x^6 - 2x^4 + 12x$

1. Plot the function.
2. Use analytical methods to prove that the function is concave for all values of x.
3. Differentiate the function and then use a root-location method to solve for the maximum $f(x)$ and the corresponding value of x.

Golden-section search method

Question 13.3 (5th Edition)
Solve for the value of x that maximize $f(x)$ in Prob. 13.2 using the golden-section search.
Employ initial guesses of $x_l = 0$ and $x_u = 2$ and perform three iterations.

Golden Ratio in Art and Architecture

Golden Ratio in Art

- ***An Old man*** by ***Leonardo Da Vinci***
- ***The Vetruvian Man*** by ***Leonardo Da Vinci***
- ***Mona-Risa*** by ***Leonardo Da Vinci***
- ***Holy Family*** by ***Micahelangelo***
- ***Crucifixion*** by ***Raphael***
- ***self-portrait*** by ***Rembrandt***
- ...

Golden Ratio in Architecture

- **The Great Pyramid**
- **Parthenon**
- **Porch of Maidens, Acropolis, Athens**
- **Chartres Cathedral**
- ....

Newton's method

Question 13.5 (5th Edition)
Repeat Prob.13.3 but use Newton's method. Employ an initial guess of $x_0 = 2$ and perform three iterations.

Let's have a try

Question 13.11 (5th Edition)
Consider the following function:
$f(x) = 3 + 6x + 5x^2 + 3x^3 + 4^4$
Locate the minimum by finding the root of the derivative of this function. Use bisection with initial guess of $x_l = -2$ and $x_u = 1$

What you need?

1. Gradient vector
2. Directional derivative.
3. Hessian matrix

Directional derivative.

Question 14.2 (5th Edition)
Find the directional derivative of
$f(x, y) = x^2 + 2y^2$
at $x = 2$ and $y = 2$ in the direction of $h = 2i + 3j$

Gradient vector and Hessian matrix.

Question 14.3 (5th Edition)
Find the gradient vector and Hessian matrix for each of the following functions:

1. $f(x,y) = 3xy^2 + 2e^{xy}$
2. $f(x,y,z) = 2x^2 + y^2 + z^2$
3. $f(x,y) = ln(x^2 + 3xy + 2y^2)$

One-dimensional Unconstrained Optimization

- Analytical method
- Golden-section search method
- Newton's method

Multi-dimensional Unconstrained Optimization

- Some basic Concepts
- Analytical method
- Gradient method - steepest ascent (descent) method
- Newton's method

Thank you