Chapter 6 (answers)

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6-1-1

(a) $\frac{1}{s}(1-e^{-s})$ (b) $\frac{1}{(s+1)^2}$ (c) $\frac{s^2-\omega_0^2}{(s^2+\omega_0^2)^2}$

(d) $\frac{1}{s-2}-\frac{2}{s+1}$ for $\sigma_0 > 2$

(e) $\frac{1}{2}\left[\frac{s}{s^2+(\omega_1+\omega_2)^2}+\frac{s}{s^2+(\omega_1-\omega_2)^2}\right]$. (f) $\frac{s}{s^2-a^2}$ $Re(s) > |a|$

(g) $\frac{a}{s^2-a^2}$ $Re(s) > |a|$ (h) $\frac{(s+2)\cos\theta - 5\sin\theta}{s^2+4s+29}$

6.1-2

(a) $\frac{1}{s^2}(1-e^{-s}-se^{-s})$ (b) $\frac{1+e^{-\pi s}}{s^2+1}$

(c) $\frac{1}{es^2}(1-e^{-s}-se^{-s})+\frac{1}{s+1}e^{-(s+1)}$

6.1-3

(a) $(e^{-2t}+e^{-3t})u(t)$ (b) $3.018e^{-2t}\cos(3t+6.34^\circ)u(t)$

(c) $\delta(t)+(3.2e^{3t}-0.2e^{-2t})u(t)$ (d) $1.25(-1+2t+e^{-2t})u(t)$.

(e) $[-e^{-t}+\sqrt{5}e^{-t}\cos(t-63.4^\circ)]u(t)$ (f) $[2-(2+t)e^{-t}]u(t)$

(g) $\left[e^{-t}-\left(1+t+\frac{t^2}{2}+\frac{t^3}{6}\right)e^{-2t}\right]u(t)$ (i) $\left[\frac{1}{4}(3-t)+1.3975\cos(2t+70^\circ)\right]e^{-t}u(t)$

(h) $\left[\frac{1}{20}-\frac{1}{4}(1-2t)e^{-2t}+\frac{\sqrt{10}}{5}e^{-2t}\cos(t+71.56^\circ)\right]u(t)$

6.2-1

(a) $\frac{1}{s}(1-e^{-s})$ (b) $\frac{1}{s+1}e^{-s\tau}$ (c) $e^{\tau}\frac{1}{s+1}$

(d) $\left(\frac{1}{s+1}\right)e^{-(s+1)\tau}$ (e) $\frac{e^{-(s+1)\tau}[1+\tau(s+1)]}{(s+1)^2}$ (f) $\left(\frac{\omega_0}{s^2+\omega_0^2}\right)e^{-s\tau}$

(g) $\frac{\omega_0\cos\omega_0\tau - s\sin\omega_0\tau}{s^2+\omega_0^2}$ (h) $\left[\cos\omega_0\tau\left(\frac{\omega_0}{s^2+\omega_0^2}\right)+\sin\omega_0\tau\left(\frac{s}{s^2+\omega_0^2}\right)\right]e^{-s\tau}$

6.2-2 (a) $\frac{1}{s^2} - \frac{1}{s^2}e^{-s} - \frac{1}{s}e^{-s}$ (b) $\frac{1}{s^2+1}(1+e^{-\pi s})$

(c) $\frac{1}{s^2}(1-e^{-s}-se^{-s}) + \frac{e^{-s}}{e(s+1)}$

6.2-3 (a) $[e^{-2(t-2)} + e^{-3(t-2)}]u(t-2)$

(b) $\sqrt{2}\; e^{-(t-3)}\cos(t-3+\frac{\pi}{4})u(t-3) + 2e^{-t}\sin t\, u(t)$

(c) $\frac{e}{2}\, e^{(t-1)}\sin 2(t-1)u(t-1) + \frac{3}{2}e^{t}\sin 2t\, u(t)$

(d) $[e^{-(t-1)} - e^{-2(t-1)}]u(t-1) + [e^{-(t-2)} - e^{-2(t-2)}]u(t-2) + (e^{-t} - e^{-2t})u(t)$

6.2-5

pair 2: $u(t) \longleftrightarrow \frac{1}{s}$ pair 3: $tu(t) \longleftrightarrow \frac{1}{s^2}$

" 4: Use successive integration of $tu(t)$ " 5: $e^{\lambda t}u(t) \longleftrightarrow \frac{1}{s-\lambda}$

" 6: $te^{\lambda t}u(t) \longleftrightarrow \frac{1}{(s-\lambda)^2}$ " 7: Apply the same arguments to $t^2u(t)$ $t^3u(t)$ ...

" 8a: $\cos bt\, u(t) \longleftrightarrow \frac{s}{s^2+b^2}$ " 9a: $e^{-at}\cos bt\, u(t) \longleftrightarrow \frac{s+a}{(s+a)^2+b^2}$

" 10a and 10b: Recognize that

$$re^{-at}\cos(bt+\theta) = re^{-at}[\cos\theta\cos bt - \sin\theta\sin bt]$$

Now use results in pairs 9a and 9 to obtain pair 10a.
Pair 10b is equivalent to pair 10a.

6.2-6 (a) (i) $\frac{1}{s}(1-e^{-2s})$ (ii) $\frac{1}{s}(e^{-2s} - e^{-4s})$

(b) $\frac{1}{s^2}(1-3e^{-2s}+2e^{-3s})$

6.3-1.

(a) $(e^{-t} - e^{-2t})u(t)$ (b) $(2+6t)e^{-2t}u(t)$

(c) $(2 + 5.836e^{-3t}\cos(4t - 99.86°)]u(t)$

6.3-2

(a) $y_{zs}(t) = (e^{-t} - e^{-2t})\,u(t)$ ← zero-state

$y_{zi}(t) = 0$ ← zero-input.

(b) $y(t) = \underbrace{(2+5t)e^{-2t}}_{\text{zero-input}} + \underbrace{te^{-2t}}_{\text{zero-state}}$.

(c) $y(t) = \underbrace{\left[\sqrt{2}\,e^{-3t}\cos\left(4t - \frac{\pi}{4}\right)\right]}_{\text{zero-input}} + \underbrace{\left[2 + 5.154\,e^{-3t}\cos(4t - 112.83^\circ)\right]}_{\text{zero-state}}$

6.3-3

(a) $y_1(t) = \left(\frac{1}{2} - \frac{1}{3}e^{-t} - \frac{1}{6}e^{-4t}\right)u(t) \quad : \quad H_1(s) = \dfrac{s+2}{s^2+5s+4}$

$y_2(t) = \left(\frac{1}{4} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{-4t}\right)u(t) \quad : \quad H_2(s) = \dfrac{1}{s^2+5s+4}$

($H_1(s)$ and $H_2(s)$ are the transfer functions relating $y_1(t)$ and $y_2(t)$ respectively to the input $f(t)$).

(b) $H_1(s) = \dfrac{s+1}{s^2+3s+1}$ and $H_2(s) = \dfrac{s+2}{s^2+3s+1}$

$y_1(t) = (1 - 0.724\,e^{-0.382t} - 0.276\,e^{-2.618t})\,u(t)$

$y_2(t) = (2 - 1.894\,e^{-0.382t} - 0.1056\,e^{-2.618t})\,u(t)$

6.3-4

$Y_1(s) = \dfrac{8}{s} + \dfrac{16s+28}{s^2+3s+2.5} \longleftrightarrow y_1(t) = \left[8 + 17.89\,e^{-1.5t}\cos\left(\frac{t}{2} - 26.56^\circ\right)\right]u(t)$

$Y_2(s) = \dfrac{20(s+2)}{(s^2+3s+2.5)} \longleftrightarrow y_2(t) = 20\sqrt{2}\,e^{-1.5t}\cos\left(\frac{t}{2} - \frac{\pi}{4}\right)u(t)$.

6.3-5 (a) $\dfrac{5s+3}{s^2+11s+24}$ (b) $\dfrac{3s^2+7s+5}{s^3+6s^2-11s+6}$ (c) $\dfrac{3s+2}{s(s^3+2)}$

6.3-6 (a) $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 8y(t) = \dfrac{df}{dt} + 5f(t)$

(b) $\dfrac{d^3y}{dt^3} + 8\dfrac{d^2y}{dt^2} + 5\dfrac{dy}{dt} + 7y(t) = \dfrac{d^2f}{dt^2} + 3\dfrac{df}{dt} + 5f(t)$

(c) $\dfrac{d^2y}{dt^2} - 2\dfrac{dy}{dt} + 5y(t) = 5\dfrac{d^2f}{dt^2} + 7\dfrac{df}{dt} + 2f(t).$

$\blacksquare$

6.3-7.

(a) (i) $y(t) = (3e^{-2t} - 3e^{-3t} - 2te^{-3t})u(t)$

(ii) $y(t) = (\frac{3}{2}e^{-2t} - 2e^{-3t} + \frac{1}{2}e^{-4t})u(t)$

(iii) $y(t) = [\frac{3}{2}e^{-2(t-5)} - 2e^{-3(t-5)} + \frac{1}{2}e^{-4(t-5)}]u(t-5)$

(iv) $y(t) = e^{20}(\frac{3}{2}e^{-2t} - 2e^{-3t} + \frac{1}{2}e^{-4t})u(t)$

(v) $y(t) = e^{-20}(\frac{3}{2}e^{-2(t-5)} - 2e^{-3(t-5)} + \frac{1}{2}e^{-4(t-5)})u(t-5)$

(b) $(D^2+2D+5)y(t) = (2D+3)f(t)$

$\blacksquare$

6.3-8

(a) $y(t) = [6 + 9.22e^{-t}\cos(2t - 130.6^\circ)]u(t)$

(b) $y(t) = \frac{1}{10}\{6 + 9.22e^{-(t-5)}\cos[2(t-5) - 130.6^\circ]\}u(t-5)$

$\blacksquare$

6.3-9

$F(S) = \dfrac{1}{S(S+1)}$; $Y(S) = \dfrac{0.1}{S+1} - \dfrac{0.1(S-1)}{S^2-9}$

$y(t) = \left(0.1e^{-t} - \dfrac{1}{3\sqrt{10}}\cos[3t + \tan^{-1}(\frac{1}{3})]\right)u(t)$

$\blacksquare$

6.4-1 $Y_2(S) = \dfrac{1}{(S+1)^2} - \dfrac{1}{(S^2+2S+2)}$; $y_2(t) = (te^{-t} - \frac{1}{2}e^{-t}\sin t)u(t) = v_o(t)$

$\blacksquare$

6.4-2 $Y(S) = \frac{5}{3}\left[\frac{3}{S} - \frac{2}{S+(2/3)}\right]$; $y(t) = (5 - \frac{10}{3}e^{-2t/3})u(t)$

$\blacksquare$

6.4-3

The impedance seen by the source $f(t)$ is: $Z(s) = \dfrac{Ls\omega_o^2}{s^2+\omega_o^2}$, and the current $Y(s)$ is given by: $Y(s) = \dfrac{F(s)}{Y(s)} = \dfrac{s^2+\omega_o^2}{Ls\omega_o^2} F(s)$

(a) $F(s) = \dfrac{As}{s^2+\omega_o^2}$; $Y(s) = \dfrac{A}{L\omega_o^2}$ and $y(t) = \dfrac{A}{L\omega_o^2}\delta(t)$

(b) $F(s) = \dfrac{A\omega_o}{s^2+\omega_o^2}$; $Y(s) = \dfrac{A}{L\omega_o s}$ and $y(t) = \dfrac{A}{L\omega_o}u(t)$

6.4-4

$$Y_1(s) = \frac{4}{s} - \frac{3/2}{s+1} - \frac{1/2}{s+3} \longleftrightarrow y_1(t) = (4 - \tfrac{3}{2}e^{-t} - \tfrac{1}{2}e^{-3t})u(t)$$

$$Y_2(s) = \frac{2}{s} - \frac{3/2}{s+1} + \frac{1/2}{s+3} \longleftrightarrow y_2(t) = (2 - \tfrac{3}{2}e^{-t} + \tfrac{1}{2}e^{-3t})u(t)$$

6.4-5

$$Y_1(s) = \frac{20}{s}\left(\frac{s+0.5}{s^2+\frac{1}{3s}+\frac{1}{3}}\right) \longleftrightarrow y_1(t) = 7.787e^{-t/6}\cos\left(\frac{\sqrt{11}}{6}t - 31.1^\circ\right)u(t)$$

$$V_s(s) = \frac{20}{3}\left(1 + \frac{3/2}{s} + \frac{1}{6}\,\frac{-8s+1}{s^2+1/3 s+1/3}\right) \longleftrightarrow v_s(t) = \frac{20}{3}\delta(t) + \left[10 + 9.045e^{-t/6}\cos\left(\frac{\sqrt{11}}{6}t - 152.2\right)\right]u(t)$$

6.4-6

$$Y_2(s) = \frac{40}{(s+0.2)} \longleftrightarrow v_o(t) = y_2(t) = 40e^{-t/5}u(t)$$

6.4-7

$$V_o(s) = \frac{3s+1}{2(s^2+4s+13)} \longleftrightarrow v_o(t) = 1.716e^{-2t}\cos(3t+29^\circ)u(t)$$

6.4-8

Impedance across terminals ab: $Z_{ab}(s) = \dfrac{3s+8}{4s+11}$

$$Y(s) = -\frac{1.61}{s+2.8} + \frac{121.61}{s+6.53} \longleftrightarrow y(t) = [121.61e^{-6.53t} - 1.61e^{-2.8t}]u(t)$$

6.4-11

(a) $Y(s) = \dfrac{6s^2+3s+10}{s(2s^2+6s+5)}$; $y(0^+) = \lim_{s\to\infty} sY(s) = 3$ (5) ; $y(\infty) = \lim_{s\to 0} sY(s) = 2$

(b) $Y(s) = \dfrac{6s^2+3s+10}{(s+1)(2s^2+6s+5)}$; $y(0^+) = \lim_{s\to\infty} sY(s) = 3$; $y(\infty) = \lim_{s\to 0} sY(s) = 0$

6.5-1 (a) $H(s) = \frac{1}{5}$ not $\frac{1}{4}$.

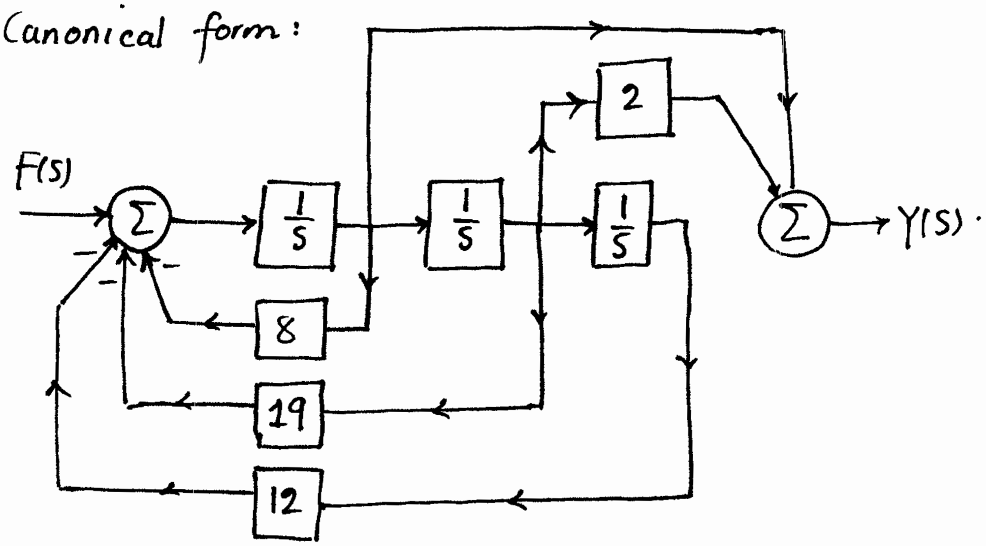
(b) $Y(s) = \dfrac{1}{3.99} F(s) \rightarrow H(s) = \dfrac{1}{3.99}$

6.5-2 For cascade combination: $h(t) = h_1(t) * h_2(t)$

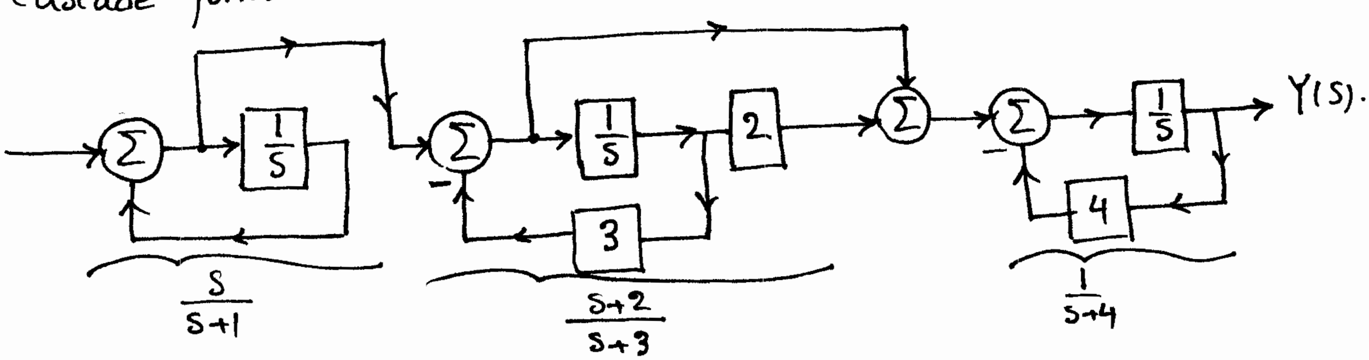
" parallel " : $h(t) = h_1(t) + h_2(t)$

6.6-1. $H(s) = \dfrac{s^2+2s}{s^3+8s^2+19s+12} = \left(\dfrac{s}{s+1}\right)\left(\dfrac{s+2}{s+3}\right)\left(\dfrac{1}{s+4}\right) = \dfrac{-1/6}{s+1} - \dfrac{3/2}{s+3} + \dfrac{8/3}{s+4}$

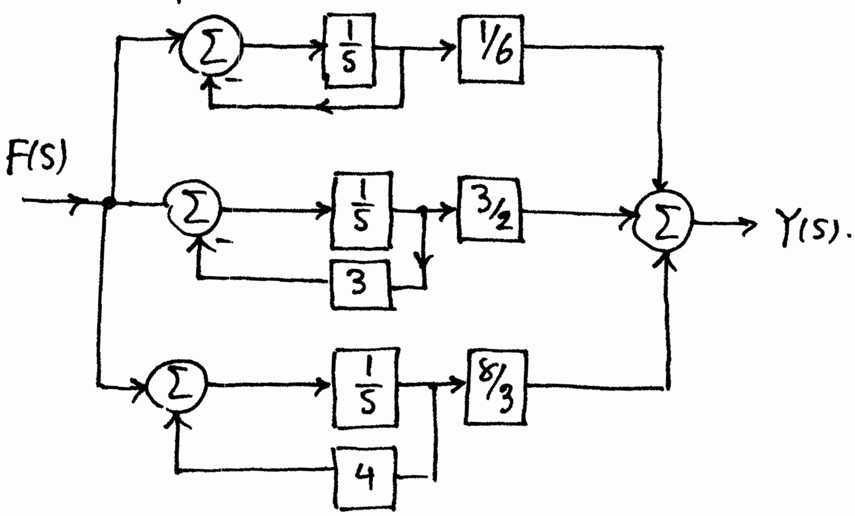
Canonical form:



Cascade form:



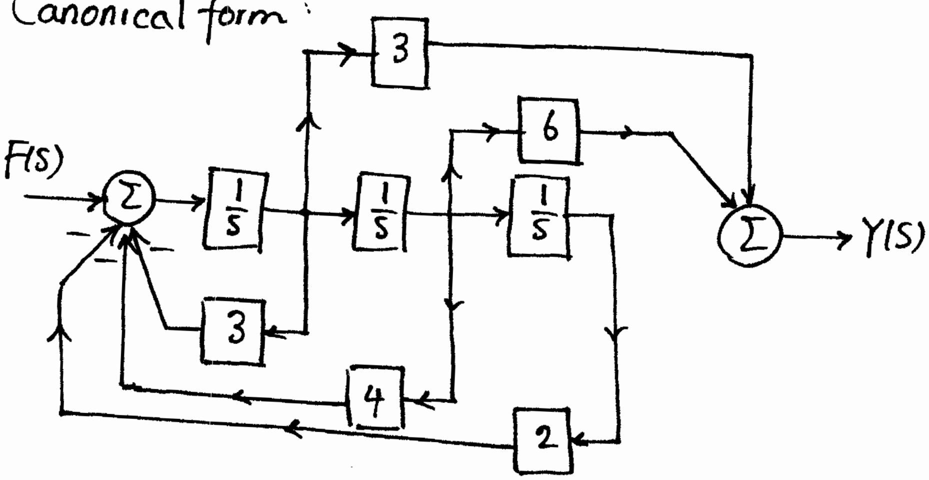
Parallel form:



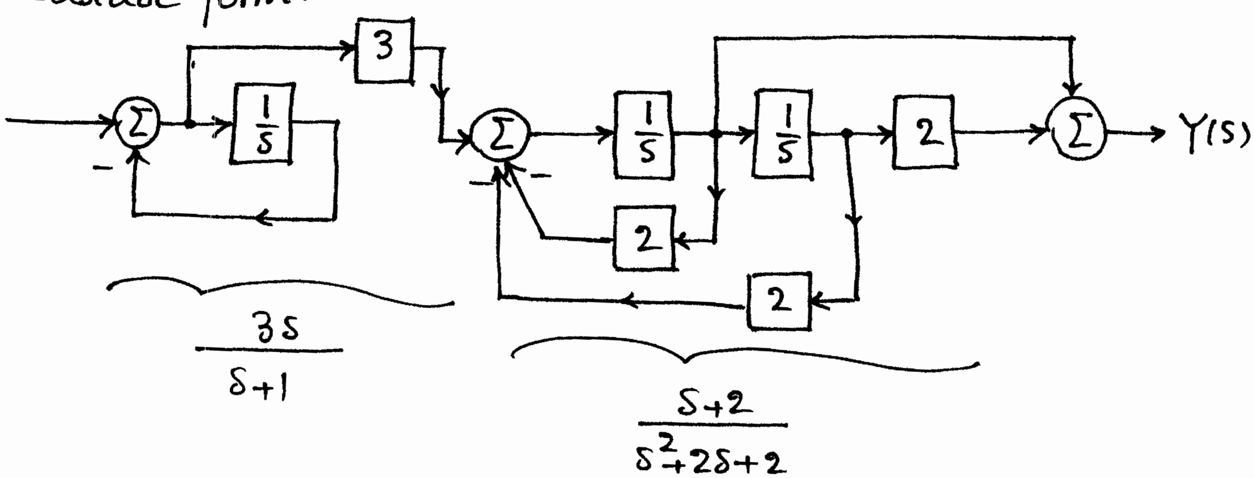
6.6.2

(a) $H(s)=\dfrac{3s(s+2)}{(s+1)(s^2+2s+2)}=\dfrac{3s^2+6s}{s^3+3s^2+4s+2}=\left(\dfrac{3s}{s+1}\right)\left(\dfrac{s+2}{s^2+2s+2}\right)=-\dfrac{3}{s+1}+\dfrac{6s+6}{s^2+2s+2}$

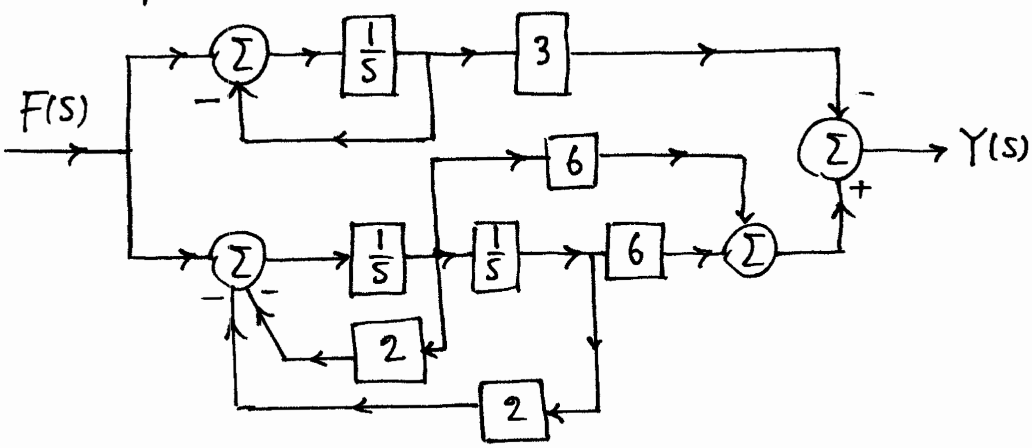
Canonical form:



Cascade form:

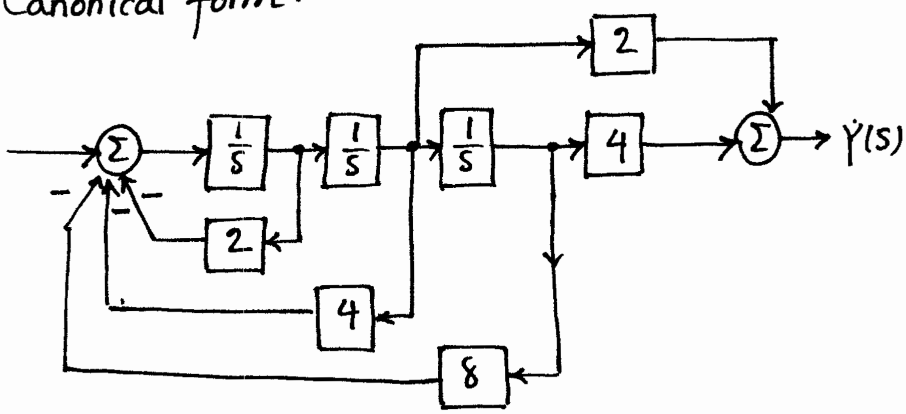


Parallel form:

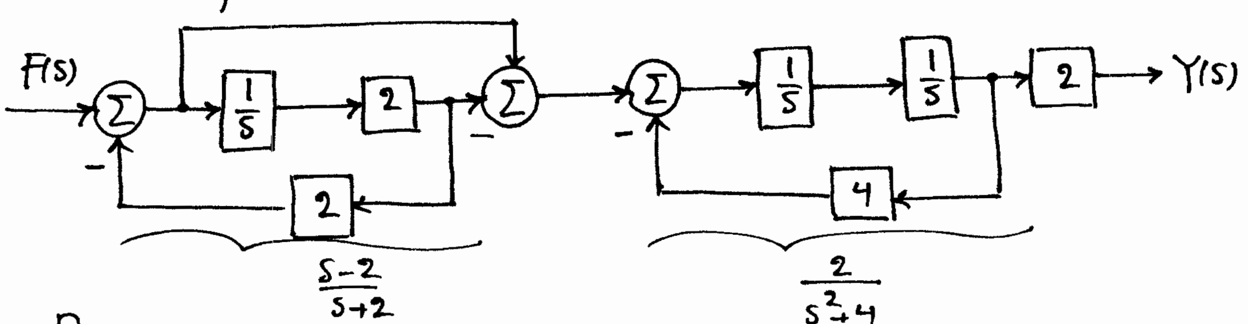


(b) $H(S) = \dfrac{2S-4}{(S+2)(S^2+4)} = \dfrac{2(S-2)}{S^3+2S^2+4S+8} = \left(\dfrac{S-2}{S+2}\right)\left(\dfrac{2}{S^2+4}\right) = -\dfrac{1}{S+2} + \dfrac{S}{S^2+4}$

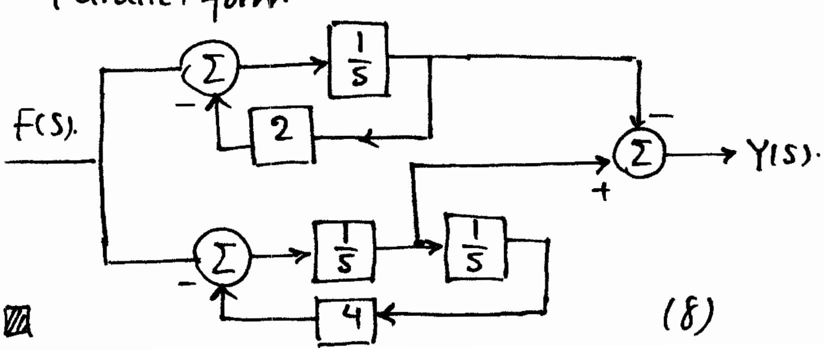
Canonical form:



Cascade form:



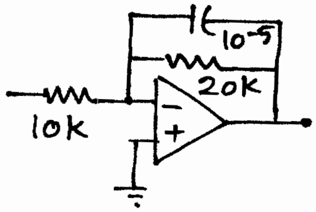
Parallel form



6.6-8

These transfer functions are readily realized by using the arrangement in Fig. 6.30 by a proper choice of $Z_f(s)$ and $Z(s)$.

(i)



$$Z_f(S) = \frac{\frac{R_f}{SC_f}}{R_f + \frac{1}{SC_f}} = \frac{1}{C_f(S+a)} \qquad a = \frac{1}{R_fC_f}$$

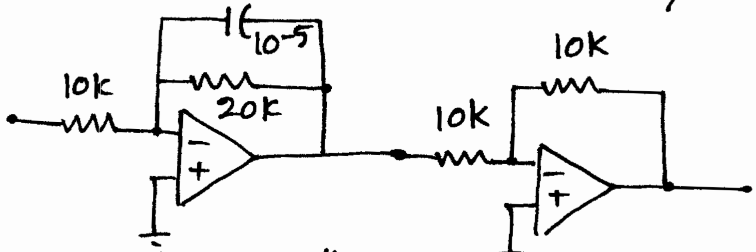
$$Z(S) = R$$

$$H(S) = -\frac{Z_f(S)}{Z(S)} = -\frac{K}{S+a} \quad ; \; K = \frac{1}{RC_f}$$

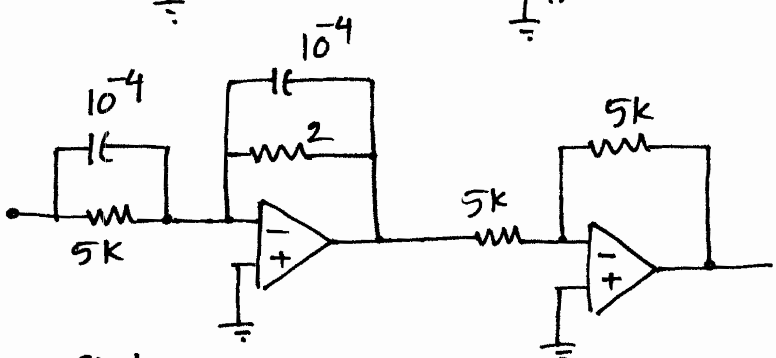
choose $R = 10{,}000$, $R_f = 20{,}000$ and $C_f = 10^{-5}$. This yeilds $K=10$ and $a=5$. Hence:

$$H(S) = \frac{-10}{S+5}.$$

(ii) This is the same as (i), followed by an amplifier of gain -1.



(iii)



For the first stage:

$$Z_f(S) = \frac{1}{C_f(S+a)} \quad ; \; a = \frac{1}{R_fC_f} \quad \text{and} \quad Z(S) = \frac{1}{C(S+b)} \quad ; \; b = \frac{1}{RC}$$

$$H(S) = -\frac{Z_f(S)}{Z(S)} = -\frac{C}{C_f}\left(\frac{S+b}{S+a}\right)$$

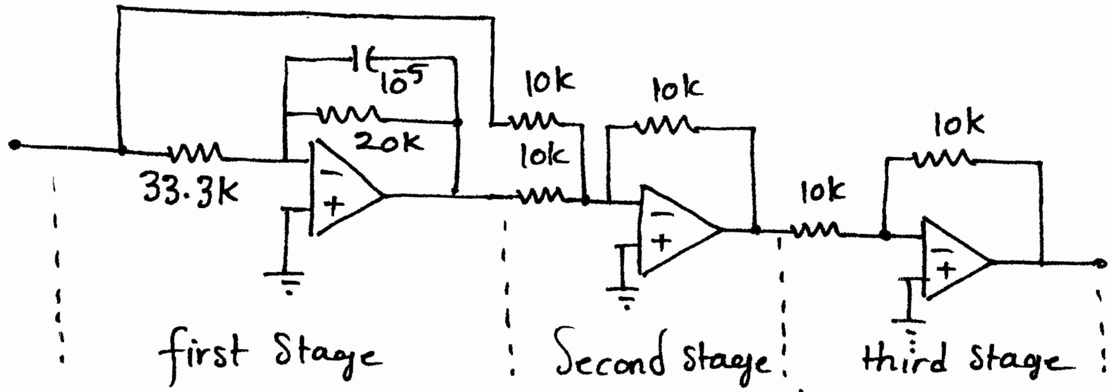
Choose $C = C_f = 10^{-4}$, $R = 5000$, $R_f = 2000$. This yeilds $H(S) = -\left(\frac{S+2}{S+5}\right)$

This is followed by an op-amp of gain -1, which yeilds:

$$H(S) = \frac{S+2}{S+5}$$

6.6-9 One realization is given in part (iii) of problem 6.6-8. For the other realization we express $H(S)$ as:

$$H(S) = \frac{S+2}{S+5} = 1 - \frac{3}{S+5}$$



The second stage serves as a summer for which the inputs are the input and output of the first stage. because the summer has a gain of -1, we need a third stage of gain -1 to obtain the desired transfer functions.

6.6-10 The transfer function here is identical to $H(S)$ in example 6.20 with a minor difference. Hence the op-amp circuit in Figure 6.32c can be used for our purpose with appropriate changes in the element values. the last summer input resistors now are $100/3$ kΩ and $100/7$ kΩ instead of 50 kΩ and 20 kΩ.

6.7-1

(a) $t_r = 0.526$; $t_s = 2.67$; $PO \approx 17\%$; $e_r = \frac{1}{3}$; $e_p = \infty$

(b) $t_r = 1.15$; $t_s = 2.67$; $PO \approx 3\%$; $e_r = 0.75$; $e_p = \infty$

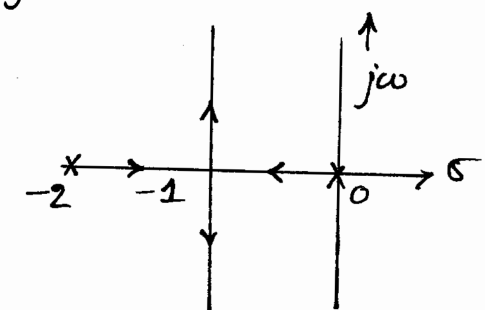
(c) $t_r = 0.163$; $t_s = 0.8$; PO 17% ; $e_r = \infty$; $e_p = \infty$

6.7-2 $K_1 = 2$; $K_2 = 25.4$; $a = 6.128$

6.7-3) The transfer function of the inner loop is $\frac{1}{S+2}$. Hence the loop transfer function of this unity feedback system is:

$$G(S) = \frac{K}{S(S+2)} \qquad T(S) = \frac{\frac{K}{S(S+2)}}{1+\frac{K}{S(S+2)}} = \frac{K}{S^2+2S+K}$$

The characteristic roots are $-1 \pm j\sqrt{K^2-1}$. The root locus is shown in the following figure:



Observe that for the characteristic polynomial S^2+2S+K, $\zeta\omega_n = 1$ and $\omega_n^2 = K$, but $t_s = 4/\zeta\omega_n = 4$. Hence we can not meet the settling time specification ($t_s \le 1$), regadless of the value of K. We now find the steady-state errors. This being a unity feedback system, we could use parameters K_p, K_v, and K_a. We have:

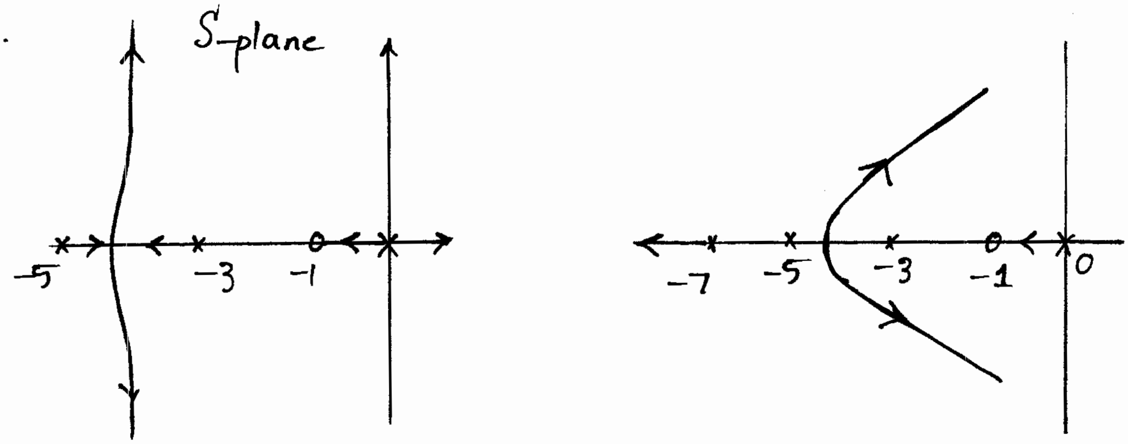
$$K_p = \lim_{S\to 0} G(S) = \infty \qquad K_v = \lim_{S\to 0}[SG(S)] = \frac{K}{2} \qquad K_a = \lim_{S\to 0}[S^2G(S)] = 0$$

Hence

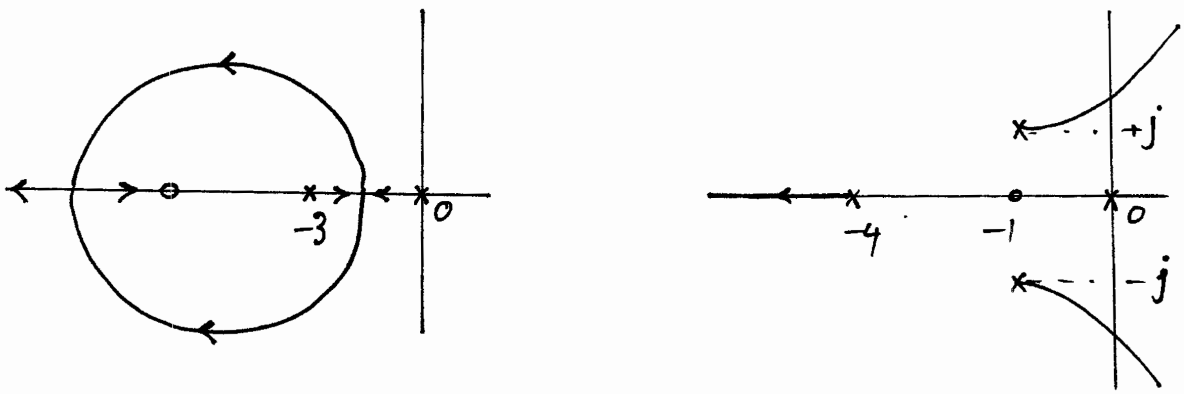
$$e_s = \frac{1}{1+K_p} = 0 \qquad e_r = \frac{1}{K_v} = \frac{2}{K} \qquad e_p = \frac{1}{K_a} = \infty$$

We already observed that we can not meet t_s specification. We can satisfy e_s. From Figure 6.40 we conclude that we can not meet both $t_r \le 0.3$ and $PO \le 30\%$. We can meet one or the other, but not both.

6.7-4.



(a). (b).



6.7-5 Please refer to detailed solutions coming next.

6.8-1. (a) No region of convergence

(b) The strip of convergence is: $-1<\sigma<0$

(c) The convergence occurs at $\sigma=0$ ($j\omega$-axis).

(d) The region of convergence is: $-1<\sigma<0$

(e) The region of convergence is the entire s-plane.

6.8-2

(a) $F(s)=\dfrac{-2}{s^2-1}$ $\quad -1<\sigma<+1$

(b) $F(s)=\dfrac{4-2s^2}{s^4-4}$ $\quad -1<\sigma<+1$.

(c) $F(s)=\dfrac{-1}{(s-1)(s-2)}$ $\quad 1<\sigma<2$

(d) $F(s)=\dfrac{-1}{s(s+1)}$ $\quad -1<\sigma<0$

(e) $F(s)=\dfrac{-1}{s(s-1)}$ $\quad 0<\sigma<1$

(f) $F(s)=\dfrac{-(s+\omega_0^2)}{(s-1)(s^2+\omega_0^2)}$ $\quad 0<\sigma<1$

6.8-3 (a) $f(t)=e^{-3t}u(t)-e^{-2t}u(-t)$

(b) $f(t)=e^{2t}u(t)-e^{3t}u(t)$

(c) $f(t)=(e^{-t}+e^{-2t})u(t)$

(d) $f(t)=-(e^{-t}+e^{-2t})u(-t)$

(e) $f(t)=(e^{-t}+e^{-3t})u(t)-e^{5t}u(-t)$

6.8-4 (a) $f(t)=(e^{-t}-e^{t}+2e^{-2t})u(t)$

(b) $f(t)=(-e^{-t}+e^{t}-2e^{-2t})u(-t)$

(c) $f(t)=(e^{-t}+2e^{-2t})u(t)+e^{t}u(-t)$

(d) $f(t)=2e^{-2t}u(t)+[-e^{-t}+e^{t}]u(-t)$

6.8-5 (a) $y(t)=(-\frac{4}{3}e^{-t}+2e^{-t/2})u(t)+\frac{2}{3}e^{t/2}u(-t)$

(b) $y(t)=(-\frac{1}{6}e^{-t}+\frac{1}{2}e^{t})u(t)+\frac{1}{3}e^{2t}u(-t)$

(c) $y(t)=(-\frac{2}{3}e^{-t}+2e^{-t/2})u(t)+\frac{4}{3}e^{-t/4}u(-t)$

(d) $y(t) = \left(\frac{1}{6}e^{-t} + \frac{1}{3}e^{2t}\right)u(t) + \frac{1}{2}e^{t}u(-t)$

(e) $y(t) = \left(\frac{2}{3}e^{-t} + \frac{4}{3}e^{-t/4}\right)u(t) + 2e^{-t/2}u(-t)$

(f) $f(t) = e^{-3t}u(t) + e^{-2t}u(-t) = f_1(t) + f_2(t)$

$F(s) = F_1(s) + F_2(s)$

where $F_1(s) = \dfrac{1}{s+3} \qquad \sigma > -3$

$F_2(s) = \dfrac{-1}{s+2} \qquad \sigma < -2$

$H(s) = \dfrac{1}{s+1}$

In this case, there is a common region of convergence for $F_1(s)$ and $H(s)$, but there is no region of convergence common to $F_2(s)$ and $H(s)$. Hence the output $y_1(t)$ will be finite, but $y_2(t)$ will be ∞.