EE750
Advanced Engineering Electromagnetics
Lecture 5

Scattering and Connection (the General Lossy Case)

- Six links exist for the general lossy case
- We, however, do not care about the value of the reflected impulses on the loss stub (energy is just being absorbed)
- Also, no incident wave appear on the loss stub because it is matched
- It follows that the scattering matrix can be reduced in dimension by 1 ($\boldsymbol{S} \in R^{5\times 5}$)
- Following a similar approach to that used in the lossless case we derive the scattering relationship

Scattering and Connection (Cont'd)

$$\begin{bmatrix} V_1^r \\ V_2^r \\ V_3^r \\ V_4^r \\ V_5^r \end{bmatrix} = \frac{1}{y} \begin{bmatrix} 2-y & 2 & 2 & 2 & 2y_o \\ 2 & 2-y & 2 & 2 & 2y_o \\ 2 & 2 & 2-y & 2 & 2y_o \\ 2 & 2 & 2 & 2-y & 2y_o \\ 2 & 2 & 2 & 2 & 2y_o - y \end{bmatrix} \begin{bmatrix} V_1^i \\ V_2^i \\ V_3^i \\ V_4^i \\ V_5^i \end{bmatrix}$$

Where $y=4+y_o+g_o$, $y_o = 4.0(\varepsilon_r - 1)$, $g_o = \dfrac{\sigma \Delta l}{\sqrt{C/L}}$

Modeling of Boundaries

- In establishing the equivalence between Maxwell's equations and a network of TLM nodes we noted that node voltage models the electric field and that link currents model the magnetic field
- It follows that the boundary resistive load represents the wave impedance
- Lossless nondispersive boundaries include open and short circuit (magnetic and electric walls, respectively)

Modeling of Boundaries (Cont'd)

- The general expression of the link reflection coefficient due to a non dispersive load R_L is

$$\Gamma = \frac{R_L - \eta_o\sqrt{2}}{R_L + \eta_o\sqrt{2}}$$

- For a magnetic wall we have $V_{k+1}^i = V_k^r$, link reflection coefficient is 1, $\Gamma_m = \lim_{R_L \to \infty} \frac{R_L - \eta_o\sqrt{2}}{R_L + \eta_o\sqrt{2}} = 1.0$

- For an electric wall we have $V_{k+1}^i = -V_k^r$, link reflection coefficient is -1, $\Gamma_e = \lim_{R_L \to 0} \frac{R_L - \eta_o\sqrt{2}}{R_L + \eta_o\sqrt{2}} = -1.0$

Modeling of Boundaries (Cont'd)

- For a lossy boundary with surface resistance R_s the link reflection coefficient $\Gamma_s = \dfrac{R_s - \eta_o\sqrt{2}}{R_s + \eta_o\sqrt{2}}, \quad V_{k+1}^i = \Gamma_s V_k^r$
- For TEM waves propagating in free space, the wave impedance is η_o regardless of the wave frequency. It follows that a wideband Absorbing Boundary Condition (ABC) has an impulse reflection coefficient

$$\Gamma = \frac{\eta_o - \eta_o\sqrt{2}}{\eta_o + \eta_o\sqrt{2}} = -0.17157, \quad V_{k+1}^i = \Gamma\ V_k^r$$

- For TEM waves propagating in a dielectric with ε_r , wave impedance is $\eta_o/\sqrt{\varepsilon_r}$ regardless of the wave frequency. It follows that $\Gamma = \dfrac{\eta_o/\sqrt{\varepsilon_r} - \eta_o\sqrt{2}}{\eta_o/\sqrt{\varepsilon_r} + \eta_o\sqrt{2}}, \quad V_{k+1}^i = \Gamma\ V_k^r$

Modeling of Boundaries (Cont'd)

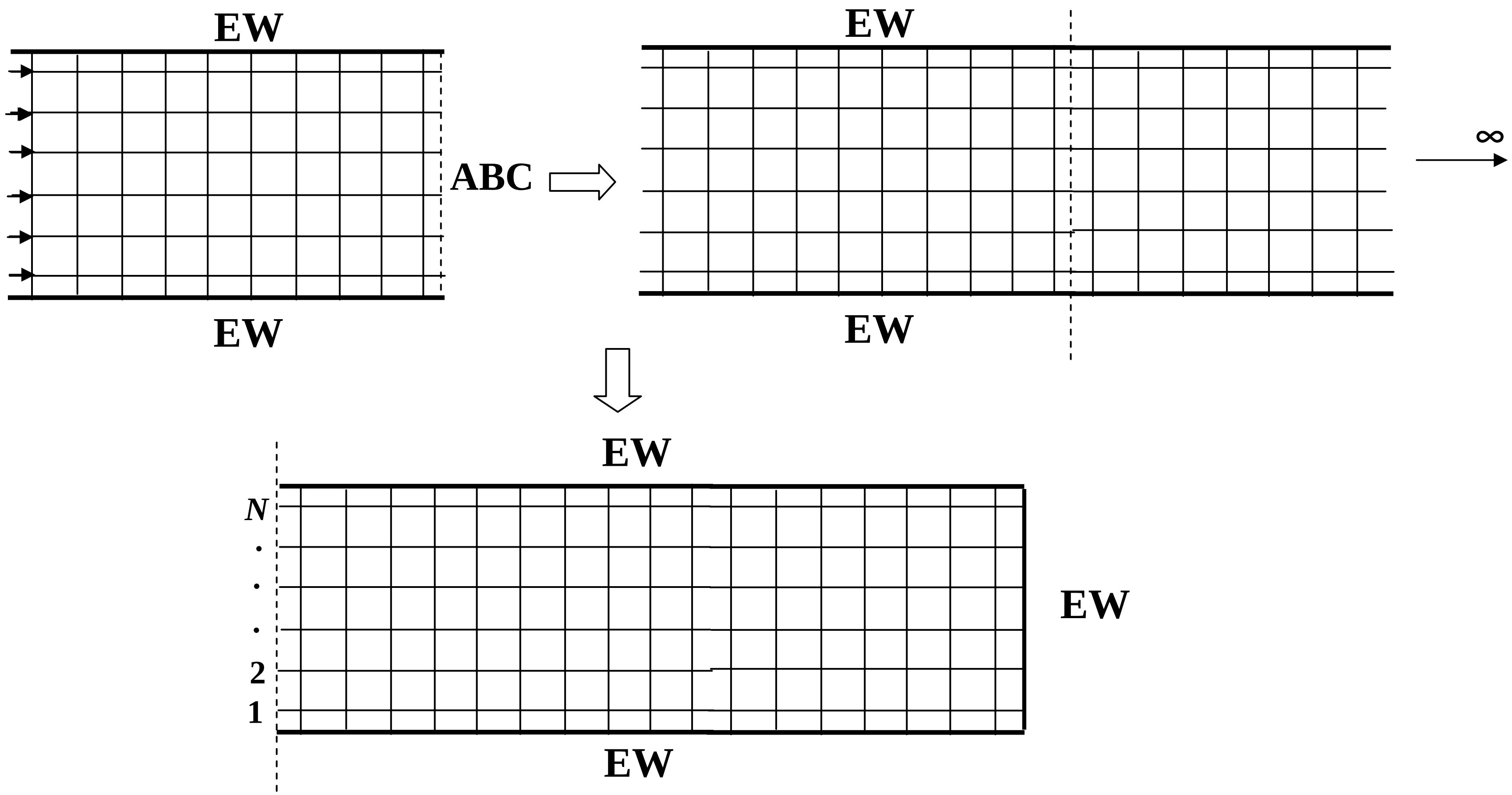
- For TEno modes in a rectangular waveguide, the wave impedance above cut-off is real but dispersive. It follows that an ABC using a real impulse reflection coefficient is feasible only at one frequency

$$\Gamma = \frac{\left(\eta_o / \sqrt{\varepsilon_r}\right)\dfrac{\lambda_g}{\lambda} - \eta_o \sqrt{2}}{\left(\eta_o / \sqrt{\varepsilon_r}\right)\dfrac{\lambda_g}{\lambda} + \eta_o \sqrt{2}}, \quad V_{k+1}^{i} = \Gamma \; V_k^{r}$$

λ_g is the guide wavelength and λ is the open medium

- A wideband ABC is obtained in this case using the John's matrix

Discrete Time-Domain Green's Function



Excite with an impulse at node *i* and register all impulses coming out at all links for all time steps

The Johns Matrix

- This matrix is also denoted as the Johns' matrix
- The Johns matrix is a three-dimensional matrix
- The *i*th row of this matrix is obtained by exciting an impulse at the *i*th node and registering all impulses coming out at all links for all time steps
- This is repeated for all links, so N TLM analyses are required
- Sequences of the form $g(m,n,k)$ are being generated. Here $g(m,n,k)$ is the reflected impulse at the *m*th node at the *k*th time step due to a unit incident impulse at the *n*th node at the 0^{th} time step

The Johns' Matrix (Cont'd)

- Using convolution summation we have

$$\left(V_m^r\right)_k = \sum_{n=1}^{N}\sum_{k'=0}^{k} g(m,n,k-k')\left(V_n^i\right)_{k'}$$

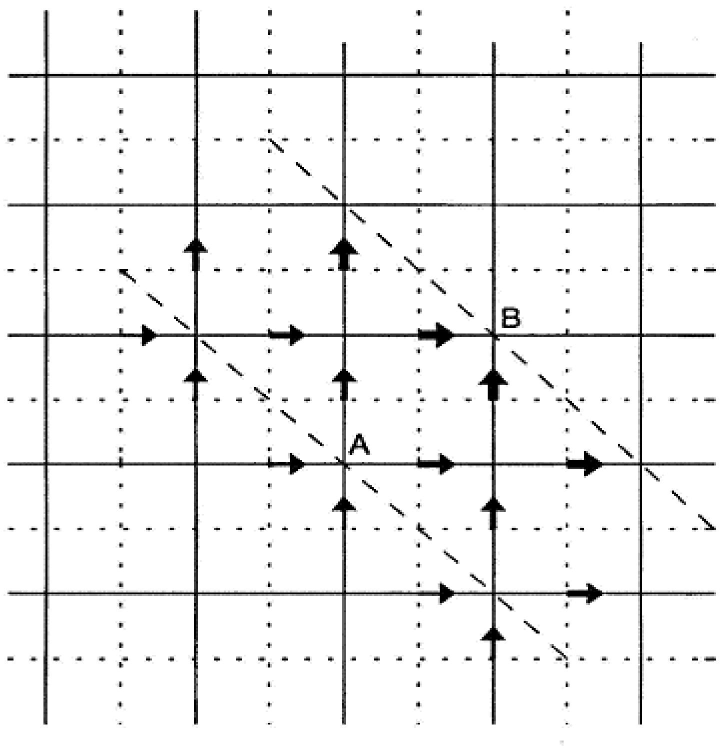
Alternatively $$\boldsymbol{G}(k) = \begin{bmatrix} g_{11}(k) & g_{12}(k) & \cdots & g_{1\boldsymbol{N}}(k) \\ g_{21}(k) & . & . & . \\ . & . & . & . \\ g_{\boldsymbol{N}1}(k) & . & . & g_{\boldsymbol{NN}}(k) \end{bmatrix}$$

$$\boldsymbol{V}^r(k) = \sum_{k'=0}^{k} \boldsymbol{G}(k-k')\boldsymbol{V}^i(k')$$

- Johns' Matrix is utilized in partioning of a large structure into small substructures and in time-domain modeling of wideband ABC in non-TEM waveguides

Dispersion in a 2D TLM Mesh

- We first study propagation at 45°



Christos, Transmission Line Modeling (TLM)

Dispersion in a 2D TLM Mesh (Cont'd)

- Exciting ports 1 and 2 by 1V results in $V_1^r = V_2^r = 0V$ and $V_3^r = V_4^r = 1V$. These reflected impulses travel to become incident on neighboring nodes at the next time step. This will give $V_1^r = V_2^r = 1V$ and $V_3^r = V_4^r = 0V$. It took 2 time steps to travel a distance of $\Delta l\sqrt{2}$

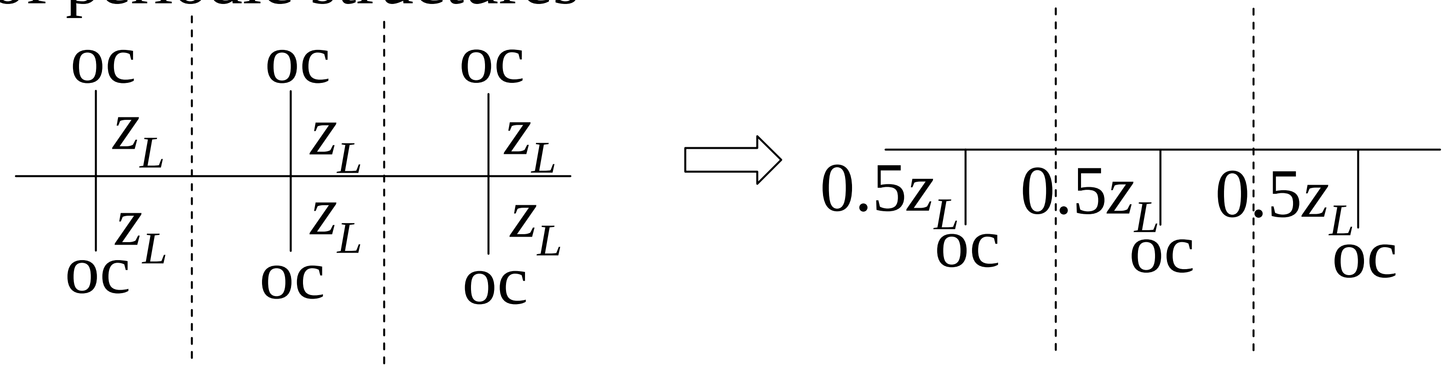
- The network velocity is

$$v_N = \frac{\Delta l\sqrt{2}}{2\Delta t} = \frac{v_L}{\sqrt{2}} = v_\text{o} \qquad \text{regardless of frequency}$$

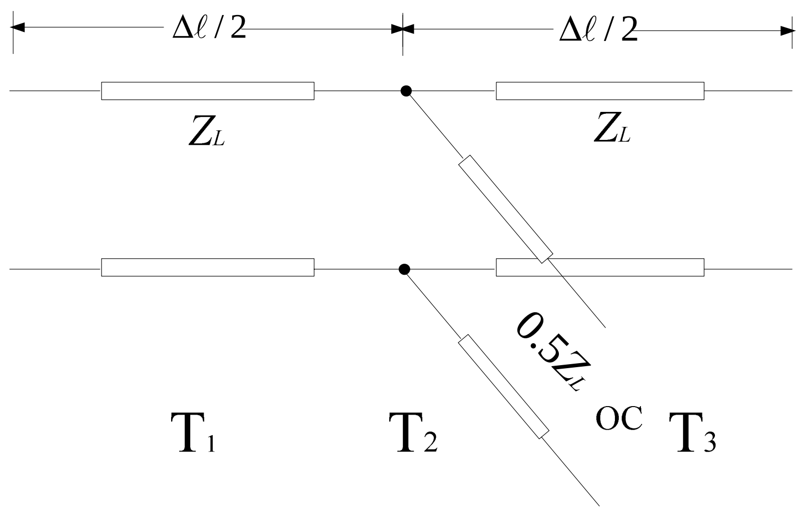
- It follows that no dispersion appears for this case

Dispersion in a 2D TLM Mesh (Cont'd)

- For propagation in the direction of one of the axis, symmetry allows us to represent the network by a cascade of periodic structures



$$[\boldsymbol{T}] = [\boldsymbol{T}_1][\boldsymbol{T}_2][\boldsymbol{T}_3]$$



Dispersion in a 2D TLM Mesh (Cont'd)

- It follows that we have

$$[\boldsymbol{T}] = \begin{bmatrix} \text{Cos}\,\theta & jZ_L\,\text{Sin}\,\theta \\ \dfrac{j\,\text{Sin}\,\theta}{Z_L} & \text{Cos}\,\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{2j\tan\theta}{Z_L} & 1 \end{bmatrix} \begin{bmatrix} \text{Cos}\,\theta & jZ_L\,\text{Sin}\,\theta \\ \dfrac{j\,\text{Sin}\,\theta}{Z_L} & \text{Cos}\,\theta \end{bmatrix}$$

$$\theta = \frac{\omega \Delta t}{2}$$

- Equating this product to the ABCD of a single section of transmission line

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \text{Cos}\,\beta\Delta l & jZ_L\,\text{Sin}\,\beta\Delta l \\ \dfrac{j\,\text{Sin}\,\beta\Delta l}{Z_L} & \text{Cos}\,\beta\Delta l \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

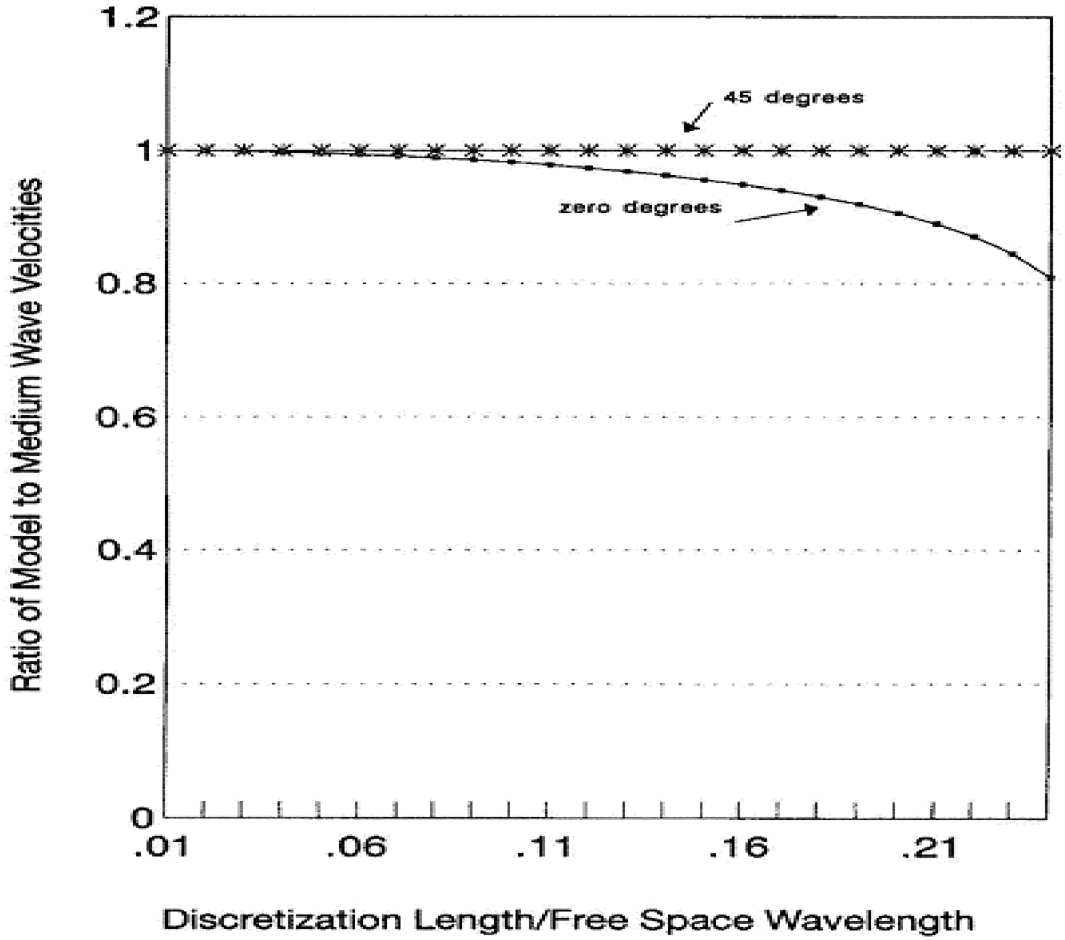
Dispersion in a 2D TLM Mesh (Cont'd)

- It follows that we have $\sin(\beta\Delta l/2) = \sqrt{2}\sin(\omega\Delta t/2)$
- But $\omega\frac{\Delta t}{2} = \frac{\omega}{2}\frac{\Delta l}{v_L} = \frac{2\pi f}{2}\frac{\Delta l}{v_L} = \pi\frac{v_L}{\lambda_o}\frac{\Delta l}{v_L} = \pi\frac{\Delta l}{\lambda_o}$

and $v_N = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi}{\beta}\frac{v_L}{\lambda_o} \Longrightarrow \beta = \frac{2\pi}{\lambda_o}\frac{v_L}{v_N}$

- Combining the above equations we obtain the dispersion relationship $\frac{v_N}{v_L} = \frac{\pi(\Delta l/\lambda_o)}{\sin^{-1}(\sqrt{2}\sin(\pi\Delta l/\lambda_o))}$
- Notice that v_N depends on the ratio $\Delta l/\lambda_o$
- Also, for $\Delta l << \lambda_o$, $v_N \approx \sqrt{2}\, v_L$

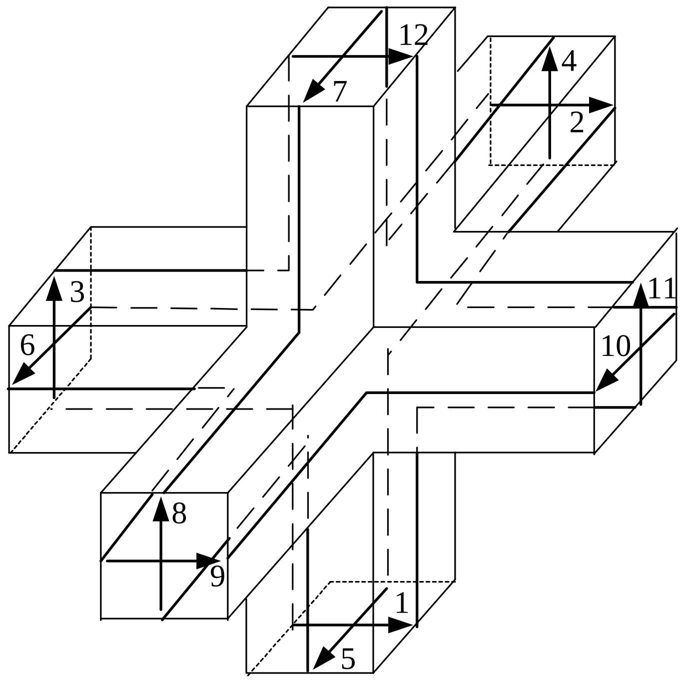
Dispersion in a 2D TLM Mesh (Cont'd)



Christos, Transmission Line Modeling (TLM)

3D TLM

- The symmetric condensed node (SCN) is the most widely used node



- The SCN has 6 branches with 2 transmission lines in each branch

3D TLM (Cont'd)

- For modeling free space $S \in \Re^{12\times 12}$
- Components of *S* are determined through conservation of energy
- Open and short circuit stubs are used to model the proper capacitance and inductance in the *x*,*y* and *z* directions
- In this case $S \in \Re^{18\times 18}$