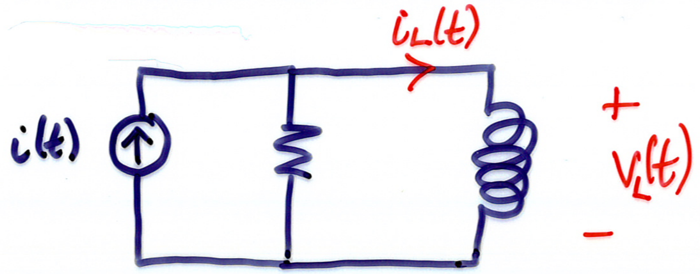
Now consider a coil of wire



- flowing current establishes magnetic field
- when current changes, field changes
- when field changes, what happens?
- Lenz & Faraday $\Rightarrow$ induced voltage

For linear coils that don't change in time,

$$v_L(t) = L \frac{di_L(t)}{dt}$$

$$\Rightarrow \quad i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(x)\,dx$$

if $i_L(-\infty) = 0$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(x)\,dx$$

Does this element store energy?

$$p(t) = v_L(t)\, i_L(t)$$

$$= L\, i_L(t) \frac{di_L(t)}{dt}$$

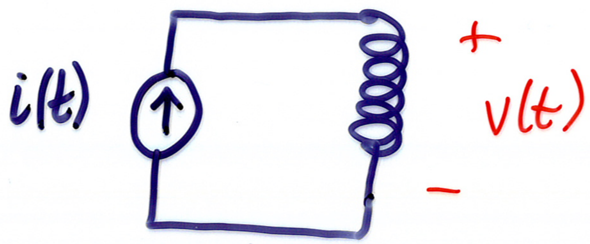
$$E(t) = \int_{-\infty}^{t} L\, i_L(x) \frac{di_L(x)}{dx}\, dx$$

$$= L \int_{i_L(-\infty)}^{i_L(t)} i_L(x)\, di_L(x)$$

if $i_L(-\infty) = 0$

$$E(t) = \frac{1}{2} L\, i_L(t)^2$$

PROPERTIES



$$v(t) = L \frac{di(t)}{dt}$$

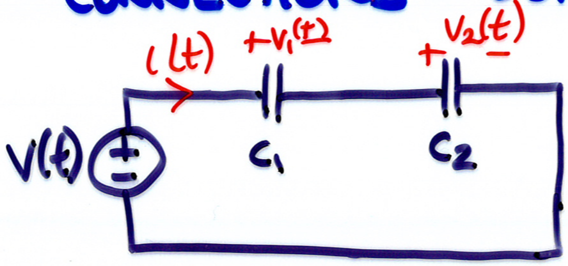
- if $i(t)$ is constant,
  $v(t) = 0$ no matter how big $i(t)$ is
  $\Rightarrow$ short circuit

- If $i(t)$ changes rapidly
  $v(t)$ can be large, even if $i(t)$ is small
  $\rightarrow$ open circuit

- $$i(t) = \frac{1}{L} \int_{-\infty}^{t} v(x)\,dx$$

$\Rightarrow$ current is continuous

Connections – Series

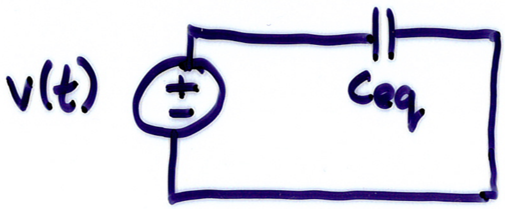


$$v_i(t) = v_i(t_0) + \frac{1}{C_i}\int_{t_0}^{t} i(x)\,dx$$

KVL: $v(t) = v_1(t) + v_2(t)$

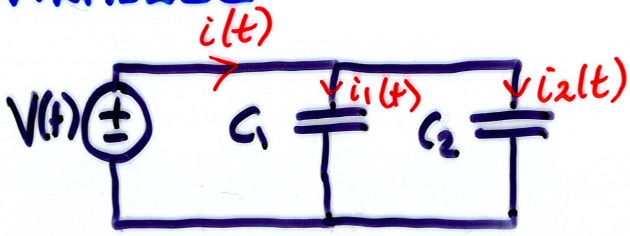
$$\Rightarrow v(t) = \left[v_1(t_0) + v_2(t_0)\right] + \left[\frac{1}{C_1} + \frac{1}{C_2}\right]\int_{t_0}^{t} i(x)\,dx$$

$\Rightarrow$ circuit is equivalent to



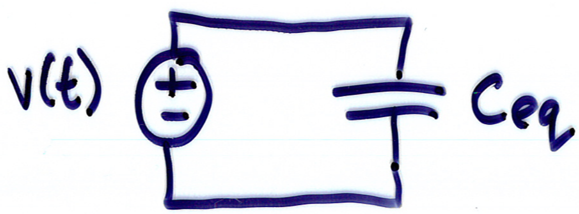
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Parallel



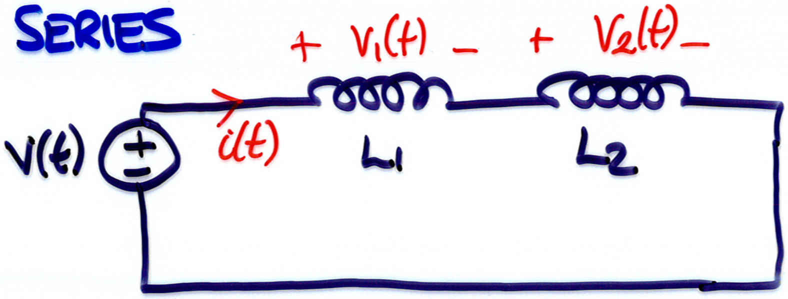
KCL $\quad i(t) = i_1(t) + i_2(t) \quad ; \quad i_k(t) = C_k \frac{dv(t)}{dt}$

$$\Rightarrow i(t) = \left[C_1 + C_2\right]\frac{dv(t)}{dt}$$



$$C_{eq} = C_1 + C_2$$

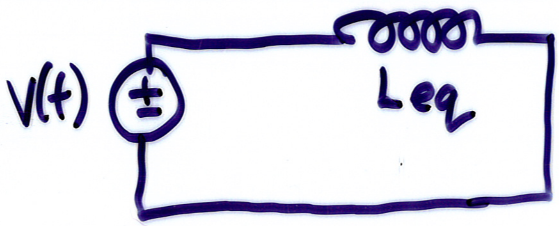
SERIES



$$v_k(t) = L_k \frac{di(t)}{dt}$$

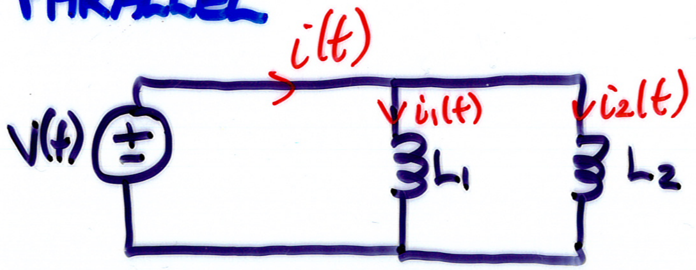
KVL: $v(t) = v_1(t) + v_2(t)$

$$= (L_1 + L_2)\frac{di(t)}{dt}$$



$$L_{eq} = L_1 + L_2$$

PARALLEL

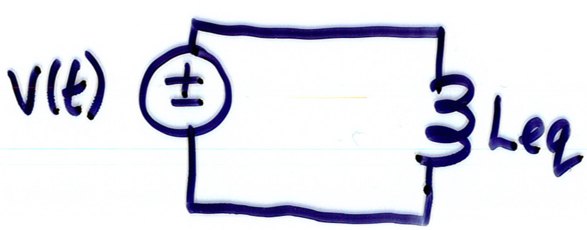


$$i_k(t) = i_k(t_0) + \frac{1}{L_k}\int_{t_0}^{t} v(x)\,dx$$

KCL:

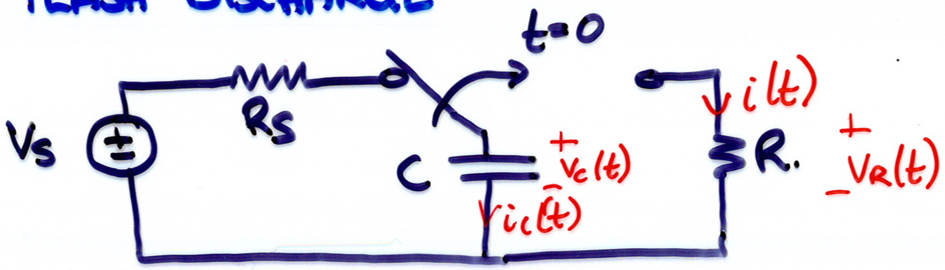
$$i(t) = i_1(t) + i_2(t)$$

$$= \left[i_1(t_0) + i_2(t_0)\right] + \left[\frac{1}{L_1} + \frac{1}{L_2}\right]\int_{t_0}^{t} v(x)\,dx$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

FLASH DISCHARGE



Assume switch has been in current position for a long time

$\Rightarrow V_c(t)\Big|_{t=0^-} = V_s$

What happens after that?

Voltage is continuous for a capacitor $\Rightarrow V_c(t)\Big|_{t=0^+} = V_s$

[redacted]

KCL: [redacted] $i_c(t) + i(t) = 0$

$$C\frac{dv_c(t)}{dt} + \frac{1}{R}v_R(t) = 0 \quad ; \text{ but } V_c(t) = V_R(t)$$

$$\Rightarrow \frac{dV_c(t)}{dt} + \frac{1}{RC}V_c(t) = 0$$

$$\Rightarrow V_c(t) = V_s\, e^{-t/RC}$$

$$\Rightarrow i(t) = +\frac{V_s}{R}\, e^{-t/RC}$$