IMPACT OF PHASOR RELATIONSHIPS.

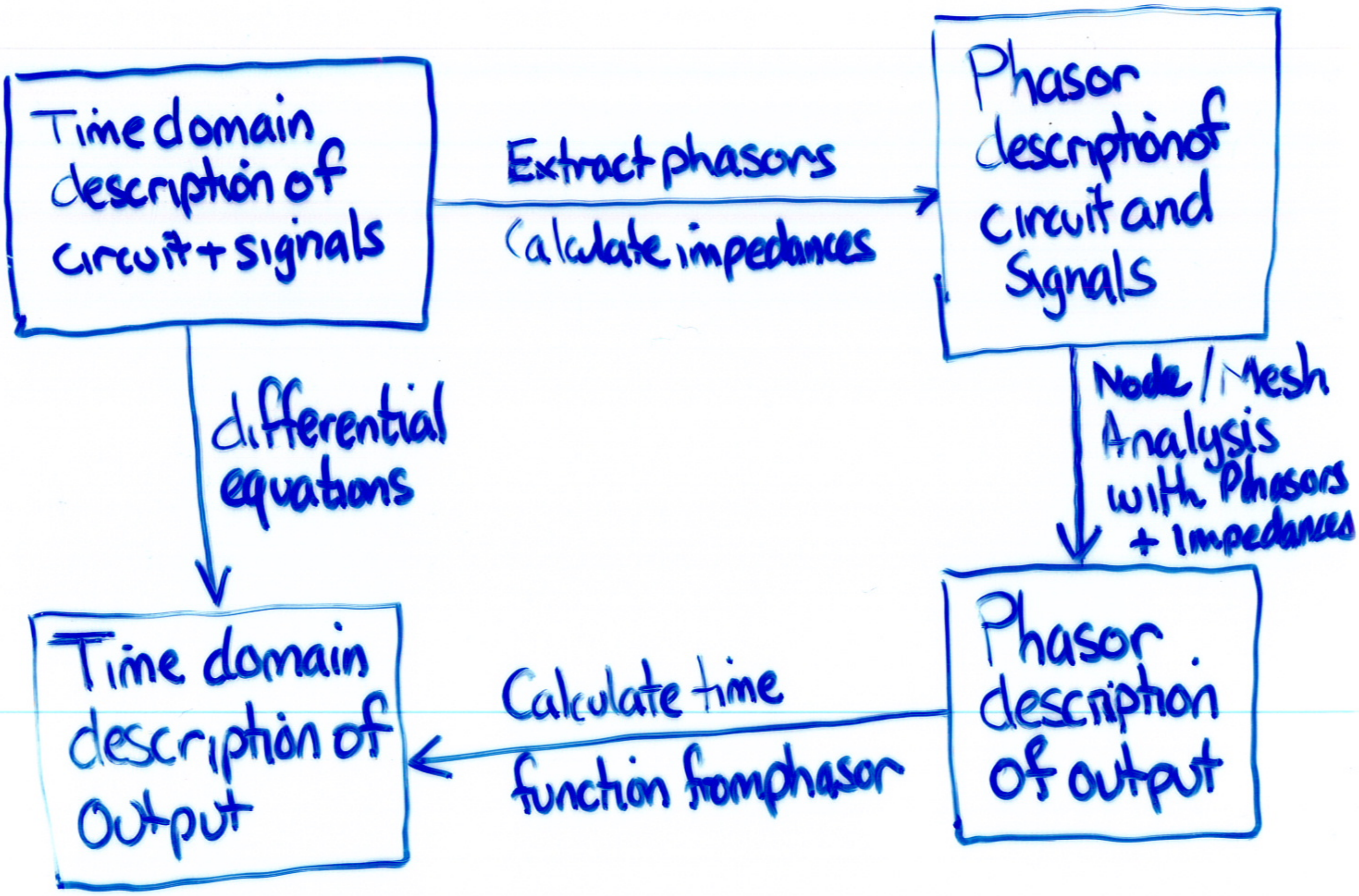
Recall

$$A\cos(\omega_0 t+\theta) = \mathrm{Re}\{A\,e^{j(\omega_0 t+\theta)}\}$$

$$= \mathrm{Re}\{Ae^{j\theta}\,e^{j\omega_0 t}\}$$

$Ae^{j\theta}$ is the phasor representation of $A\cos(\omega_0 t+\theta)$

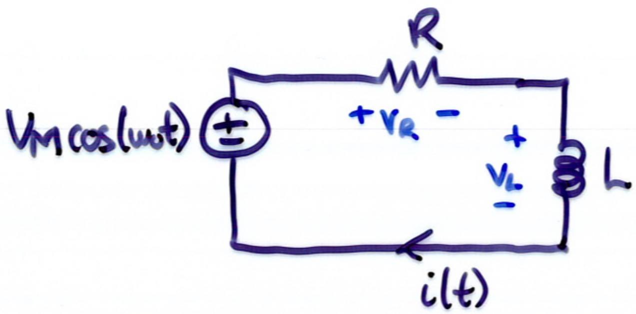
Simplified Analysis of Linear Circuits with Sinusoidal inputs in steady-state using phasors



Why does this work?

How do phasors allow us to avoid differential equations?

FIND THE FORCED RESPONSE:



KVL $\quad -V_M\cos(\omega_0 t) + v_R(t) + v_L(t) = 0$

$$\Rightarrow \quad v_L(t) + v_R(t) = V_M\cos(\omega_0 t)$$

$$\Rightarrow \quad L\frac{di(t)}{dt} + R\,i(t) = V_M\cos(\omega_0 t)$$

Postulate form of forced response

$$i(t) = A_1\cos(\omega_0 t) + A_2 \cancel{\cos}\sin(\omega_0 t)$$

Substitute into differential equation

$$-A_1 L\omega_0\sin(\omega_0 t) + A_2\omega_0 L\cos(\omega_0 t) + RA_1\cos(\omega_0 t) + RA_2\sin(\omega_0 t)$$

$$= V_M\cos(\omega_0 t)$$

Equating coeffs of sin and cosine

$$-A_1 L\omega_0 + RA_2 = 0$$

$$A_2\omega_0 L + RA_1 = V_M$$

Solution:

$$A_1 = \frac{R V_M}{R^2 + \omega_0^2 L^2} \quad ; \quad A_2 = \frac{\omega_0 L V_M}{R^2 + \omega_0^2 L^2}$$

$\Rightarrow$

$$i(t) = \frac{R V_M}{R^2 + \omega_0^2 L^2} \cos(\omega_0 t) + \frac{\omega_0 L V_M}{R^2 + \omega_0^2 L^2} \sin(\omega_0 t)$$

$$= \frac{V_M}{\sqrt{R^2 + \omega_0^2 L^2}} \cos\left(\omega_0 t - \operatorname{atan}\left(\frac{\omega_0 L}{R}\right)\right)$$

$\Rightarrow$ Forced response is also a sinusoid with same frequency, but

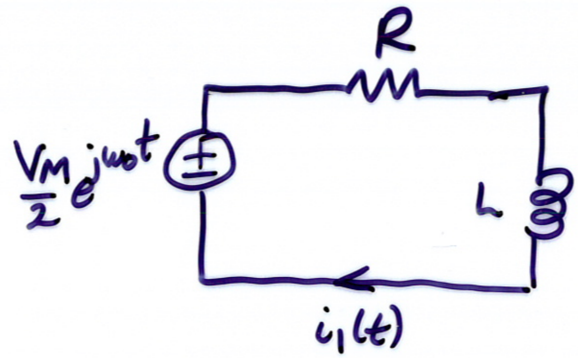
1. Different amplitude
2. Different phase

Phasors capture amplitude and phase.

Can we use them to make this easier?

$$V_M \cos(\omega_0 t) = \frac{V_M}{2} e^{j\omega_0 t} + \frac{V_M}{2} e^{-j\omega_0 t}$$

Let's try a super position approach



$$L \frac{di_1(t)}{dt} + R i_1(t) = \frac{V_M}{2} e^{j\omega_0 t}$$

Postulate form of forced response

$$i_1(t) = I_M e^{j(\omega_0 t + \phi)}$$

Substitute into differential equation

$$j\omega_0 L I_M e^{j(\omega_0 t + \phi)} + R I_M e^{j(\omega_0 t + \phi)} = \frac{V_M}{2} e^{j(\omega_0 t)}$$

Divide by $e^{j\omega_0 t}$

$$j\omega_0 L I_M e^{j\phi} + R I_M e^{j\phi} = \frac{V_M}{2}$$

$$\Rightarrow \quad I_M e^{j\phi} = \frac{V_M/2}{R + j\omega_0 L}$$

$$= \frac{V_M/2}{\sqrt{R^2 + \omega_0^2 L^2}\; e^{j\,\mathrm{atan}(\omega_0 L/R)}}$$

$$= \frac{V_M/2}{\sqrt{R^2 + \omega_0^2 L^2}}\; e^{-j\,\mathrm{atan}(\omega_0 L/R)}$$

$$\Rightarrow \quad i_1(t) = \frac{V_M/2}{\sqrt{R^2 + \omega_0^2 L^2}}\; e^{j(\omega_0 t - \mathrm{atan}(\omega_0 L/R))}$$

Similarly

$$i_2(t) = \frac{V_M/2}{\sqrt{R^2 + \omega_0^2 L^2}}\; e^{-j(\omega_0 t - \mathrm{atan}(\omega_0 L/R))}$$

$$\Rightarrow \quad i(t) = \frac{V_M}{\sqrt{R^2 + \omega_0^2 L^2}}\; \cos(\omega_0 t - \mathrm{atan}(\omega_0 L/R))$$

Now let's try by phasor analysis

$$L\frac{di(t)}{dt} + Ri(t) = V_M\cos(\omega_0 t)$$

$$V_M\cos(\omega_0 t) = \mathrm{Re}\{\mathbf{V}e^{j\omega_0 t}\} \qquad \mathbf{V} = V_M$$

Postulate

$$i(t) = \mathrm{Re}\{\mathbf{I}e^{j\omega_0 t}\} \qquad \mathbf{I} = I_M e^{j\phi}$$

Approach ① Solve differential equation in complex case

② Take the real part at the end.

Substitute $\mathbf{I}e^{j\omega_0 t}$ into differential equation

$$j\omega_0 L\mathbf{I}e^{j\omega_0 t} + R\mathbf{I}e^{j\omega_0 t} = \mathbf{V}e^{j\omega_0 t}$$

Divide by $e^{j\omega_0 t}$

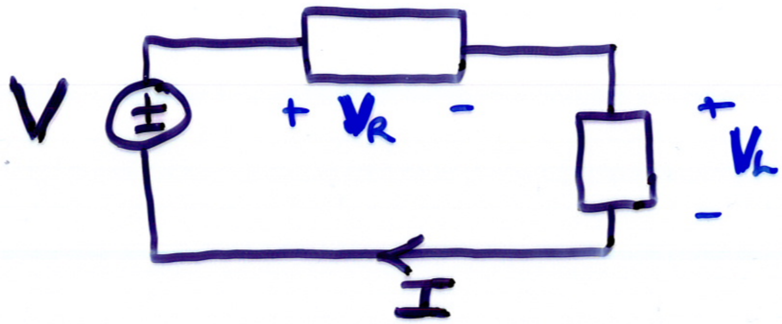
$$j\omega_0 L\mathbf{I} + R\mathbf{I} = \mathbf{V}$$

Differential eqn has become algebraic!

$$\Rightarrow \mathbf{I} = \frac{V_M}{R+j\omega_0 L} = \frac{V_M}{\sqrt{R^2+\omega_0^2L^2}}e^{-j\,\mathrm{atan}(\omega_0 L/R)}$$

$$\Rightarrow i(t) = \frac{V_M}{\sqrt{R^2+j\omega_0^2L^2}}\cos(\omega_0 t - \mathrm{atan}(\omega_0 L/R))$$

Can we use phasors directly?



KVL $\quad -\mathbf{V} + \mathbf{V}_R + \mathbf{V}_L = 0$

$$\mathbf{V} = V_M e^{j0}$$

$$\mathbf{V}_R = R\mathbf{I}$$

$$\mathbf{V}_L = j\omega_0 L\mathbf{I}$$

$$\Rightarrow \quad (R + j\omega_0 L)\mathbf{I} = V_M$$

$$\Rightarrow \quad \mathbf{I} = \frac{V_M}{R + j\omega_0 L} = \frac{V_M}{\sqrt{R^2 + j\omega_0^2 L^2}} e^{-j\,\mathrm{atan}(\omega_0 L/R)}$$

$$\Rightarrow \quad i(t) = \frac{V_M}{\sqrt{R^2 + \omega_0^2 L^2}} \cos(\omega_0 t - \mathrm{atan}(\omega_0 L/R))$$