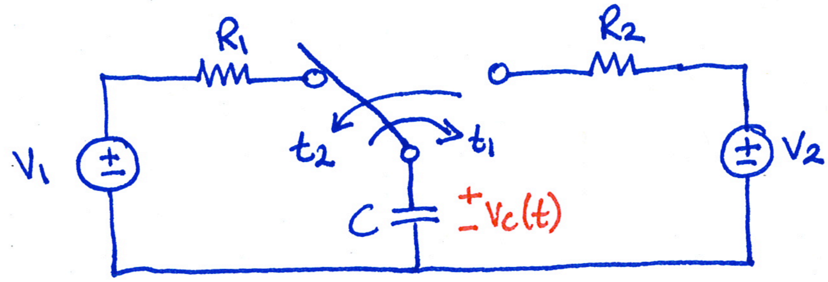
EE2CI5 SWITCHING CIRCUIT EXAMPLE

Prior to time t_1 the switch in the following circuit was in the illustrated position for a long time

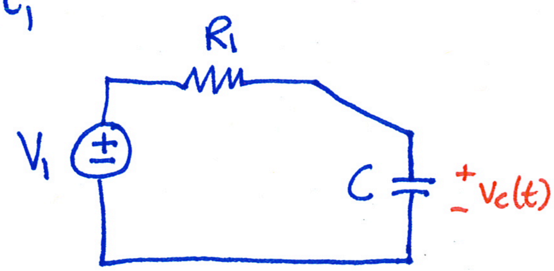
At time t_1 the switch flips and at time $t_2 > t_1$ it flips back

Find $V_C(t)$ for $t \geq t_1$

Sketch $V_C(t)$. For your sketch, assume $V_1 < V_2$, $R_1 < R_2$



For $t < t_1$



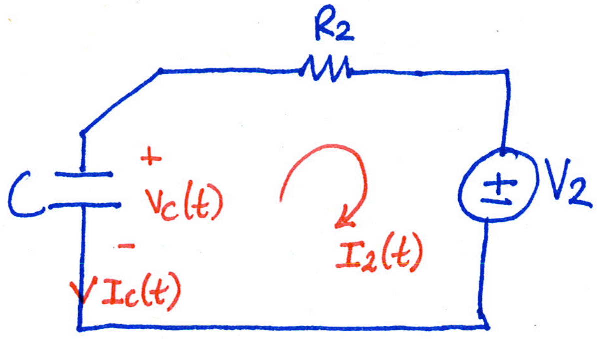
Switch has been in this position for a long period of time
Since source is constant, there is no current

$$\Rightarrow \quad V_c(t_1^-) = V_1$$

But capacitor voltage is continuous $\Rightarrow$ $V_c(t_1^+) = V_1$

For $t_1 < t < t_2$

$V_C(t_1^+) = V_1$



KVL: $\quad -V_C(t) + R_2 I_2(t) + V_2 = 0$

Capacitor: $\quad I_C(t) = C \dfrac{dV_C(t)}{dt}$

Currents: $\quad I_C(t) = -I_2(t)$

$$\Rightarrow \quad R_2 I_C(t) + V_C(t) = V_2$$

$$\Rightarrow \quad R_2 C \frac{dV_C(t)}{dt} + V_C(t) = V_2$$

$$\Rightarrow \quad \frac{dV_C(t)}{dt} + \frac{V_C(t)}{R_2 C} = \frac{V_2}{R_2 C}$$

Let $\tau_2 = R_2 C$

$$\Rightarrow \quad V_C(t) = K_2 e^{-t/\tau_2} + N_2$$

Initial condition $V_C(t_1) = V_1 \quad \Rightarrow \quad K_2 e^{-t_1/\tau_2} + N_2 = V_1$

Final condition, using the fact that source is constant

If the circuit would not change, as $t \to \infty$ $V_C(t) \to V_2$

$$\Rightarrow N_2 = V_2$$

So, we have, for $t_1 < t < t_2$

$$V_c(t) = K_2 e^{-t/\tau_2} + N_2$$

with $$K_2 e^{-t_1/\tau_2} + N_2 = V_1$$

$$N_2 = V_2.$$

$$\Rightarrow \quad K_2 = (V_1 - V_2) e^{t_1/\tau_2}$$

$$\Rightarrow \quad V_c(t) = (V_1 - V_2) e^{-(t-t_1)/\tau_2} + V_2$$

for $t_1 < t < t_2$.

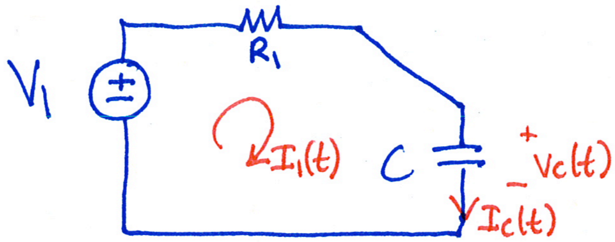
Note that at $t = t_2^-$ we have.

$$V_c(t_2^-) = (V_1 - V_2) e^{-(t_2 - t_1)/\tau_2} + V_2$$

For convenience let V_x denote $V_c(t_2^-)$

For $t > t_2$

$$V_c(t_2^+) = V_x$$



KVL: $\quad -V_1 + R_1 I_1(t) + V_c(t) = 0$

Capacitor: $\quad I_c(t) = C \dfrac{dV_c(t)}{dt}$

Currents. $\quad I_c(t) = I_1(t)$

$$\Rightarrow \quad \frac{dV_c(t)}{dt} + \frac{V_c(t)}{R_1 C} = \frac{V_1}{R_1 C}$$

Let $\tau_1 = R_1 C$

$$\Rightarrow \quad V_c(t) = K_1 e^{-t/\tau_1} + N_1$$

Initial condition $\quad V_c(t_2^+) = V_x \quad \Rightarrow \quad K_1 e^{-t_2/\tau_1} + N_1 = V_x$

Final condition, using the fact that the source is constant,

$$V_c(t)\Big|_{t\to\infty} = V_1$$

$$\Rightarrow \quad N_1 = V_1$$

So, for $t > t_2$.

$$V_c(t) = K_1 e^{-t/\tau_1} + N_1$$

with $$K_1 e^{-t_2/\tau_1} + N_1 = V_X$$

$$N_1 = V_1$$

$$\Rightarrow V_c(t) = (V_X - V_1)\, e^{-(t-t_2)/\tau_1} + V_1$$

Summary

Let $$V_x = (V_1 - V_2)\, e^{-(t_2 - t_1)/\tau_2} + V_2 .$$

$$V_c(t) = \begin{cases} V_1 & t \le t_1 \\ (V_1 - V_2)\, e^{-(t-t_1)/\tau_2} + V_2 & t_1 \le t \le t_2 \\ (V_x - V_1)\, e^{-(t-t_2)/\tau_1} + V_1 & t \geq t_2 . \end{cases}$$

Sketch, with $V_1 < V_2$, $R_1 < R_2$.

Note $R_1 < R_2 \Rightarrow \tau_1 < \tau_2$.