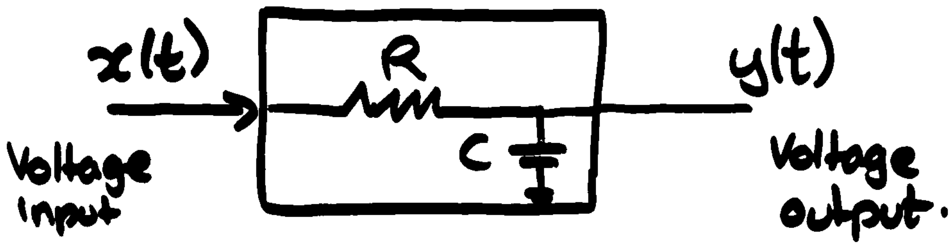
Convolution Integral

A Recipe –

$$y(t) = \int x(\tau)\, h(t-\tau)\, d\tau$$

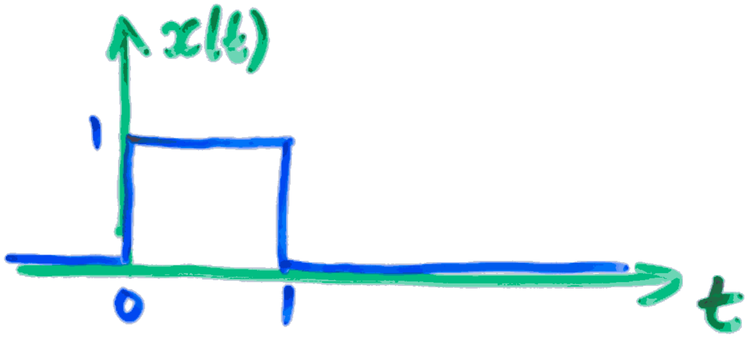
1. Plot $x(\tau)$ $h(\tau)$ and $\tilde{u}(\tau) = h(-\tau)$
2. Start with t large and negative
3. Plot $\tilde{u}(\tau - t) = h(t-\tau)$ and use your graph to help you find a functional form for $w_t(\tau) = x(\tau)\, h(t-\tau)$
4. Increase t until your formula is no longer valid. Record this value as one of the t_j's in the previous formula for $y(t)$
5. Repeat 3 and 4 until all functional forms of $w_t(\tau)$ have been identified and all t_j's identified. Usually this requires t to be large and positive
6. On each interval find $f_j(t) = \int w_t(\tau)\, d\tau$
7. Sanity check. If $x(t)$ and $h(t)$ have no "infinite" discontinuities then

Realistic Example (Similar to Ex 2.7)



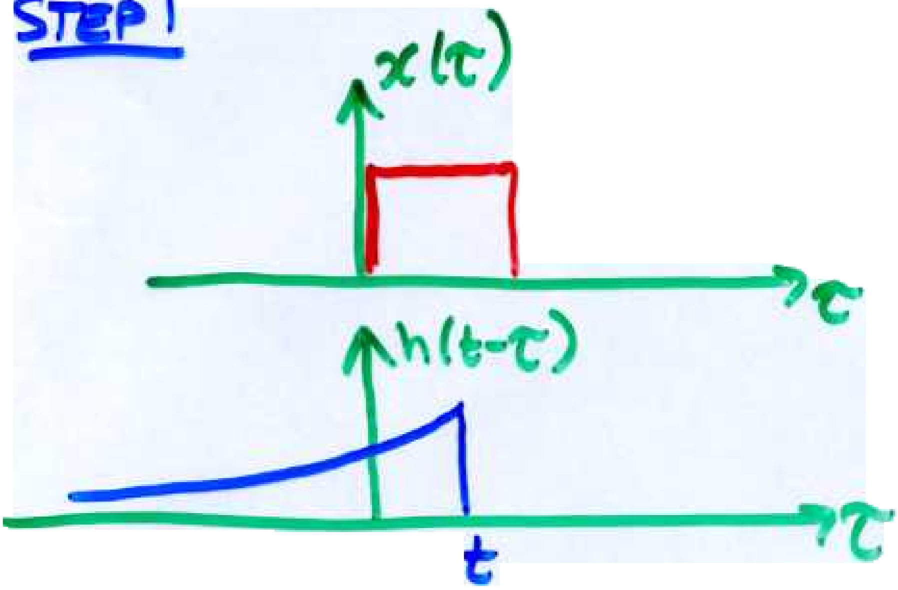
The impulse response of this circuit is

What is the output $y(t)$, when the input is a square pulse



The answer to questions such as this is fundamental to the determination of how fast chips can run + how fast we can communicate

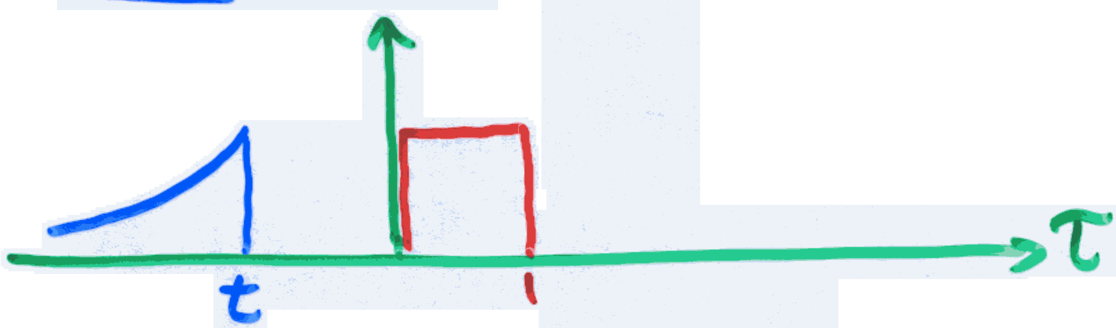
STEP 1



$h(t) = e^{-at}u(t)$

note $h(t-\tau) = e^{-a(t-\tau)}$ u(t-

STEP 3, for t large and negative

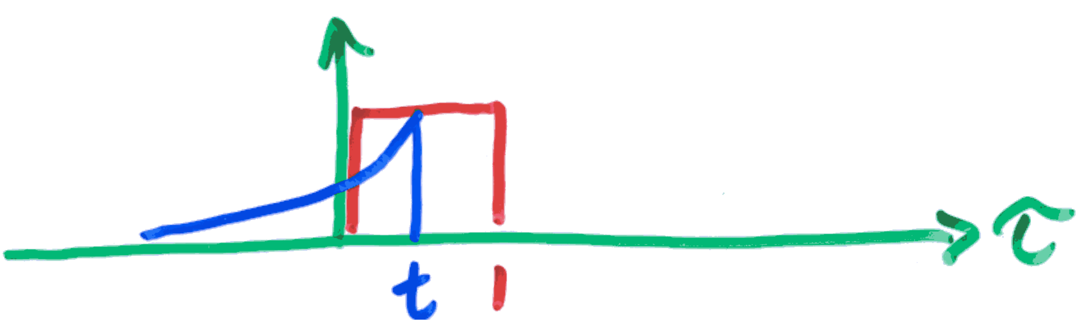


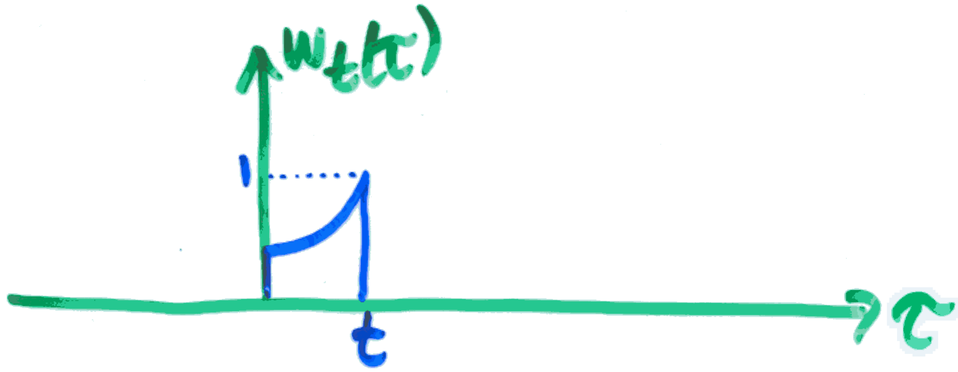
Hence $w_t(\tau) = 0$

STEP 4

This formula is valid until $t = 0$

STEP 3, for $t \geq 0$



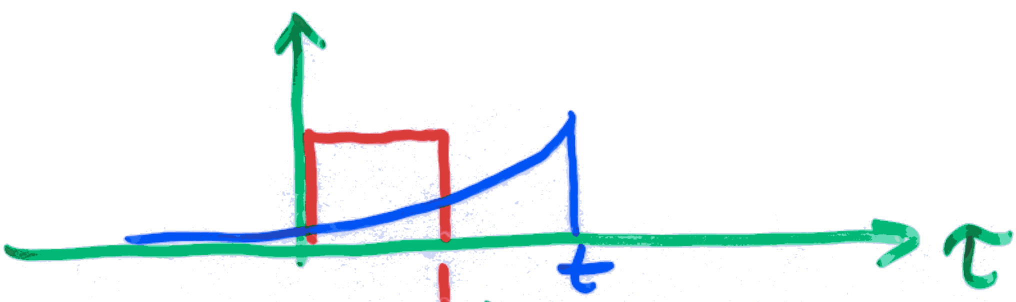


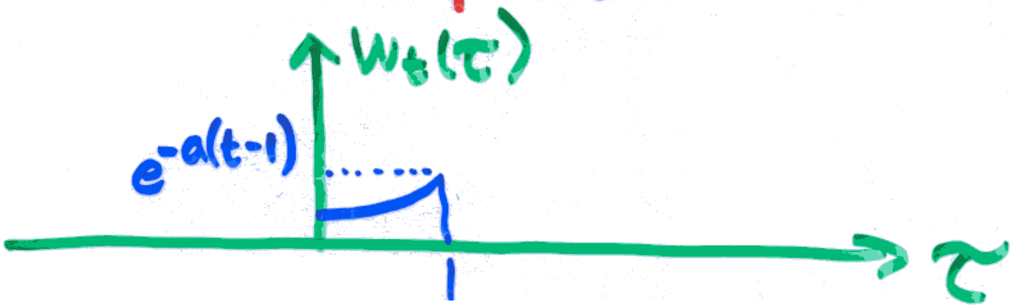
Hence

$$w_t(\tau) = \begin{cases} 0 & \tau < 0 \\ e^{-a(t-\tau)} & 0 \le \tau < t \\ 0 & \tau \geq t \end{cases}$$

Formula is valid until t

Step3, for $t \geq$





$$\Rightarrow w_t(\tau) \begin{cases} 0 & \tau < 0 \\ e^{-a(t-\tau)} & 0 \le \tau < \\ 0 & \tau \geq 1 \end{cases}$$

This formula 's va id for a $t \geq$

Hence

$$w_t(\tau) = \begin{cases} 0 \text{ for a } \tau & \text{when } t < 0 \\ \begin{cases} e^{-a(t-\tau)} & 0 \leq \tau < t \\ 0 & \text{otherwise} \end{cases}, & \text{when } 0 \leq t < \\ \begin{cases} e^{-a(t-\tau)} & 0 \leq \tau < \\ 0 & \text{otherwise} \end{cases} & \text{whe } t \geq \end{cases}$$

Step 5

$$y(t) = \int_{-\infty}^{\infty} w_t(\tau)\, d\tau$$

[redacted]

when $t < 0$ $\quad w_t(\tau) = 0 \Rightarrow y\ t) = 0$

when $0\ t <$ $\quad y\ t) = \int_0^t e^{-a\ t-\tau)}\, d\tau$

$$= e^{-at} \int_0^t e^{a\tau}\, d\tau$$

$$= e^{at} \left[\frac{e^{a\tau}}{a} \right]_0^t$$

$$= \frac{-e^{-at}}{a}$$

when $t \geqslant 1$

$$y(t) = \int_0^1 \ _{-a(t-\tau)} \, d\tau$$

$$= \frac{(e^a - 1)}{a} e^{-at}$$

Summarise

$$(t) = \begin{cases} 0 & t < 0 \\ \frac{1-e^{-at}}{a} & 0 \leq t \leq 1 \\ \left(\frac{e^a-1}{a}\right) e^{-at} & t \geqslant 1 \end{cases}$$

ty checks

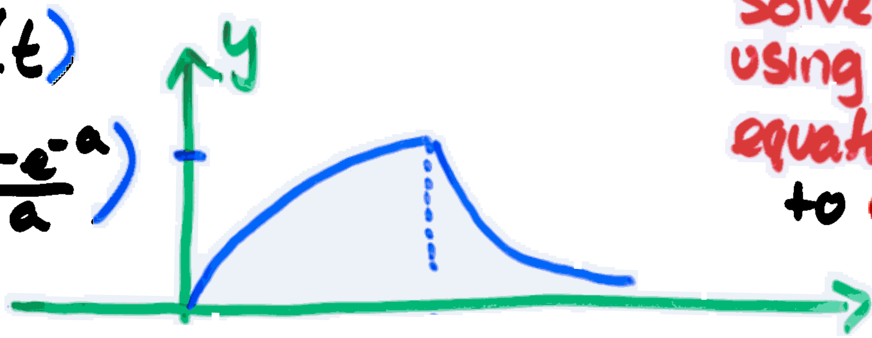
$y(0^- = 0$; $y(\ )$ $\overline{\ \ a\ \ }$

$y(1^-) =$ $y(1)$ $a))$

$$= \frac{1-e^{-a}}{a}.$$

Passed!

Pl $y(t)$ $\left(\frac{1-e^{-a}}{a}\right)$



Solve using diffential equations (2CI4 to confirm resul

SUGGESTED HOMEWORK

DRILL PROBLEMS

2.2, 2.3

2.4 2.5

These problems have answers in the book