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Non-linear Transceiver Design for Broadcast Channels with Statistical Channel State Information

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Abstract— **We consider the design of Tomlinson-Harashima (TH) transceivers for the downlink of a multiuser communication system in the presence of uncertain channel state information (CSI) at the base station. We consider systems in which the base station has multiple antennas and each user has a single antenna, and we consider a stochastic model for the uncertainty in the CSI. We study the joint design of a Tomlinson-Harashima precoder at the base station and the equalizing gains at the receivers so as to minimize the average, over channel uncertainty, of the total mean-square-error (MSE). By generalizing the MSE duality between the broadcast channel (BC) with TH precoding and the multiple access channel (MAC) with decision feedback equalization (DFE) to scenarios with uncertain CSI, we obtain a relation between the desired robust broadcast transceivers and the corresponding transceivers that optimize the same performance metric for the dual multiple access channel. Our simulations indicate that the proposed approach can significantly reduce the sensitivity of the downlink to uncertainty in the CSI, and can provide improved performance over that of existing robust designs.**

I. Introduction

A key advantage of using multiple antennas on the downlink of multiuser systems is the ability to transmit independent data messages to decentralized users. In these broadcasting scenarios, the transmitter typically employs spatial multiplexing techniques to precode the users' messages in a way that mitigates the effect of multiuser interference at the receivers created by the channel propagation. One of the available spatial multiplexing techniques is to apply Tomlinson-Harashima (TH) precoding at the transmitter jointly with linear equalization at each receiver. Tomlinson-Harashima precoding works by precoding the users' messages sequentially by pre-subtracting the interference that previously precoded messages would otherwise create at the receivers. A fundamental assumption of TH precoding is the availability of perfect Channel State Information (CSI) at the transmitter. Perfect CSI enables the transmitter to precisely pre-subtract the terms that would interfere at the receivers. Based on the assumption of perfect CSI at the transmitter, several different approaches for designing TH precoders for broadcast channels have been proposed, including zero-forcing designs [1], [2], [3], [4], and minimum mean square error (MMSE) designs [5], [6].

In practical broadcasting schemes, the CSI available at the transmitter suffers from inaccuracies that arise from sources such as channel estimation errors. These inaccuracies can result in serious degradation of the performance of broadcast systems, e.g., [7]. Furthermore, the performance of TH precoding is particularly sensitive to inaccuracies in CSI, e.g., [8]. Motivated by the sensitivity of both broadcast channels and TH precoding to channel uncertainty, we design, herein, a robust TH transceiver that explicitly takes into account the CSI uncertainty. We will use a stochastic model for the CSI uncertainty. This model is particularly suitable for systems in which the CSI uncertainty is dominated by the effects of channel estimation errors. Examples of such systems include those with uplink-downlink reciprocity, such as time division duplex systems with short "ping-pong" time. Using the stochastic model for channel uncertainty, we consider the joint design of a Tomlinson-Harashima precoder and the users' equalizing gains to minimize the average, over the channel uncertainty, of the total MSE. Previous attempts to solve this problem have considered a simpler design problem by restricting all the users' equalizing gains to be equal [9], [10], or by using a simpler detection model [11]. In our approach we will preserve all the degrees of freedom, and will exploit the duality, derived herein, between the broadcast with TH precoding and the multiple access channel (MAC) with decision feedback equalization (DFE), under a statistical model of CSI. More generally, the duality result that we will derive will enable us to obtain robust designs for broadcast channels with TH precoding that optimize functions of the the average MSEs, by solving the same design problem for a dual MAC with a DFE. By doing so, we extend to the case of imperfect CSI earlier work on the duality, in the MSE sense, of the BC with TH precoding and MAC with a DFE assuming perfect CSI [6], [12]. Our results indicate that the proposed approach can significantly reduce the sensitivity of the downlink to uncertainty in the CSI, and can provide improved performance over that of existing robust designs.

II. System Model

We consider the downlink of a multiuser cellular communication system with N_t antennas at the transmitter and K users, each with one receive antenna. We consider downlink systems in which Tomlinson-Harashima (TH) precoding is used at the transmitter for multi-user interference pre-subtraction. As shown in Fig. 1, interference pre-subtraction and channel spatial equalization are performed at the transmitter using a strictly lower triangular feedback precoding matrix $\mathbf{B} \in \mathbb{C}^{K\times K}$ and a feedforward precoding matrix $\mathbf{P} \in \mathbb{C}^{N_t\times K}$. The vector $\mathbf{s} \in \mathbb{C}^{K}$ contains the data symbol destined for each user, and we assume that s_k is chosen from a square QAM

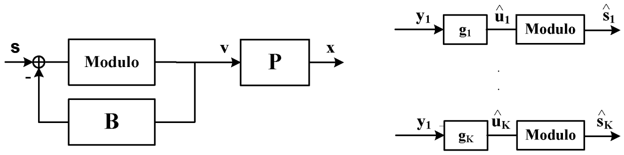

Fig. 1. BC with Tomlinson-Harashima precoding.

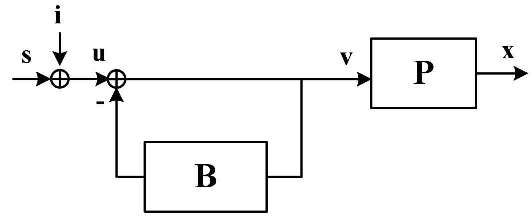

Fig. 2. Equivalent linear model for the transmitter.

constellation $\mathcal{S}$ with cardinality M. The Voronoi region of the constellation $\mathcal{V}$ is a square whose side length is D; i.e., $D = \sqrt{M}\, d$, where d is the distance between two successive constellation points along any of the basis directions.

In absence of the modulo operation, the output symbols of the feedback loop in Fig. 2, v_k, would be generated successively according to the following relation:

$$v_k = s_k - \sum_{j=1}^{k-1} B_{k,j} v_j, \tag{1}$$

where at the k^{th} step, only the previously precoded symbols $v_1, .., v_{k-1}$ are subtracted. Hence, $\mathbf{B}$ is a strictly lower triangular matrix. The modulo operation is used to prevent the magnitude of v_k in (1) from growing outside the boundaries of $\mathcal{V}$. For square QAM symbols, the modulo operation corresponds to performing separate modulo-D operations on the real and imaginary parts of v_k, and this is equivalent to the addition of the complex quantity $i_k = i_k^{re}\, D + j\; i_k^{imag}\, D$ to v_k, where $i_k^{re},\ i_k^{imag} \in \mathbb{Z}$, and $j = \sqrt{-1}$. Using this observation, we obtain the standard linear model of the transmitter that does not involve a modulo operation, as shown in Fig. 2; e.g., [13]. In this model, the constellation of the modified data symbols in the vector $\mathbf{u} = \mathbf{s} + \mathbf{i}$ is simply the periodic extension of the original constellation $\mathcal{S}$ along the real and imaginary axes. For this equivalent model, the vector $\mathbf{v}$ is linearly related to the modified data vector $\mathbf{u}$,

$$\mathbf{v} = (\mathbf{I} + \mathbf{B})^{-1}\mathbf{u}. \tag{2}$$

Following the feedback processing, the vector $\mathbf{v}$ is then linearly precoded to produce the vector of transmitted signals

$$\mathbf{x} = \mathbf{P}\mathbf{v}. \tag{3}$$

As a result of the modulo operation, the elements of $\mathbf{v}$ are almost uncorrelated and uniformly distributed over the Voronoi region $\mathcal{V}$, [13, Th. 3.1]. Therefore, the symbols of $\mathbf{v}$ will have slightly higher average energy than the input symbols $\mathbf{s}$. This slight increase in the average energy is termed precoding loss [13]. For example, for square M-ary QAM we have $\mathrm{E}\{|v_k|^2\} = \frac{M}{M-1}\mathrm{E}\{|s_k|^2\}$ for $k = 2, \ldots, K$, and $\mathrm{E}\{|v_1|^2\} = \mathrm{E}\{|s_1|^2\}$, [13]. For moderate to large values of M this power increase can be neglected and the approximation $\mathrm{E}\{\mathbf{v}\mathbf{v}^H\} = \mathbf{I}$ is often used; e.g., [1], [14]. If we assume negligible precoding loss, the average transmitted power constraint can be written as $\mathrm{E}_{\mathbf{v}}\{\mathbf{x}^H\mathbf{x}\} = \mathrm{tr}(\mathbf{P}^H\mathbf{P}) \leq P_{\text{total}}$.

The signal received by the k^{th} user, y_k, can be written as

$$y_k = \mathbf{h}_k\mathbf{x} + n_k, \tag{4}$$

where $\mathbf{h}_k \in \mathbb{C}^{1\times N_t}$ is a row vector representing the channel gains from the transmitting antennas to the k^{th} receiver, and n_k is the additive zero-mean white noise at the k^{th} receiver whose variance is σ_n^2. Collecting the received signals in the vector $\mathbf{y}$, we can write

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

where $\mathbf{H}$ is the broadcast channel matrix whose k^{th} row is $\mathbf{h}_k$, and $\mathbf{n}$ is the noise vector whose covariance matrix is $\mathrm{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2\mathbf{I}$. Due to the decentralized nature of the receivers, joint processing of the received vector $\mathbf{y}$ is not possible. Instead, each receiver will process its received signal y_k independently using a single equalizing gain g_k to obtain the estimate, $\hat{u}_k = g_k y_k$, followed by a modulo operation to obtain $\hat{s}_k$. In terms of the modified data symbols, the error signal $\hat{u}_k - u_k$ can be used to define the mean square error,

$$\begin{aligned}\mathrm{MSE}_k = \mathrm{E}_{\mathbf{v}}\{|\hat{u}_k - u_k|^2\} &= \sum_{j=1}^{K} |g_k|^2\mathbf{p}_j^H(\mathbf{h}_k^H\mathbf{h}_k)\mathbf{p}_j \\ &+ \sigma_n^2|g_k|^2 - g_k\mathbf{h}_k\mathbf{p}_k - \mathbf{p}_k^H\mathbf{h}_k^H g_k^H - \sum_{j=1}^{k-1}\mathbf{p}_j^H\mathbf{h}_k^H g_k^H B_{kj} \\ &- \sum_{j=1}^{k-1} B_{kj}^H g_k\mathbf{h}_k\mathbf{p}_j - \sum_{j=1}^{k-1}|B_{kj}|^2 + 1. \end{aligned} \tag{5}$$

Assuming negligible precoding loss and that the vector $\mathbf{i}$ is eliminated by the receivers modulo operation, the error signal $\hat{u}_k - u_k$ is equivalent to $\hat{s}_k - s$.

A. Stochastic Channel Uncertainty Model

We consider the following additive model for the CSI uncertainty at the transmitter:

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k, \tag{6}$$

where $\mathbf{h}_k$ is the true channel for the k^{th} user, $\hat{\mathbf{h}}_k$ is the transmitter's estimate of $\mathbf{h}_k$, and the error $\mathbf{e}_k$ is modeled as Gaussian random variable with zero-mean and a covariance matrix $\mathrm{E}\{\mathbf{e}_k^H\mathbf{e}_k\} = \sigma_{e_k}^2\mathbf{I}$. This model is particularly suitable for communication schemes with reciprocity between the uplink and the downlink in which the transmitter can use this reciprocity to estimate the users' channels.

Using this uncertainty model, the average over the channel estimation errors of MSE_k is given by:

$$\begin{aligned}\overline{\mathrm{MSE}}_k &= \sum_{j=1}^{K} |g_k|^2\mathbf{p}_j^H(\hat{\mathbf{h}}_k^H\hat{\mathbf{h}}_k + \sigma_{e_k}^2\mathbf{I})\mathbf{p}_j + \sigma_n^2|g_k|^2 \\ &- g_k\hat{\mathbf{h}}_k\mathbf{p}_k - \mathbf{p}_k^H\hat{\mathbf{h}}_k^H g_k^H - \sum_{j=1}^{k-1}\mathbf{p}_j^H\hat{\mathbf{h}}_k^H g_k^H B_{kj}\end{aligned}$$

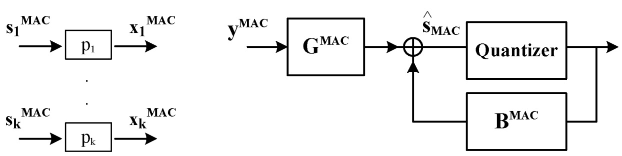

Fig. 3. The Dual MAC with decision feedback equalization.

$$- \sum_{j=1}^{k-1} B_{kj}^H g_k \hat{\mathbf{h}}_k \mathbf{p}_j - \sum_{j=1}^{k-1} |B_{kj}|^2 + 1. \quad (7)$$

III. Robust design Via BC-MAC duality

For the uncertainty model under consideration, our objective is to jointly design the feedback and precoding matrices, $\mathbf{B}$ and $\mathbf{P}$, and the receivers' equalizing gains, g_k, so as to minimize the average, over the channel estimation error, of the total MSE:

$$\overline{\text{MSE}} = \sum_{k=1}^{K} \overline{\text{MSE}}_k. \quad (8)$$

Previous attempts to solve this problem have considered a simpler design problem by restricting all g_k to be equal [9], [10], or by using a simpler detection model [11]. In our approach we will preserve all degrees of freedom, and will exploit the duality, derived herein, between the broadcast channel with TH precoding and the multiple access channels with DFE, under a statistical model of CSI. Using this duality, we will jointly design the transceiver parameters $\mathbf{B}$, $\mathbf{P}$, and g_k so as to minimize (8). More generally, the duality result that we will derive will enable us to obtain robust designs of broadcast channels with TH precoding that optimize objectives that are functions of the the average MSEs, by solving the same design problem for a dual MAC with a DFE. We will start by briefly introducing a dual MAC for the BC presented in Section II.

A. Dual Multiple Access Channel

By switching the roles of the transmitter and the receiver in the broadcast channel, we obtain a dual MAC that consists of K transmitters, each with a single antenna, and a receiver with N_t antennas. The channel matrix between the transmitters and the receiver of the dual MAC is $\mathbf{H}^H$; e.g., [15]. Interference subtraction in the dual MAC is implemented using decision feedback equalization (DFE) in which detection of a given user is preceded by subtraction of the interference resulting from previously detected users; see Fig 3. To obtain a dual MAC, the users are detected using the reverse order of the BC precoding order; i.e., detection starts with the K^{th} user.

Similar to the MSE expressions obtained for the BC in (7), we will be interested in obtaining corresponding expressions for individual MSEs in the dual MAC with linear precoding and DFE. Because the transmitters in the dual MAC are decentralized and each have only one transmit antenna, linear precoding reduces to power loading:

$$x_k^{\text{MAC}} = p_k^{\text{MAC}} s_k^{\text{MAC}}, \quad (9)$$

where s_k^{MAC} and x_k^{MAC} are the data symbol and the transmitted signal of the k^{th} transmitter. Without loss of generality, we will assume that $\text{E}\{\mathbf{s}^{\text{MAC}} \mathbf{s}^{\text{MAC}H}\} = \mathbf{I}$. Hence, a total power constraint on all the transmitters can be written as $\sum_{k=1}^{K} |p_k^{\text{MAC}}|^2 \leq P_{\text{total}}$.

The vector of received signals $\mathbf{y}^{\text{MAC}}$ is given by:

$$\mathbf{y}^{\text{MAC}} = \mathbf{H}^H \mathbf{x}^{\text{MAC}} + \mathbf{n}^{\text{MAC}}, \quad (10)$$

where $\mathbf{n}^{\text{MAC}}$ is the zero-mean receiver noise vector whose covariance matrix is $\text{E}\{\mathbf{n}^{\text{MAC}} \mathbf{n}^{\text{MAC}H}\} = \sigma_n^2 \mathbf{I}$. As shown in Fig. 3, the DFE is implemented using feedforward matrix $\mathbf{G}^{\text{MAC}} \in \mathbb{C}^{K \times n_r}$ and a strictly upper triangular feedback matrix $\mathbf{B}^{\text{MAC}} \in \mathbb{C}^{K \times K}$. In this scenario, the detection of the k^{th} symbol is preceded by subtracting the effect of previously detected symbols. Assuming correct previous decisions, the input to the quantizer, $\hat{s}^{\text{MAC}}$, can be written as

$$\hat{\mathbf{s}}^{\text{MAC}} = (\mathbf{G}^{\text{MAC}} \mathbf{H}^H \mathbf{P}^{\text{MAC}} - \mathbf{B}^{\text{MAC}}) \mathbf{s}^{\text{MAC}} + \mathbf{G}^{\text{MAC}} \mathbf{n}, \quad (11)$$

where $\mathbf{P}^{\text{MAC}} = \text{Diag}(p_1^{\text{MAC}}, \ldots, p_K^{\text{MAC}})$. Using the channel uncertainty model in (6), the average over channel estimation errors of the MSE associated with the estimation $\hat{s}_k^{\text{MAC}}$ can be written as:

$$\begin{aligned} \overline{\text{MSE}}_k^{\text{MAC}} &= \sum_{j=1}^{K} |p_j^{\text{MAC}}|^2 \mathbf{g}_k^{\text{MAC}} (\hat{\mathbf{h}}_j^H \hat{\mathbf{h}}_j + \sigma_{e_j}^2 \mathbf{I}) \mathbf{g}_k^{\text{MAC}H} \\ &+ \sigma_n^2 \mathbf{g}_k^{\text{MAC}} \mathbf{g}_k^{\text{MAC}H} - p_k^{\text{MAC}H} \hat{\mathbf{h}}_k \mathbf{g}_k^{\text{MAC}H} - \mathbf{g}_k^{\text{MAC}} \hat{\mathbf{h}}_k^H p_k^{\text{MAC}} \\ &- \sum_{j=k+1}^{K} \left(p_j^{\text{MAC}H} \hat{\mathbf{h}}_j \mathbf{g}_k^{\text{MAC}H} B_{kj}^{\text{MAC}} + B_{kj}^{\text{MAC}H} \mathbf{g}_k^{\text{MAC}} \hat{\mathbf{h}}_j^H p_j^{\text{MAC}} \right) \\ &- \sum_{j=k+1}^{K} |B_{kj}^{\text{MAC}}|^2 + 1, \end{aligned} \quad (12)$$

where $\mathbf{g}_k^{\text{MAC}}$ is the k^{th} row of $\mathbf{G}^{\text{MAC}}$.

B. BC-MAC Duality with Stochastic Channel Uncertainty

In this section, we will present the MSE duality result between the broadcast channel with TH precoding and the multiple access channel with DFE subject to the stochastic channel uncertainty model described in Section II-A. This duality result generalizes the MSE duality between BC with TH precoding and MAC with DFE for the perfect channel knowledge case [6], [12] to scenarios with uncertain CSI. This duality relation will be useful in obtaining a robust design of the BC transceiver that minimizes the average total MSE in terms of the corresponding transceiver of the dual MAC that minimizes the same objective.

Theorem 1: Assuming no precoding loss in the BC and no error propagation in the dual MAC, the sets of individual average MSEs for the BC, $\{\overline{\text{MSE}}_k\}$, and for the dual MAC, $\{\overline{\text{MSE}}_k^{\text{MAC}}\}$, are equal under the same total transmitted power constraint when one uses the following transceiver designs:

$$\mathbf{p}_k = \omega_k \mathbf{g}_k^{\text{MAC}H}, \quad g_k = \omega_k^{-1} p_k^{\text{MAC}H}, \quad B_{kj} = \frac{\omega_j}{\omega_k} B_{jk}^{\text{MAC}}, \quad (13)$$

where the vector of positive constants $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_K)$ is given by:

$$\boldsymbol{\omega}^2 = \mathbf{M}^{-1} \left[\; |p_1^{\text{MAC}}|^2, \ldots, |p_K^{\text{MAC}}|^2 \; \right]^T, \quad (14)$$

and the matrix $\mathbf{M}$ is given by:

$$M_{ki} = \begin{cases} -\frac{|p_k^{\text{MAC}}|^2}{\sigma_n^2}\mathbf{g}_i^{\text{MAC}}(\hat{\mathbf{h}}_k^H\hat{\mathbf{h}}_k + \sigma_{e_k}^2\mathbf{I})\mathbf{g}_i^{\text{MAC}^H} & k < i \\ -\frac{|p_k^{\text{MAC}}|^2}{\sigma_n^2}\mathbf{g}_i^{\text{MAC}}(\hat{\mathbf{h}}_k^H\hat{\mathbf{h}}_k + \sigma_{e_k}^2\mathbf{I})\mathbf{g}_i^{\text{MAC}^H} \\ -\frac{|B_{ik}^{\text{MAC}}|^2}{\sigma_n^2} + \frac{1}{\sigma_n^2}\mathbf{g}_i^{\text{MAC}}\hat{\mathbf{h}}_k^H p_k^{\text{MAC}} B_{ik}^{\text{MAC}^H} \\ +\frac{1}{\sigma_n^2} B_{ik}^{\text{MAC}} p_k^{\text{MAC}^H}\hat{\mathbf{h}}_k\mathbf{g}_i^{\text{MAC}^H} & k > i \\ \mathbf{g}_k^{\text{MAC}}\mathbf{g}_k^{\text{MAC}^H} - \sum_{j\neq k} M_{jk} & k = i. \end{cases} \quad (15)$$

A sketch of the proof of this result is provided in the appendix. It is a generalization of the result in [6], [12] to scenarios with channel uncertainty. Using Theorem 1, an optimal design of the BC transceiver that jointly minimize $\overline{\text{MSE}}$ under a total power constraint can be obtained by first obtaining the optimal MAC transceiver that jointly minimize the average of the total MSE, and then applying the transformation (14) to obtain the optimal BC transceiver.

IV. Statistically Robust Transceiver Design for the Dual MAC

In this section, we will obtain a statistically robust design of the dual MAC transceiver that jointly minimizes the average, over channel estimation errors, of the total MSE,

$$\overline{\text{MSE}}^{\text{MAC}} = \textstyle\sum_{k=1}^{K} \overline{\text{MSE}}_k^{\text{MAC}}. \quad (16)$$

First, we will obtain an analytic expression for the optimal receiver, $\mathbf{B}^{\text{MAC}}$ and $\mathbf{g}_k^{\text{MAC}}$, for a given set of transmitters p_k^{MAC}. Using these expressions we will then obtain an optimization formulation for the optimal p_k^{MAC} under a total power constraint.

From equation (12), we observe that each $\overline{\text{MSE}}_k^{\text{MAC}}$ is convex function of the k^{th} row of $\mathbf{B}_k^{\text{MAC}}$ and is independent of the other rows. Hence, each term in the above summation can be minimized independently. Setting the derivative of $\overline{\text{MSE}}_k^{\text{MAC}}$ with respect to k^{th} row of $\mathbf{B}_k^{\text{MAC}}$ to zero, we obtain the following expression:

$$B_{kj}^{\text{MAC}} = \mathbf{g}_k^{\text{MAC}}\hat{\mathbf{h}}_k^H p_j^{\text{MAC}}. \quad (17)$$

Substituting the expression for the optimal $\mathbf{B}^{\text{MAC}}$ in (12), each resulting $\overline{\text{MSE}}_k^{\text{MAC}}$ is a convex function of $\mathbf{g}_k^{\text{MAC}}$ and is independent of $\mathbf{g}_j^{\text{MAC}}$, for $j \neq k$. Hence, it is optimized by setting

$$\mathbf{g}_k^{\text{MAC}} = p_k^{\text{MAC}^H}\hat{\mathbf{h}}_k\mathbf{S}_k, \quad (18)$$

where

$$\mathbf{S}_k = \Big(\sum_{i=1}^{k} |p_i^{\text{MAC}}|^2(\hat{\mathbf{h}}_i^H\hat{\mathbf{h}}_i + \sigma_{e_i}^2\mathbf{I}) + \sigma_n^2\mathbf{I}\Big)^{-1} \quad (19)$$

Using this optimal value, the average total MSE reduces to:

$$\overline{\text{MSE}}^{\text{MAC}} = K - \sum_{k=1}^{K} |p_i^{\text{MAC}}|^2\hat{\mathbf{h}}_k\mathbf{S}_k\hat{\mathbf{h}}_k^H. \quad (20)$$

Similar to scenarios in which channel state information is available [12], the expression in (20) is differentiable function of $|p_i^{\text{MAC}}|^2$, and hence a (locally) optimal solution to the minimization of (20) under a total power constraint can be found by applying a standard iterative algorithm.

V. Simulation Studies

In order to compare the performance of the proposed robust design with the existing approaches, we have simulated these methods for the cases of QPSK transmission over independent Rayleigh fading channels. We will plot the average bit error rate (BER) over all users against the signal-to-noise-ratio, which is defined as $\text{SNR} = P_{\text{total}}/(K\sigma_n^2)$. In our simulations, the coefficients of the channel matrix $\mathbf{H}$ are modeled as being independent circularly symmetric complex Gaussian random variables with zero mean. All TH precoding strategies assume a given ordering of the users. Since finding an optimal ordering will involve an exhaustive search over $K!$ possible arrangements, a suboptimal ordering is usually employed. We will choose the suboptimal ordering proposed for MMSE Tomlinson-Harashima transceiver design in [5], using the transmitter's channel estimate $\hat{\mathbf{H}}$. This ordering will be used for all methods, including the proposed robust transceiver. To model the error $\mathbf{e}_k$ between the actual channel $\mathbf{h}_k$ and the estimated channel at the transmitter $\hat{\mathbf{h}}_k$, $\mathbf{e}_k$ is generated from a zero-mean Gaussian distribution with $\text{E}\{\mathbf{e}_k^H\mathbf{e}_k\} = \sigma_{e_k}^2\mathbf{I}$. In our simulation, we will use the same $\sigma_{e_k}^2$ for all users. This model is appropriate for a scenario in which the uplink power is controlled so that the received SNRs on the uplink are equal and independent from the downlink SNR. For convenience, we define $\epsilon^2 = \text{E}\{\mathbf{e}_k\mathbf{e}_k^H\} = N_t\sigma_{e_k}^2$.

In Fig. 4 we compare the performance of the statistically robust Tomlinson-Harashima transceiver proposed in Section III with that of the zero-forcing Tomlinson-Harashima transceiver design (ZF-THP) in [3], [4], and the MMSE Tomlinson-Harashima transceiver design (MMSE-THP) in [5] for a system with 4 transmit antennas, 4 users, and QPSK signalling. In Fig. 4, the performance of each method is plotted for values of $\epsilon^2 = 0.05, 0.1$. It can be seen that the performance of Tomlinson-Harashima precoding in the broadcast channel is rather sensitive to the mismatch between the actual CSI and the transmitter's estimate of CSI. It can be also seen that while the effect of noise is dominant at low SNR, the channel uncertainty dominates at high SNR, where the proposed robust transceiver design performs significantly better than the other two approaches. Fig. 4 also shows that in the presence of channel uncertainty, both the ZF-THP and MMSE-THP designs have the same performance limit at high SNR. This is due to the fact that the MMSE method involves the addition of a regularization term whose value is inversely proportional to $P_{\text{total}}/(K\sigma_n^2)$; see [5].

For Fig. 5 we consider a system with $N_t = 4$ antennas and $K = 4$ users. In addition to the previous two designs, ZF-THP [3], [4] and MMSE-THP [5], that assume precise CSI, we will also compare the performance of the statistically robust transceiver proposed in Section III with that of the robust zero-forcing Tomlinson-Harashima (Robust ZF-THP)

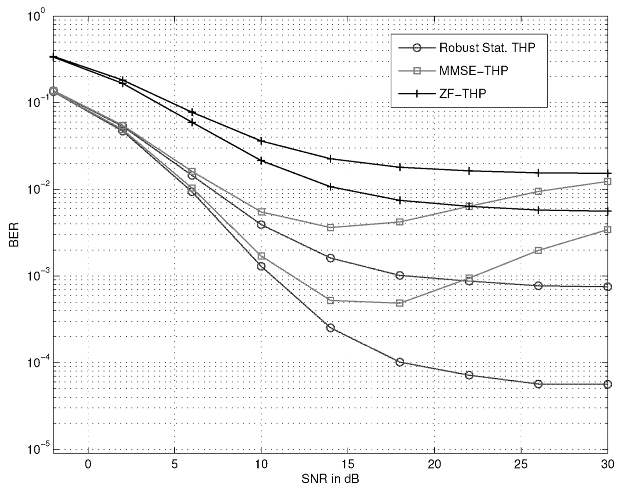

Fig. 4. Comparison between the performance of the proposed statistically robust Tomlinson-Harashima transceiver, zero-forcing Tomlinson-Harashima transceiver design (ZF-THP) in [3], [4], and the MMSE Tomlinson-Harashima transceiver design (MMSE-THP) in [5] for values of channel uncertainty $\epsilon^2 = 0.05, 0.1$ for a system with $N_t = 4$ and $K = 4$ using QPSK signalling. The upper performance curve of each method corresponds to channel uncertainty $\epsilon^2 = 0.1$

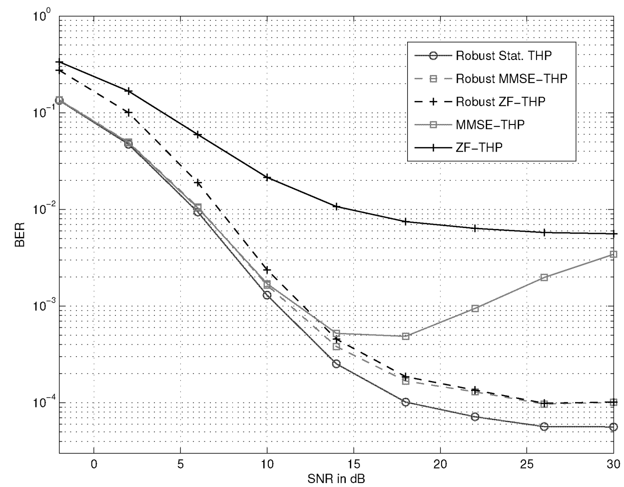

Fig. 5. Comparison between the performance of the proposed statistically robust TH transceiver, zero-forcing TH transceiver design (ZF-THP) in [3], [4], and the MMSE TH transceiver design (MMSE-THP) in [5], robust zero-forcing TH (Robust ZF-THP) approach introduced in [10], and the robust MMSE Tomlinson-Harashima (Robust MMSE-THP) approach introduced in [9], for values of channel uncertainty $\epsilon^2 = 0.05$ for a system with $N_t = 4$ and $K = 4$ using QPSK signalling.

approach introduced in [10], and the robust MMSE Tomlinson-Harashima (Robust MMSE-THP) approach introduced in [9]. These two approaches restrict the all gains g_k to be equal. It can be seen from Fig. 5 that improvement in the performance can be achieved by the proposed robust design as it offers more degrees of freedom in the choice of the gains g_k.

VI. Conclusion

We have presented a robust design for Tomlinson-Harashima transceivers for broadcast channels that jointly minimizes the average, over channel estimation errors, of the sum of the MSEs of each user. By generalizing the MSE duality between the broadcast channel with Tomlinson-Harashima precoding and the multiple access channel with decision feedback equalization to schemes with channel estimation errors, we have obtained the desired robust broadcast transceivers in terms of the robust transceivers that optimize the same performance metric for the dual multiple access channel. The proposed approach can significantly reduce the sensitivity of the downlink to uncertainty in the CSI, and can provide improved performance over that of existing robust designs.

Appendix

We start by considering linearly related transceivers for BC and dual MAC:

$$\mathbf{p}_k = \omega_k \mathbf{g}_k^{\mathrm{MAC}^H}, \quad g_k = \omega_k^{-1} p_k^{\mathrm{MAC}^H}, \quad B_{kj} = \frac{\omega_j}{\omega_k} B_{jk}^{\mathrm{MAC}}, \tag{21}$$

and we find the necessary conditions for ω_k such that set of MSEs in BC and dual MSE are equal. By setting $\overline{\mathrm{MSE}}_k = \overline{\mathrm{MSE}}_k^{\mathrm{MAC}}$ and substituting the values $\mathbf{p}_k$, g_k, and B_{kj} from (21), we obtain a set of K linear equations:

$$\mathbf{M}\,\boldsymbol{\omega}^2 = \left[\ |p_1^{\mathrm{MAC}}|^2, \ldots, |p_K^{\mathrm{MAC}}|^2\ \right]^T, \tag{22}$$

where $\mathbf{M}$ was defined in (15). We observe that $\mathbf{M}$ has strictly dominant diagonal elements and negative off-diagonal elements, hence it is non-singular and the elements of $\mathbf{M}^{-1}$ are non-negative. Adding all equations in the linear system in (22) results in $\sum_{k=1}^{K} \omega_k^2 \mathbf{g}_k^{\mathrm{MAC}} \mathbf{g}_k^{\mathrm{MAC}^H} = \sum_{k=1}^{K} |p_k^{\mathrm{MAC}}|^2$, i.e., total transmitted power in BC and dual MAC are the same.

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