# ELEC ENG 3BB3:

# Cellular Bioelectricity

## Notes for Lecture 18

Thursday, February 13, 2014

# 7. EXTRACELLULAR FIELDS

*We will look at:*

- Generation and measurement of extracellular fields
- Single-fiber distributed-source models
- Lumped-source models

*Generation and measurement of extracellular fields:*

The activity of excitable cells leads to flow of currents into the extracellular space.

These currents can be measured by extracellular electrodes or even electrodes on the body surface. Examples include:

- **ECG** (electrocardiogram)
- **EMG** (electromyogram)
- **EEG** (electroencephalogram)

## *Generation and measurement of extracellular fields (cont.):*

Measurement of extracellular fields depends on:

- the spatial and temporal characteristics of the locally-generated extracellular fields, and
- the conductive characteristics of the tissue between the excited cell(s) and the electrodes, referred to as a *volume conductor*.

# *Observed extracellular potentials:*

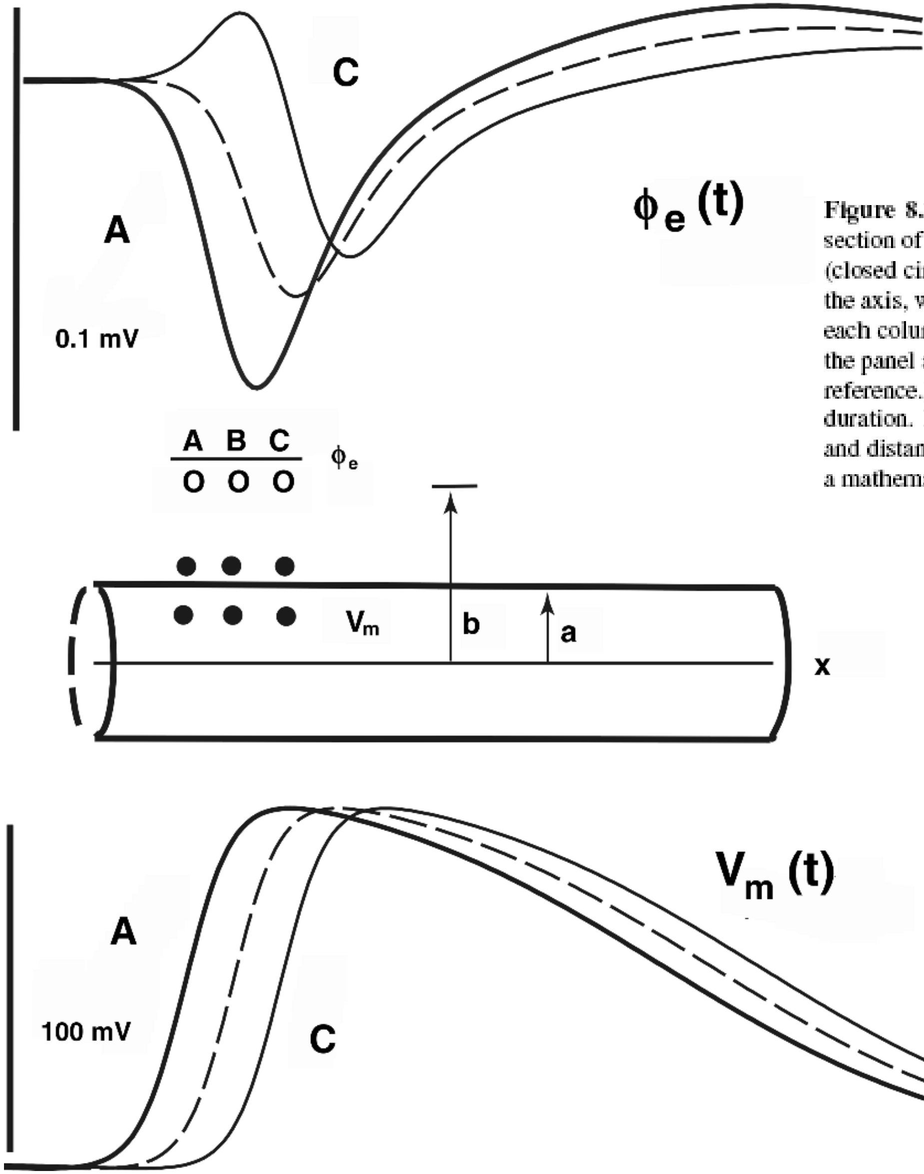


**Figure 8.1.** Intracellular and Extracellular Temporal Waveforms. A sketch of a short section of a long cylindrical fiber is shown in the middle panel. Transmembrane electrodes (closed circles) and extracellular electrodes (open circles) are drawn at three points along the axis, with their respective columns labeled A–C. Transmembrane potentials $V_m(t)$ for each column are shown in the panel below. Extracellular waveforms $\Phi_e(t)$ are shown in the panel at the top. (These are unipolar waveforms, i.e., potential with respect to a distant reference.) The vertical bars (top, bottom) give a voltage calibration. Each trace has a 10-ms duration. Locations A, B, and C are at $x = 0, 1, 2$ mm, respectively. Radius $a = 0.1$ mm and distance $b = 1.0$ mm. These waveforms are based on a computer simulation that uses a mathematically defined template function for $V_m(t)$. $R_i = 100$ and $R_e = 30$ $\Omega$cm.

## *Observed extracellular potentials (cont.):*

The temporal waveforms of extracellular potentials cannot be determined directly from the temporal waveforms for transmembrane potentials. Instead the following steps must be taken:

1. Convert transmembrane potential temporal waveform into spatial waveform.
2. Find transmembrane current and/or axial extracellular current.
3. Find extracellular spatial waveform produced by transmembrane current (or alternatively axial extracellular current).
4. Convert extracellular spatial waveform into temporal waveform.

# *Observed extracellular potentials (cont.):*

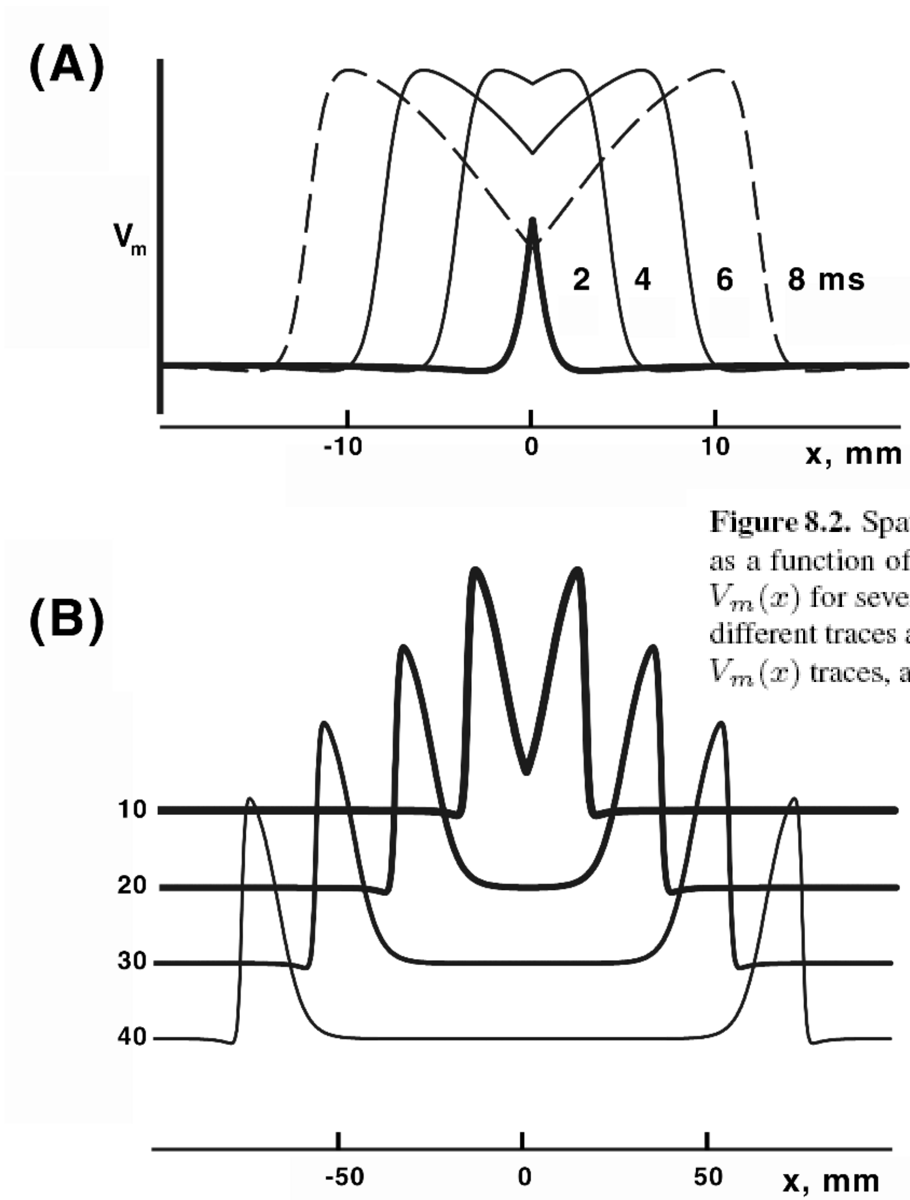


**Figure 8.2.** Spatial Transmembrane Potentials. Panel A shows the transmembrane potential as a function of distance along the fiber, for early times after excitation. Panel B shows $V_m(x)$ for several later times, given by the number beside each trace. For illustration, the different traces are displaced vertically. Note the multiplicity of wave shapes present in the $V_m(x)$ traces, as compared to $V_m(t)$, which is shown in Figure 8.1.

*Source density* $i_m(x)$*:*

The transmembrane current $i_m(x)$ acts as a current source in the extracellular electrolyte.

From the cable equations:

$$i_m = \frac{1}{r_i} \frac{\partial^2 \Phi_i}{\partial x^2}. \qquad (8.3)$$

For an axon with radius a and axoplasmic resistivity $R_i$, Eqn. (8.3) becomes:

$$i_m = \frac{\pi a^2}{R_i} \frac{\partial^2 \Phi_i}{\partial x^2} = \pi a^2 \sigma_i \frac{\partial^2 \Phi_i}{\partial x^2}. \qquad (8.5)$$

where ¾$_i$= 1/ $R_i$ is the conductance per unit length.

# *Source density* $i_m(x)$*:*

However, one normally knows $V_m(x)$ but not ©$_i(x)$.

In the case where $r_e$ ¿ $r_i$, $V_m(x)$ ¼ ©$_i(x)$ and thus:

$$i_m = \frac{1}{r_i}\frac{\partial^2 \Phi_i}{\partial x^2} \approx \frac{1}{r_i}\frac{\partial^2 V_m}{\partial x^2}. \qquad (8.6)$$

Under this approximation:

$$I_i = -\frac{\pi a^2}{R_i}\frac{\partial \Phi_i}{\partial x} \approx -\pi a^2 \sigma_i \frac{\partial V_m}{\partial x} \qquad (8.7)$$

$$i_m = \frac{\pi a^2}{R_i}\frac{\partial^2 \Phi_i}{\partial x^2} \approx \pi a^2 \sigma_i \frac{\partial^2 V_m}{\partial x^2}. \qquad (8.8)$$

# *Source density* $i_m(x)$ *(cont.):*

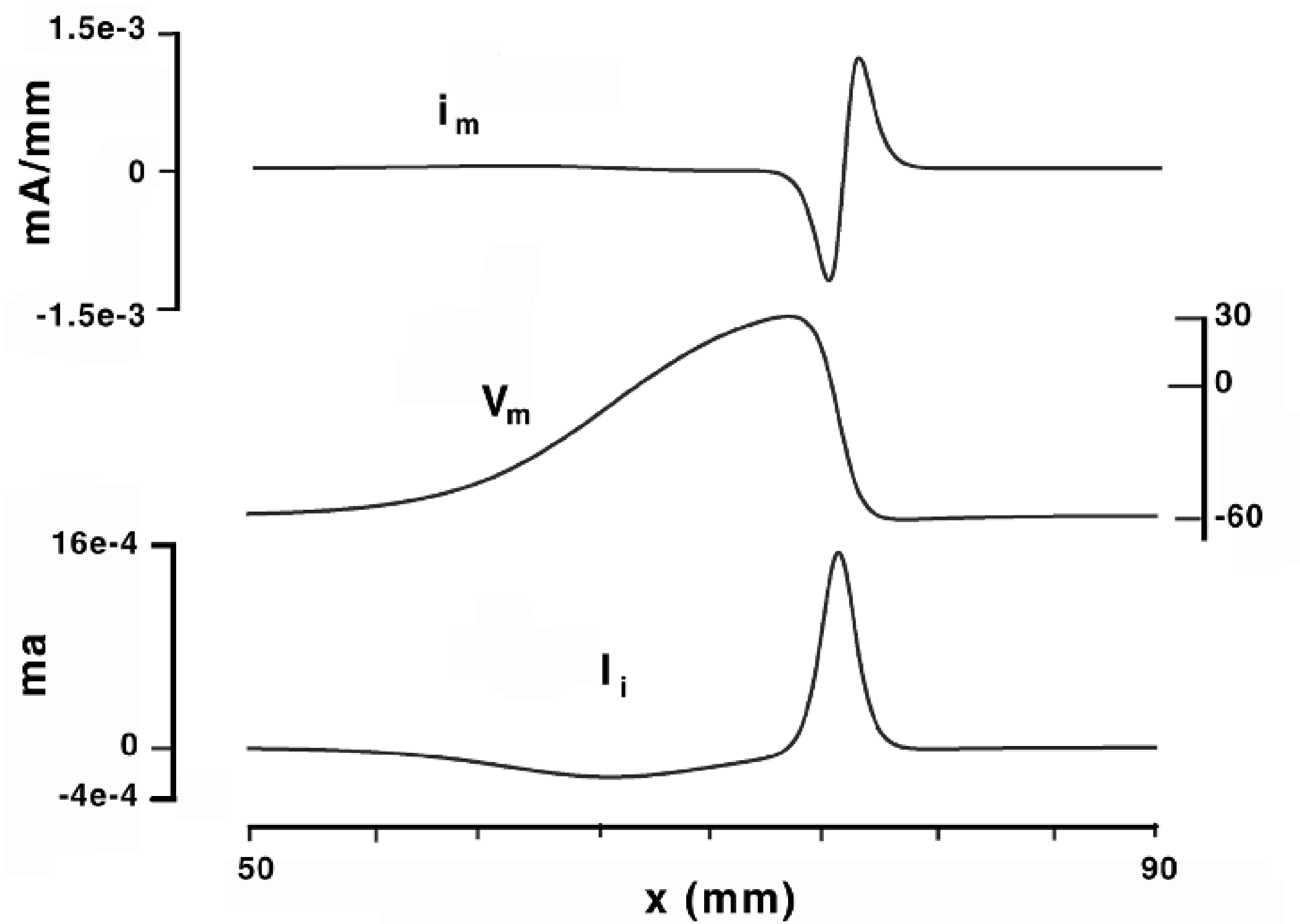


**Figure 8.3.** Transmembrane Potential $V_m$, Intracellular Current $I_i$, and Transmembrane Current $i_m$. The Figure shows a transmembrane potential (middle). Other traces give the transmembrane (top) and intracellular axial (bottom) currents, as determined from the transmembrane potential using Eqs. (8.7) and (8.8).

# *Observed extracellular potentials (cont.):*

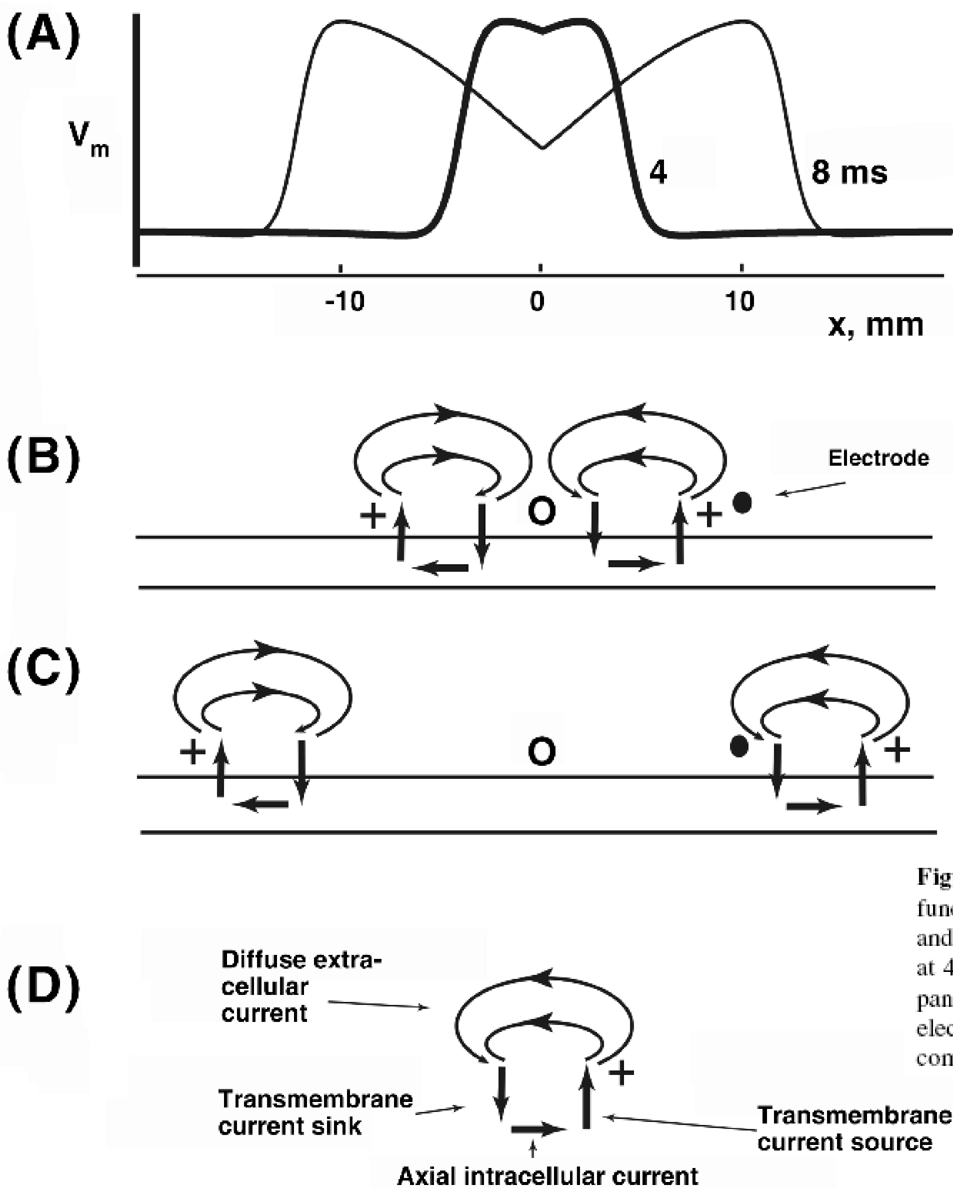


**Figure 8.4.** Action Current Cartoon. Panel A shows the transmembrane potential as a function of distance along the fiber for 4 and 8 milliseconds. Excitation began at $x = 0$ and spread from there in both directions. Panel B shows a cartoon of the current flow at 4 msec, and Panel C for 8 msec. Labels identifying elements of the current loops of panels B and C are given in panel D. The open and closed dots in B and C are hypothetical electrode positions. In B–D, for purposes of illustration the source–sink distance is widened, compared to that implied by the upstrokes of panel A.

## *Single-fiber source model:*

Consider a cylindrical fiber lying in an extensive conducting medium and carrying an action potential propagating in the x direction:

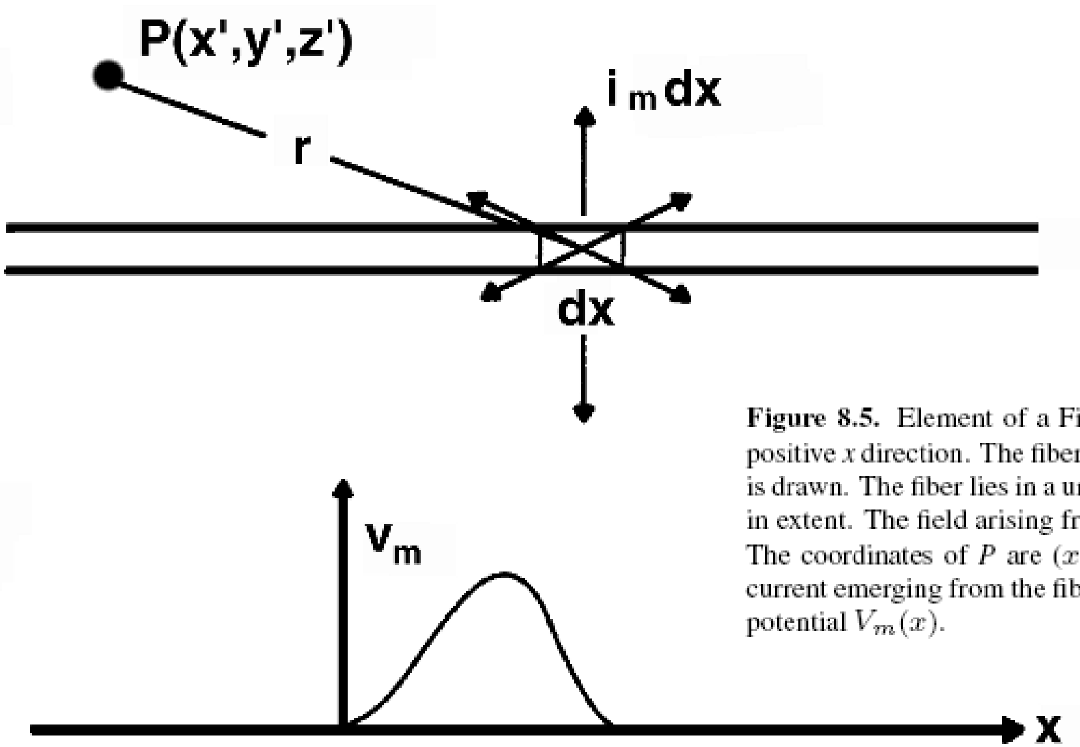


**Figure 8.5.** Element of a Fiber. (a) An action potential is propagating on a fiber in the positive $x$ direction. The fiber is divided into mathematical segments, and one such segment is drawn. The fiber lies in a uniform extracellular medium of conductivity $\sigma_e$ that is infinite in extent. The field arising from the action currents at an arbitrary field point $P$ is desired. The coordinates of $P$ are $(x', y', z')$. The source element is at $(x, y, z)$. Shown is the current emerging from the fiber element $dx$ (magnitude $i_m dx$). (b) The monophasic action potential $V_m(x)$.

*Single-fiber source model (cont.):*

For a fiber element of length dx, the current $I_0$ passed into the extracellular space is:

$$I_0 = i_m \mathrm{d}x,$$

where $i_m$ is the transmembrane current per unit length.

Although the element dx of fiber membrane is actually ring shaped, at distances that are large relative to the fiber's diameter, $I_0$ can be approximated as a point source.

*Single-fiber source model (cont.):*

For a point source, the extracellular field is:

$$\Phi_e = \frac{1}{4\pi\sigma_e}\frac{I_0}{r}, \qquad (8.9)$$

where $I_0 = i_m dx$, $\sigma_e$ is the conductivity of the extracellular medium, and $r$ is the distance from the point source to an arbitrary field point.

Note that once again the effect of the fiber on the field is typically ignored.

*Single-fiber source model (cont.):*

The extracellular field for the fiber element dx is:

$$\mathrm{d}\Phi_e = \frac{1}{4\pi\sigma_e}\frac{i_m}{r}\mathrm{d}x\,, \tag{8.10}$$

and the total extracellular potential at point P is:

$$\Phi_e(P) = \frac{1}{4\pi\sigma_e}\int_L \frac{i_m}{r}\mathrm{d}x\,, \tag{8.11}$$

where L is the length of the fiber over which $i_m \neq 0$.

*Single-fiber source model (cont.):*

Substituting (8.8) into (8.11) gives:

$$\Phi_e = \frac{a^2\sigma_i}{4\sigma_e}\int \frac{\partial^2 V_m/\partial x^2}{r}\mathrm{d}x\,. \qquad (8.12)$$

The potential difference between two extracellular electrodes at positions a and b is then:

$$V_{ab} = \Phi_e(a) - \Phi_e(b)\,. \qquad (8.13)$$

*Single-fiber source model (cont.):*

If the element dx is located at point (x, y, z) and the field potential is measured at ($x^0$, $y^0$, $z^0$), then:

$$r = \sqrt{\left(x - x'\right)^2 + \left(y - y'\right)^2 + \left(z - z'\right)^2}. \quad (8.14)$$

Normally the coordinate origin is placed on the fiber so that y = z = 0. Since the source element is approximated by a point source, it too lies on the fiber's axis.

*Single-fiber source model (cont.):*

With this expression for r, the field potential is given by:

$$\Phi_e\left(x',y',z'\right)=\frac{1}{4\pi\sigma_e}\int_L \frac{i_m(x)\,\mathrm{d}x}{\sqrt{(x-x')^2+(y')^2+(z')^2}}\,. \quad (8.15)$$

Eqn. (8.15) can be written as the convolution:

$$\Phi_e(x',y',z')=\frac{1}{4\pi\sigma_e}\int_L H(x-x')\,i_m(x)\,\mathrm{d}x\,, \qquad (8.16)$$

where $$H(x-x')=\frac{1}{\sqrt{(x-x')^2+(y')^2+(z')^2}}\,. \qquad (8.17)$$

# *Single-fiber source model (cont.):*

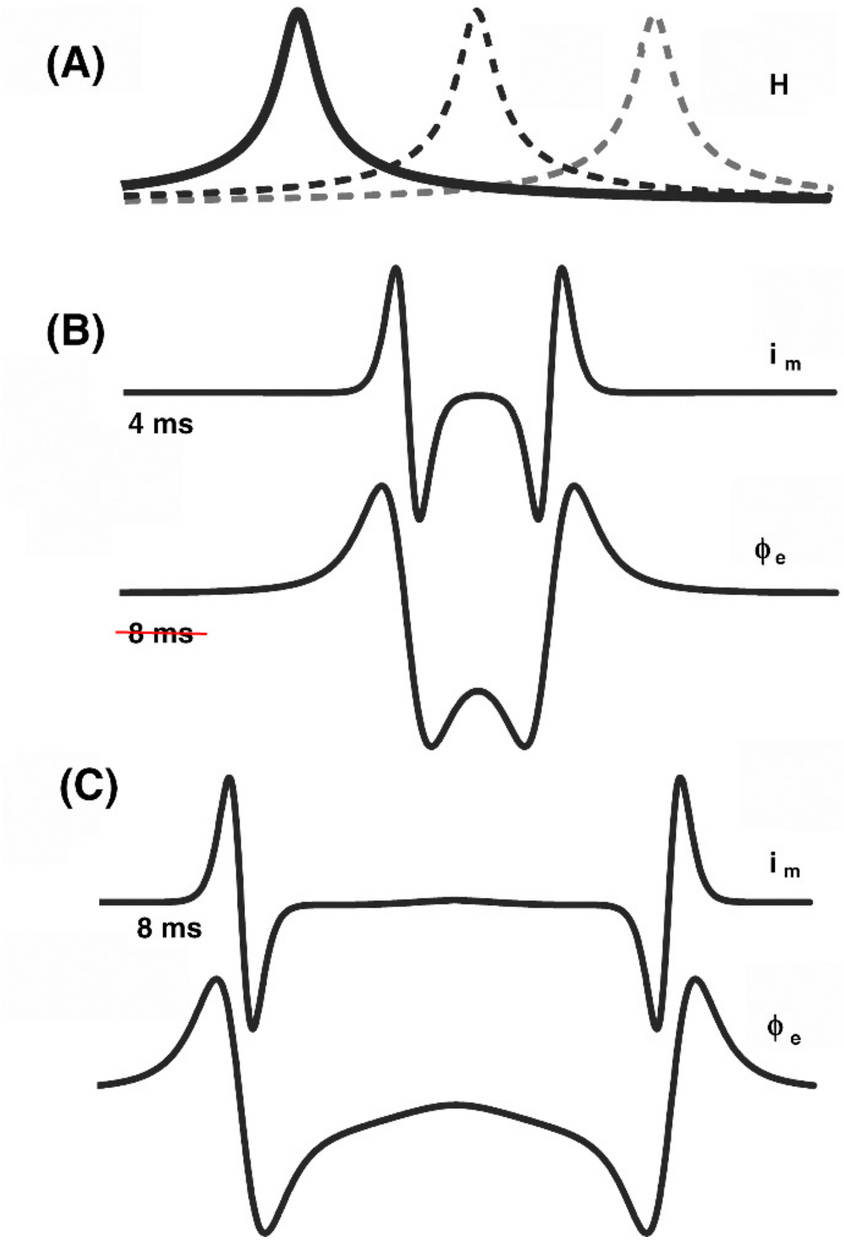


**Figure 8.6.** Transfer Function $H$, Membrane Current $i_m(x)$, and Extracellular Potentials $\Phi_e(x)$. Panel **A**: Plots of the transfer function $H$. Transfer function $H(x - x')$ is given for three values of $x'$. The solid line is for $x' = -10$ mm, while the two dashed lines are for $x' = 0$ (centered) and $x' = 10$ (on right). Panel B: Membrane current $i_m(x)$ at 4 milliseconds and extracellular potential $\Phi_e(x)$ along a line a distance of 1 mm from the fiber axis. Panel C: Membrane current $i_m(x)$ at 8 milliseconds and extracellular potential $\Phi_e(x)$ along a line a distance of 1 mm from the fiber axis. The extracellular potential distribution $\Phi_e(x)$ for 4 ms (thick line) and 8 ms (thin line). The 4-ms potential function comes from the convolution of $H$ with $i_m$ for 4 ms, and similarly for 8 ms.

# *Single-fiber source model (cont.):*

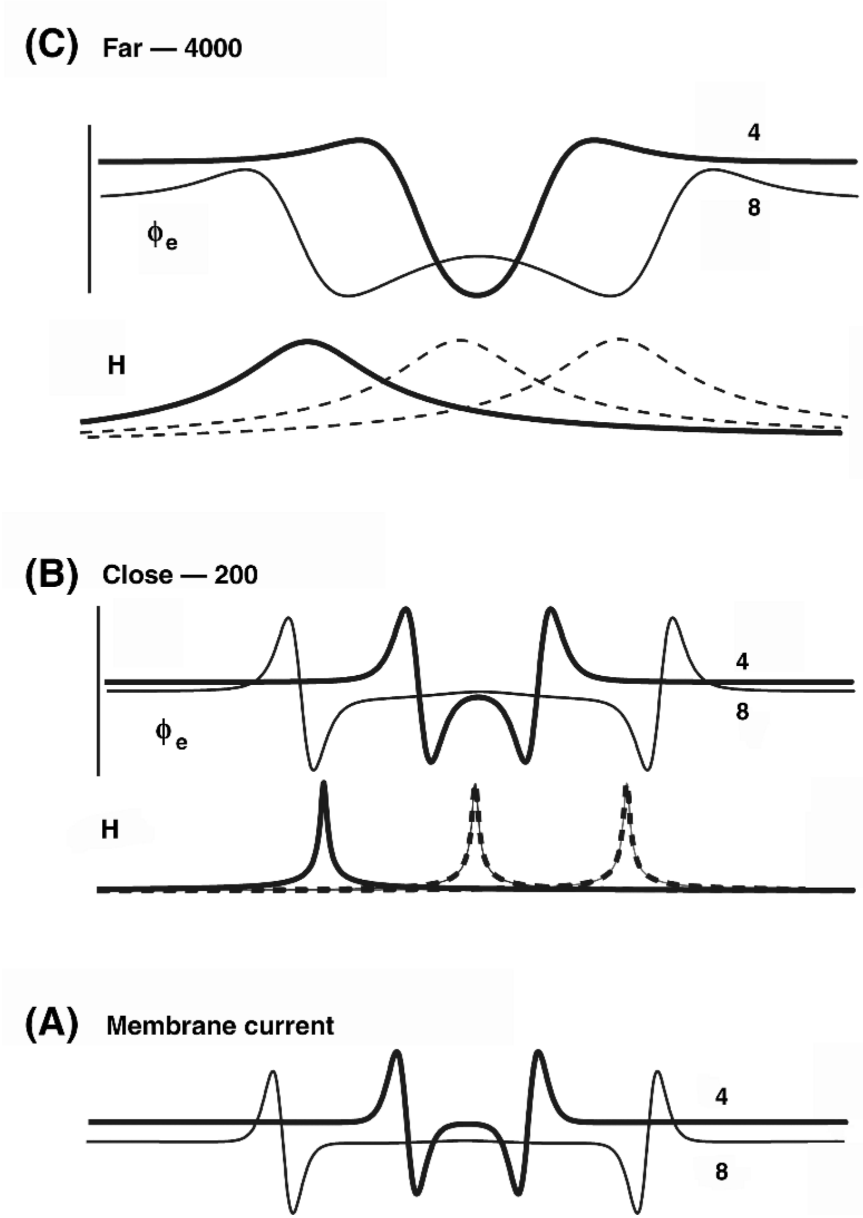


**Figure 8.7.** Changes with Distance of Extracellular Waveforms. Panel A shows the transmembrane current waveform. Panel B shows data for a distance of 200 micrometers from the fiber axis. Data plotted is $H$ for positions –10, 0, and 10 mm, and $\phi_e(x)$. Panel C shows the same data at a distance of 4000 micrometers (4 mm). In all panels, the small numbers 4 and 8 are to identify curves for 4 milliseconds and 8 milliseconds after excitation begins at the center. (For illustration, the 8-ms plot is slightly displaced downward.)

*Single-fiber source model (cont.):*

Eqn. (8.12) can be considered the integral:

$$\Phi_e = \frac{1}{4\pi\sigma_e}\int \frac{I_\ell}{r}\,\mathrm{d}x \tag{8.18}$$

of the **monopole source density** $I_\ell$ (with units of current per unit length):

$$I_\ell = \pi a^2 \sigma_i \frac{\partial^2 V_m}{\partial x^2}\,. \tag{8.19}$$