ELEC ENG 3BB3:

Cellular Bioelectricity

Notes for Lecture #3

Friday, January 10, 2014

2. BIOELECTRIC POTENTIALS AND CURRENTS

We will look at:

- Ionic composition of excitable cells
- Nernst-Planck equation
- Membrane structure
- Nernst potential
- Parallel-conductance model

Ionic composition of excitable cells:

- *Sodium* (Na^{+}) and *potassium* (K^{+}) are the most important ions for the electrical activity of the majority of excitable cells.
- *Calcium* (Ca^{2+}) and *chloride* (Cl^{i}) play a significant role in some circumstances.
- Many of the fundamental properties of ionic movement are the same no matter which ion is being considered.

  Consequently, we will often derive mathematical expressions for “the p^{th} ion”.

Ionic composition (cont.):

- Example intra- and extra-cellular ionic concentrations are given below.

Table 3.1. Ionic Concentrations[a]

	Concentration (nM/l)			
	Muscle (frog)		Nerve (squid axon)	
	Intracellular	Extracellular	Intracellular	Extracellular
K^+	124	2.2	397	30
Na^+	4	109	50	437
Cl^-	1.5	77	40	556
A^-	126.5			

[a]The A^- ion is large and impermeable.

- Note that the particular ratios of intra- to extra-cellular ionic concentrations are similar across different types of excitable cells.

Nernst-Planck Equation:

- The *Nernst-Planck* equation describes the effects of spatial differences in *concentration* and/or *electric potential* on ion flow.
- The individual effect of a concentration gradient is described by *Fick’s law of diffusion*.
- The individual effect of an electric potential gradient is described by *Ohm’s law of drift*.

Fick's law of diffusion:

$$\bar{j}_d = -D\nabla C, \qquad (3.1)$$

where:

$\bar{j}_d$ is the flux due to diffusion
D is the diffusion coefficient
C is the concentration
as a function of position
∇ is the *Del operator*:

$$\nabla \equiv \bar{a}_x \frac{\partial}{\partial x} + \bar{a}_y \frac{\partial}{\partial y} + \bar{a}_z \frac{\partial}{\partial z} \qquad (1.17)$$

The diffusion coefficient:

The diffusion coefficient is also known as *Fick's coefficient*, or alternatively as the diffusion or Fick's *constant*. However, it is not strictly a constant because it varies with temperature and somewhat with C.

D is normally determined empirically.

Table 3.2. Numerical Values for Several Diffusion Coefficients

Ion	*D*	Units	Conditions	Reference
Na^+	1.33×10^{-5}	cm^2/sec	at 25 °C	3
K^+	1.96×10^{-5}	cm^2/sec	at 25 °C	3
Cl^-	2.03×10^{-5}	cm^2/sec	at 25 °C	3
KCl	2.03×10^{-5}	cm^2/sec	0.002 mole/l, 25 °C	4
NaCl	1.58×10^{-5}	cm^2/sec	0.002 mole/l, 25 °C	4

Ohm's law of drift:

$$\bar{j}_e = -u_p \frac{Z_p}{|Z_p|} C_p \nabla \Phi, \qquad (3.2)$$

where:

$\bar{j}_e$	is the ionic flux due to an electric field
$-\nabla\Phi$	is the electric field
u_p	is the mobility of the pth ion
$Z_p/\|Z_p\|$	is the sign of the valence of the pth ion
C_p	is the concentration of the pth ion

Relating diffusion and drift:

Diffusion and drift are impeded by the same molecular processes, i.e., collisions with solvent molecules, and consequently a physical connection exists between the parameters u_p and D.

The mathematical expression for this relationship is known as *Einstein's equation*.

Einstein's equation:

$$D_p = \frac{u_p RT}{|Z_p|\, F}, \qquad (3.3)$$

where:

R is the gas constant
T is the absolute temperature
F is Faraday's constant

Table 3.3. Faraday's Constant *F* and the Gas Constant R

Constant	Value
F	96,487 Coulombs/mole
R	8.314 Joules/degree K-mole
RT/F	$8.314 \times .300/96487 = 25.8$ mV at $27\,^{\circ}$C

Total ion flow:

The total flux when both diffusional and electric field forces are present is:

$$\bar{j}_p = \bar{j}_d + \bar{j}_e \tag{3.4}$$

$$= -D_p \left(\nabla C_p + \frac{Z_p C_p F}{RT} \nabla \Phi \right), \tag{3.5}$$

which is known as the **Nernst-Planck equation**.

Electric current density:

The electric current density can be found by multiplying the ionic flux by FZ_p, giving:

$$\bar{J}_p = -D_p F Z_p \left(\nabla C_p + \frac{Z_p C_p F}{RT} \nabla \Phi \right), \quad (3.6)$$

Alternatively, substituting for D_p using Einstein's equation, one has:

$$\bar{J}_p = -u_p \left(RT \frac{Z_p}{|Z_p|} \nabla C_p + |Z_p| C_p F \nabla \Phi \right). \; (3.7)$$

Resistance and conductance:

The linear relationship between the current density and the strength of the electric field applied to an electrolyte suggests that an expression can be derived for the *resistance* (or its reciprocal, *conductance*) of the intra- or extra-cellular space.

Considering a standard form for Ohm's law:

$$\bar{J} = \sigma \bar{E},$$

how does the electrical conductivity σ relate to parameters such as mobility and concentration?

Conductivity:

The electric current density arising solely under the influence of an electric field is:

$$\bar{J}_p^e = -u_p\,|Z_p|C_pF\nabla\Phi. \tag{3.8}$$

For example, a KCl electrolyte in which there is complete dissociation has the total current density:

$$\bar{J}_{\mathrm{KCl}}^e = FC_{\mathrm{KCl}}\left[u_{\mathrm{K}} + u_{\mathrm{Cl}}\right]\bar{E}, \tag{3.9}$$

giving the electrolyte conductivity:

$$\sigma = FC_{\mathrm{KCl}}\left[u_{\mathrm{K}} + u_{\mathrm{Cl}}\right]. \tag{3.10}$$